Inverse Problems in Engineering: Theory and Practice
3rd. Int. Conference on Inverse Problems in Engineering
June 13-18, 1999, Port Ludlow, WA, USA

EXP03

EXPERIMENTAL DESIGN: IMPORTANCE IN PETROLEUM ENGINEERING

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ABSTRACT

The inverse problem associated with data measured to characterize systems and processes are frequently ill-posed, and, consequently, small errors in the measured quantities may transfer into large errors in the desired estimates. Better accuracy may be obtained by improving the experimental design. This paper deals with experimental design and its importance in the field of petroleum engineering. The experimental design studies described in this paper are based on a linearized covariance analysis. We utilize measures such as confidence intervals, contribution of information, and optimal design as ways of ranking different competing measurement strategies (or designs). Among a series of inverse problem applications within petroleum engineering, we consider two cases: The first application deals with estimation of properties defined within a set of partial differential equations, namely those describing flow of fluids through porous media. We show how experimental design can be utilized to select flow experiments (both flow rates, types of data and when and where to measure) to obtain accurate estimates in saturation regions important for reservoir exploitation. The second example deals with identification of the inflow profile of oil, water, and gas, for a production well drilled into an underground reservoir.

1 INTRODUCTION

Inverse problems play an important role in petroleum engineering. Several of the major problems associated with exploration and exploitation of oil reserves, involve solution of inverse problems. For exploration purposes, seismic surveys are utilized to localize potential petroleum reserves, see, e.g, Jahn et al. (Jahn et al, 1998). Furthermore, techniques based on repeated seismic surveys are now emerging for determination of the movements of the fluid in the reservoirs over time (socalled 4D seismic) (Jahn et al, 1998). Such information may be important for selecting optimal exploitation strategies. However, 4D seismic alone will not be sufficent – typically the inverse problem associated with the matematical model of fluid flow through porous media and the historical data from the exploitation of the reserves (typically time series of phase production, pressure drop, etc.), is solved for this purpose. This usually involves solving a regularized, linearly constrained non-linear least-squares problems for some

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uknown coefficients in the flow model (usually the permeability and porosity). This process is known in the petroleum industry as "history matching." A huge number of other examples exists – only within the integral equation niche of inverse problems, we find several important petroleum engineering cases. These include determination of porous media pore size distribution from NMR data (Chen et al., 1992), determination of the capillary pressure function from centrifuge data (Nordtvedt and Kolltveit, 1991), estimation of the characteristics of non-stationary flow in layered reservoirs (Virnovsky, 1979). For the formulation of these integral equations as well as for additional examples, we refer to Subbey (Subbey, 1997).

The inverse problem associated with data measured to describe systems and processes are frequently ill-posed, and, consequently, small errors in the measured quantities may transfer into large errors in the desired estimates. This characterizes several of the inverse problems within petroleum engineering, as continuous fuctions are estimated from discrete data (Watson et al, 1993). For some cases the formulation of the inverse problem is fixed, and bounded solutions are only obtained through a regularization in the parameter (and/or state) space. In other cases, the types, amount, and accuracy of the data, can be varied. Thus, the design of the experiment may be altered in such a manner that more accurate estimates results. Also, as the different data will have an associated cost, engineering applications typically aim at obtaining adequate solutions balancing accuracy and cost considerations. In this paper we study quantitative criterions for designing experiments to meet the objectives of high accuracy and low cost.

We will be investigating into experimental design for two inverse problems arising in petroleum engineering. The first problem is that of characterization of multiphase flow in porous media. Multiphase flow is typically modelled by a continuum representation in space and time based on local volume averaging (Slattery, 1981). Several properties will have to be specified in such models, including the fluid viscosity and rock properties such as porosity and permeability. In multiphase situations, e.g., when two fluid phases flow simultaneously in a porous medium, some multiphase properties – the relative permeability and capillary pressure functions – are introduced (see next section for details). These functions (collectively referred to as the flow functions) are defined through the mathematical model for fluid flow in porous media. They can thus not be measured directly, but have to be determined through solutions of the inverse problems associated with the model and the data. This is done utilizing observations of the state variables (or functions thereof) measured during some displacement experiments. A series of different types of data can be utilized for this purpose, and the experiments may be conducted at different rates of injected fluids, and also at different rate fractions when several fluids are injected simultaneously. To select a design of the experiment leading to accurate and simultaneous determination of the flow function is an issue of great concern, as these functions provide the basis for reservoir simulation used in selection of exploitation strategies.

