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# APP04

# ESTIMATION OF ELECTRONIC CIRCUIT TEMPERATURE SOLVING AN INVERSE PROBLEM

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## ABSTRACT

This paper presents an approach to estimation of electronic circuit temperature solving an inverse problem. The main goal of the simulations presented in this paper is to investigate the possibility of improving the quality of circuit temperature estimates applying the function specification algorithm. Given information on circuit layout as well as typical working conditions and material thermal properties, a mathematical model of the circuit is created. Based on the model, sensitivity coefficients relating the amount of dissipated power to the temperature rise at chosen temperature sensor locations are determined. For arbitrary values of power dissipated in the circuit, the coefficients are employed to compute temperature rise values at chosen temperature sensor locations. Next, these values are contaminated with noise and used as input data for the function specification algorithm, which produces estimates of the power dissipated in heat sources. In the simulations, different configurations of temperature sensors and algorithm parameter values are considered.

## NOMENCLATURE

$A$ – vector of identity matrices
$J$ - number of sensors
$T$ - sensor temperatures
$\hat{T}$ - structure temperature
$X$ - coefficients relating temperature to heat fluxes
$g$ - interlayer thermal conductance
$h$ - heat exchange coefficient
$q$ - heat flux
$\hat{q}$ - unknown heat flux estimate
$p$ – number of heat sources
$r$ - number of "future values"
$x, y, z$ – co-ordinates
$\phi$ - temperature rise for unitary heat flux change
$\lambda$ - thermal conductivity
$i, j$ - series indexes
$k$ - sampling instant

## INTRODUCTION

The growth in the density of power dissipated in modern electronic circuits has induced still increasing interest in circuit thermal analysis. In many applications, continuous monitoring of circuit temperature is required. The best solution would be to place temperature sensors, e.g. p-n junctions directly where heat is generated. This solution, however, usually cannot be realised in practice. Therefore, it is necessary to estimate the temperature of heat sources from remote temperature sensor measurements solving an inverse problem.

As the thermal response is damped, the problem is extremely sensitive to measurement errors and special techniques must be applied to obtain robust estimates of circuit temperature. In most cases encountered in electronics, heat is generated only on structure surface, thus the problem of determining heat generation density can be reduced to the problem of unknown surface heat flux estimation.

Then, having the number of temperature sensors greater than the number of heat sources, the corrupted with noise sensor temperature readings are supplied to the function specification algorithm, which is adapted for multiple surface

 

heat flux estimation. Owing to the space and time averaging, obtained estimates are more accurate than in case of single-step estimation. Once the power dissipated in heat sources is known, it is possible to determine the temperature anywhere in the structure solving a direct problem.

The next section of the paper presents briefly the analytical thermal modelling method applied by the authors for the analysis of a hybrid circuit containing several power devices. Next, a short description of the function specification algorithm is given. This is followed by numerical simulations illustrating the application of the algorithm for circuit temperature estimation. In the simulations density of power dissipated in heating resistors is estimated from remote sensor temperature readings.

## THERMAL ANALYSIS METHOD

The heat conduction process in solids is governed by the second order Fourier-Kirchhoff partial differential equation. In most cases, it is extremely difficult to obtain exact analytical solutions of the equation. However, since typical electronic circuits have fairly regular shapes and their boundary conditions can be approximated in relatively simple manner, analytical solutions usually model such type of circuits with satisfactory accuracy. Therefore, in all the simulations presented in this paper an analytical circuit thermal model will be used.

The simple but accurate analytical model proposed by the authors has been already tested for different types of electronic circuits. In the model, it is assumed that the heat is generated only in power devices, e.g. transistors, placed on the top surface of the structure, which are modelled by surface heat fluxes penetrating into the structure. The heat exchange coefficient "h" represents the cooling of the circuit at the bottom surface. Furthermore, since electronic circuits are relatively thin in comparison to their area, the heat flows mainly vertically and adiabatic conditions can be imposed on lateral surfaces. The interlayer heat conductance "g" models the imperfect contacts between structure layers. The heat removal to the ambient through radiation at the top surface is not taken into account because for the temperatures encountered in electronics this phenomenon does not play an important role. Concluding, the entire model for steady states can be summarised as follows (Ozisik, 1993):

Heat equation: $$\nabla^2 T_i = 0 \qquad (1)$$

Boundary conditions:

Top: $$-\lambda_1 \frac{\partial T_1}{\partial z} = q \qquad (1a)$$

Bottom: $$-\lambda_2 \frac{\partial T_2}{\partial z} = h T_2 \qquad (1b)$$

Sides: $$\frac{\partial T_i}{\partial x} = 0 \; ; \quad \frac{\partial T_i}{\partial y} = 0 \qquad (1c)$$

Contact: $$\lambda_1 \frac{\partial T_1}{\partial z} = \lambda_2 \frac{\partial T_2}{\partial z} \; ; \quad -\lambda_1 \frac{\partial T_1}{\partial z} = g\left(T_1 - T_2\right) \qquad (1d)$$

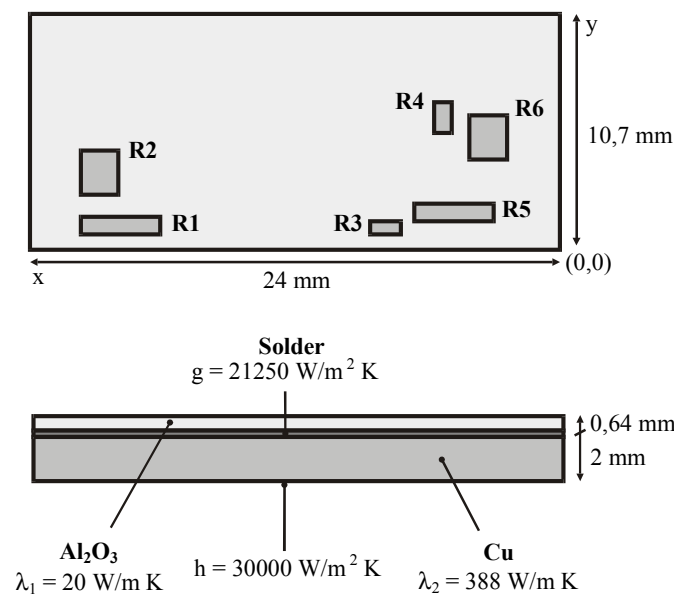


**Figure 1 - The test hybrid circuit**

For the solution of the problem the separation of variables technique was chosen. This technique rendered possible to obtain the solution in the below presented form, which is suitable for estimation purposes.

$$\mathbf{T} = \mathbf{X} * \mathbf{q} \qquad (2)$$

The analytical thermal model was applied for the analysis of a hybrid circuit. The circuit comprised six heating resistors placed on an alumina substrate overlaid on a copper heat sink. The dimensions of the resistors, power dissipated in them and the thermal properties of materials, which are necessary to solve the heat conduction equation, are given in figure 1 and table 1.

In order to verify the model accuracy, for the load conditions specified in table 1, the temperature map of the top surface was captured using the Hughes TVS 4100 infrared camera (Leturcq et al., 1987). The measurement accuracy was ±3 K. Next, for the same load conditions, the temperature values in the middle of each of the heating resistors were calculated and compared with the measurement. The comparison results are presented in table 2. Additionally, the values of deviation from measurement are included.

As can be seen the simulation is in good accordance with measurement. The slightly larger discrepancy for resistor R3 Leturcq et al. (1987) explain by the possible presence of air cushion under this resistor in non-uniform solder layer connecting alumina substrate to the heat sink. Nevertheless, in all cases, the error does not exceed 10 %, which should be regarded rather satisfactory for thermal simulations.

**Table 1 – Information on resistors**

| Resistor | Power [W] | Dimensions [mm] | | Center coordinates [mm] | |
|---|---|---|---|---|---|
| | | x | y | x | y |
| R1 | 2,50 | 3,6 | 0,8 | 19,9 | 1,1 |
| R2 | 4,12 | 1,7 | 2,0 | 20,85 | 3,5 |
| R3 | 2,44 | 1,4 | 0,6 | 7,9 | 1,0 |
| R4 | 2,38 | 0,8 | 1,4 | 5,3 | 6,0 |
| R5 | 2,76 | 3,6 | 0,8 | 4,8 | 1,7 |
| R6 | 4,28 | 1,7 | 2,0 | 3,25 | 5,1 |

**Table 2 –Thermal simulation results**

| Resistor | Temperature rise in center [K] | |
|---|---|---|
| | Measurement | Fourier solution |
| R1 | 37,0 | 37,3 |
| R2 | 57,0 | 57,7 |
| R3 | 81,5 | 73,4 |
| R4 | 64,0 | 63,9 |
| R5 | 40,0 | 38,0 |
| R6 | 64,0 | 60,3 |
| Maximal deviation from measurement | | 10 % |
| Average deviation from measurement | | 4 % |

## FUNCTION SPECIFICATION ALGORITHM

Many electronic applications, as mentioned earlier, do not allow direct temperature measurement where heat is generated. Therefore, the calculation of power dissipated in heat sources, and consequently their temperature, has to be performed based on remote temperature sensor readings solving an inverse heat conduction problem (IHCP). Although the solution of an IHCP usually exists and is unique, the obtained surface heat flux estimates are not numerically stable because the inversion of an ill-conditioned matrix is required. In order to improve problem stability, in the first approach authors tried to reduce the sensitivity to input data errors by introducing redundant temperature sensors and thus rendering the problem overdetermined. Then, a pseudo-inverse matrix is computed using the LMS method and, consequently, the power dissipation can be estimated. (Janicki et al., 1998c). Developing further the method, the authors focused on specialised algorithms for solving inverse problems. These algorithms are commonly used in metallurgy or geology, but in electronics they have not found widespread application yet. Since there exists no universal algorithm, its choice should depend on a particular application. Finally, the authors decided to choose from a large variety of algorithms the sequential version of the function specification algorithm, in which the values of unknown quantity in time are found successively with each new arriving data.

The function specification algorithm is based on the assumption that the variation with time of the surface heat flux to be estimated can be described in the functional form. In the simplest and the most commonly used sequential estimation algorithm, r "future" surface heat flux samples are assumed to be equal, i.e. the functional form is constant. In this way, through the time averaging process, the stability of an IHCP algorithm is increased. The estimation accuracy can be further improved through the space averaging by introducing redundant temperature sensors, i.e. more than the number of heat sources. Taking into account the above assumptions and using the well-known least squares method, the surface heat flux in each k-th iteration can be estimated from the following formula (Beck, 1985):

$$\hat{q}_k = \frac{\sum_{i=1}^{r}\sum_{j=1}^{J}\phi_{ji}\left(T_{j,k+i-1}-\hat{T}_{j,k+i-1}\right)}{\sum_{i=1}^{r}\sum_{j=1}^{J}\phi_{ji}^2} \tag{3}$$

The surface heat flux $\hat{q}$ in each iteration is calculated knowing current sensor temperature value and r-1 “future” temperature values T. $\hat{T}$ is the temperature which the circuit would have if the current and “future” heat flux values were equal to 0. $\phi_i$ is the temperature rise caused by unitary heat flux after the time corresponding to i iterations.

The authors adapted further the method, as suggested by Beck (1985), for multiple heat flux estimation so as to render possible simultaneous estimation of all unknown heat fluxes. Then, for p unrelated heat sources, under the assumption of constant heat flux, the vector $\hat{q}$ containing estimates of all unknown heat fluxes can be found using the following equation (Janicki et al., 1998b):

$$\hat{q}_k = \left(\left(X\,A\right)^T X\,A\right)^{-1}\left(X\,A\right)^T\left(T-\hat{T}\right) \tag{4}$$

## NUMERIACAL SIMULATIONS

Due to the diffusive nature of heat, thermal processes in semiconductor structures are relatively slow comparing to the high frequencies of electronic signals generating the heat. Thus the resulting power distribution in the whole structure is rather the function of mean dissipated power and not its instantaneous value. Therefore, when estimating the mean power dissipated in a circuit from remote temperature sensor readings, the steady state model can be used. Then, all the transient states are neglected and the function in time of mean power dissipated is obtained as a series of interpolated steady state estimates.

In the numerical simulations presented in this section, three different temperature sensor configurations are considered. The first of them constitutes the optimal solution when temperature sensors are placed directly in heat source locations. Although this case is somewhat unrealistic from technological reasons, it is useful for configuration comparison purposes. For this configuration, the condition number of sensitivity coefficient matrix is equal to 2. The second sensor configuration considered consists of six temperature sensors placed close but outside heat sources. The third configuration comprises two additional sensors, which improve significantly problem conditioning. The matrix condition numbers are 19 and 5 respectively. The position of heat sources and remote temperature sensors are presented in fig. 2.

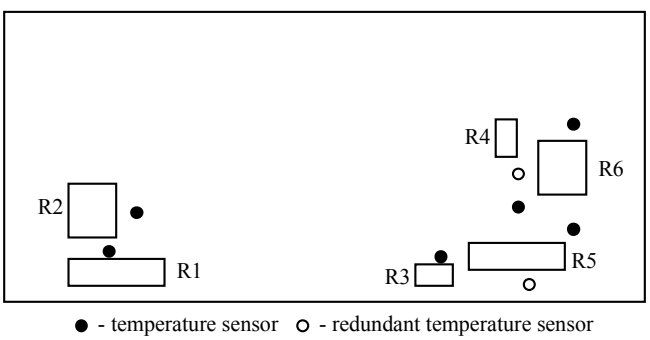


**Figure 2 – Temperature sensor configuration**

In order to test the function specification algorithm, for each sensor configuration, the quasi steady state responses at sensor locations were calculated using the analytical model. The assumed values of mean power dissipated in resistors were equal to those specified in Table 1. All the calculated temperature values were within the infrared measurement error margin. Then, the values were contaminated with additive noise. The mean value of the noise was equal to 0 and its standard deviation 1 K or 5 K. Next, the noisy data were used as input test sets for the function specification algorithm. The multiple heat source power density (surface heat flux) estimation was performed averaging 1, 3 or 5 "future" values. The mean value and the standard deviation of the difference between real signal and its estimates were chosen as the measure of estimation quality. The mean values were almost exactly equal to 0. The standard deviations, related for each heat source to its maximal power signal value, are presented in Table 3.

**Table 3 – Estimation results**

| 6 center sensors– condition number 2 | | | | |
|---|---|---|---|---|
| Error deviation [K] | Number of future values | Estimate deviation [%] | | |
| | | Min. | Max. | Mean |
| 1 | 1 | 2 | 2 | 2 |
| | 3 | 5 | 5 | 5 |
| | 5 | 7 | 7 | 7 |
| 5 | 1 | 7 | 16 | 11 |
| | 3 | 6 | 11 | 8 |
| | 5 | 7 | 10 | 8 |
| 6 remote sensors – condition number 19 | | | | |
| Error deviation [K] | Number of future values | Estimate deviation [%] | | |
| | | Min. | Max. | Mean |
| 1 | 1 | **10** | **49** | 22 |
| | 3 | 8 | 28 | 13 |
| | 5 | 8 | 22 | 12 |
| 5 | 1 | 48 | 253 | 109 |
| | 3 | 29 | 144 | 62 |
| | 5 | 24 | 100 | 48 |
| 8 remote sensors – condition number 5 | | | | |
| Error deviation [K] | Number of future values | Estimate deviation [%] | | |
| | | Min. | Max. | Mean |
| 1 | 1 | 6 | 19 | 10 |
| | 3 | **6** | **12** | 8 |
| | 5 | 7 | 11 | 8 |
| 5 | 1 | 28 | 95 | 52 |
| | 3 | 17 | 55 | 30 |
| | 5 | 14 | 43 | 24 |

As could be expected the estimates of the best quality were obtained when temperature sensors were placed directly in the middle of heat sources. Then, even for significant error standard deviation values greater than 10 % of heat source temperature rise value, the resulting temperature estimates are of excellent quality and for each heat source their standard deviation does not exceed 16 % of the real signal value.

When the sensors are placed outside heat sources, the problem becomes ill-conditioned and the estimation error increases considerably. Relatively significant improvement in the quality of estimates can be achieved thorough the increase

of the number of averaged temperature values, but still for large input data errors the estimation error remains unacceptable.

Far better solution to improve conditioning of the problem is to place additional temperature sensors rendering the system overdetermined. Then, in comparison to the previous configuration, the maximal standard deviation of estimates was reduced by more than 50 %. Comparing with the optimal sensor configuration, similar values of estimation error can be obtained if the sensor temperature measurement accuracy is maintained within a reasonable margin.

In order to assess better the benefits coming from the application of the function specification algorithm, the original signals and their estimates for the cases highlighted in table 3 are visualised in figures 3-6. Figures 3 and 5, on the left hand side, show the best and the worst quality estimates (for resistor R3 and R4 respectively) in case of 6 remote sensors with no averaging. Figures 4 and 6, on the right hand side, show the improved estimates for the same resistors obtained for 8 temperature sensors and averaging 3 successive temperature measurements. As can be seen, owing to the space and time averaging processes, the estimate standard deviation value can be reduced employing the function specification algorithm even to one fourth of its original value.

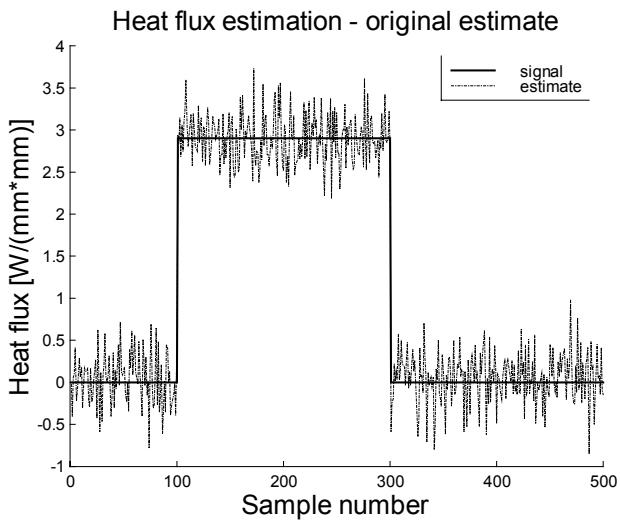


**Figure 3 – Ordinary estimate – best case**

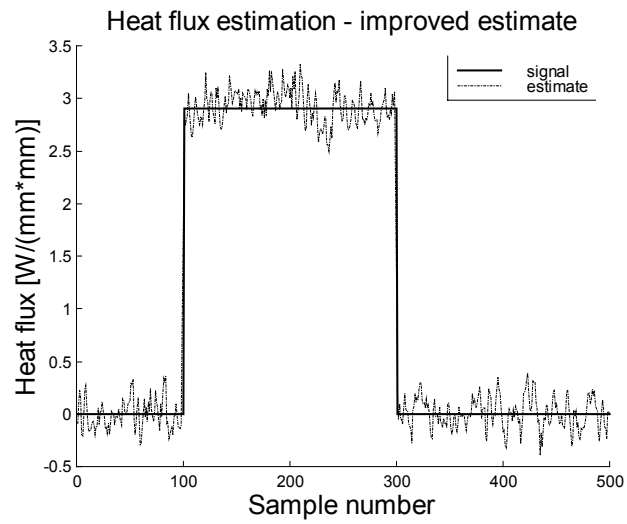


**Figure 4 – improved estimate – best case**

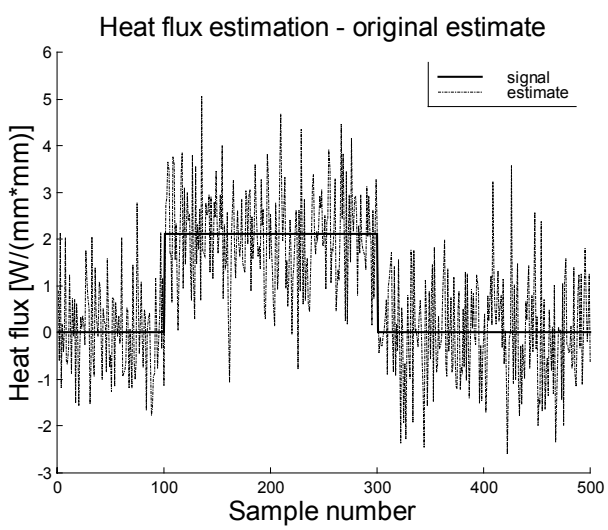


**Figure 5 – Ordinary estimate – worst case**

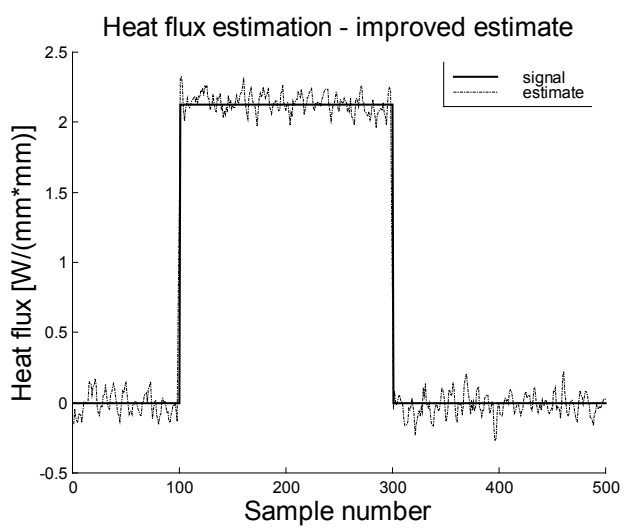


**Figure 6 – Improved estimate – worst case**

## CONCLUSIONS

The numerical simulations presented in this paper proved that the function specification algorithm could be an efficient instrument for electronic circuit temperature estimation. Although theoretically the optimal sensor configuration for the estimation purposes is when the temperature sensors are placed directly where the heat is generated, it is possible to optimise temperature sensor locations outside heat sources obtaining almost optimal estimation conditions. Further significant improvement in the quality of estimates, beside the correct sensor positioning, can be achieved by the combined use of space and time averaging processes. This can by realised by placing redundant temperature sensors and assuming a few subsequent temperature value samples to be equal (Janicki et al., 1998a).

Considering the number of "future" values, the greater is the noise the more samples should be averaged. However, one should bear in mind that in the presence of rapidly changing heat fluxes excessive averaging could increase the estimation error. The number of averaged values can be reduced if the redundant sensors are present because the benefits from the placement of each new temperature sensor decrease with their number.

Although, in experiments the analytical steady state solution of heat conduction equation was used, it does not affect the overall scope of the analysis concerning the applicability of the function specification algorithm and the observation should remain valid also for transient states.

Additional advantage of the function specification algorithm is that for the calculation of the sensitivity coefficients different methods of thermal analysis can be applied, including numerical ones.

If the generated heat density in a heat source is not uniform, this heat source can be divided into several elements of uniform heat generation density and then each of these heat flux sub-components has to be estimated separately.

## ACKNOWLEDGMENTS

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