The second problem is connected to fluid flow in wells. Well technology used for oil production has been advancing rapidly over the last decade. New technology now give the possibility of controlling the production of hydrocarbons using surface adjustable downhole valves, and to obtain more information about the production characteristic using downhole instrumentation. The cost of the downhole instrumentation is varying, and extensive instrumentation is a source of operational problems. It is therefore important to be able to evaluate the usefulness of the downhole instrumentation. By combining the information provided from measurements in the well with a wellflow simulator it is possible to estimate the inflow profiles for each of the three phases oil, water, and gas. If this inflow estimate is reliable it can be useful for instance to detect water producing zones. The accuracy of an estimate of the inflow profile depends on the quantities measured, where in the well the quantities are measured, and on the accuracy of the measurements.

In this paper we will describe methodologies for designing experiments leading to accurate determination of the porous media flow functions and for assessing the necessary instrumentation for estimating the inflow profile to a well from downhole measurements (such as pressure, temperature, etc.).

2 FORWARD MODELS

We first briefly describe the two forward models, namely that of fluid flow through porous media and in the wells.

2.1 Fluid flow through porous media

We study a macroscopic model for immiscible two-phase fluid flow in porous media. In porous media there exists, as the name indicates, an interconnected matrix of pores and channels through which fluids may flow. The fraction of space available for fluid flow is called the porosity ϕ of the medium. An empirical relation describing the connection between flow rate and pressure drop for single-phase flow was discovered by Darcy (Darcy, 1856):

$$\mathbf{q} = -\frac{k}{\mu}(\nabla p + \rho g \nabla z). \tag{1}$$

In this equation, $\mathbf{q}$ is the fluid flow vector, giving the flow rate across a cross-section with area A and normal $\mathbf{n}$ as $A\mathbf{q}\cdot\mathbf{n}$. Furthermore, p is the fluid pressure, ρ the fluid density, μ the viscosity, g the acceleration of gravity, and z the vertical coordinate. The proportionality factor k is called the permeability.

The single phase equation is intuitively extended to immiscible two-phase flow by postulating that the individual fluid

phases follow a relation similar to Eq. 1 (Dullien, 1992):

$$\mathbf{q}_i = -\frac{k k_{ri}(S_i)}{\mu_i}(\nabla p_i + \rho_i g \nabla z). \tag{2}$$

The quantity k_{ri}, called the relative permeability, is introduced to account for the observed fact that the presence of one phase reduces the effective permeability of the other. Relative permeability is a function of the respective fluid saturation, S_i, i.e., the fraction of pore space occupied by phase i.

Due to interfacial tension, two phases may coexist in the porous medium at different phase pressures, p_i. The pressure difference is given by the capillary pressure, $P_c = p_2 - p_1$, which is a function of fluid saturation. With a nonzero P_c, the process of fluid flow in porous media is not symmetrical with respect to the two phases. The flow functions are monotonic in S_1; typical examples are given in Figure 1.

In the model used in this work, we assume constant densities and viscosities of the phases, and constant porosity and permeability of the porous medium. Furthermore, we study one-dimensional flow in the horizontal direction. We combine the one-dimensional version of Eq. 2 with one-dimensional continuity equations for each phase,

$$\phi \frac{\partial S_i}{\partial t} = -\frac{\partial q_i}{\partial x}, \tag{3}$$

and add the capillary pressure relation and an equation stating that the two fluids fill the pore space completely: $S_1 + S_2 = 1$. This leads to

$$\phi \frac{\partial S_i}{\partial t} = \frac{\partial}{\partial x}\left(\frac{k k_{ri}(S_1)}{\mu_i}\frac{\partial p_i}{\partial x}\right), \quad i = 1,2, \tag{4}$$

$$p_2 - p_1 = P_c(S_1), \tag{5}$$

$$S_1 + S_2 = 1. \tag{6}$$

The solution of these equations for given boundary conditions constitutes the forward model used in this work.

2.2 Fluid flow in wells

Calculation of fluid flow in the well is based on conservation of mass, momentum, and energy (Alves et al., 1992). We utilize a steady-state formulation. The conservation of mass is given by the following system of equations

$$\frac{\partial}{\partial s}\left(A\alpha_o \frac{\rho_{oSC}}{B_o} u_o + A\alpha_g R_v \frac{\rho_{oSC}}{B_g} u_g\right) = \frac{\rho_{oSC}}{B_o} q_o + R_v \frac{\rho_{oSC}}{B_g} q_g,$$

$$\frac{\partial}{\partial s}\left(A\alpha_g \frac{\rho_{gSC}}{B_g} u_g + A\alpha_o R_l \frac{\rho_{gSC}}{B_o} u_o\right) = \frac{\rho_{gSC}}{B_g} q_g + R_l \frac{\rho_{gSC}}{B_o} q_o, \tag{7}$$

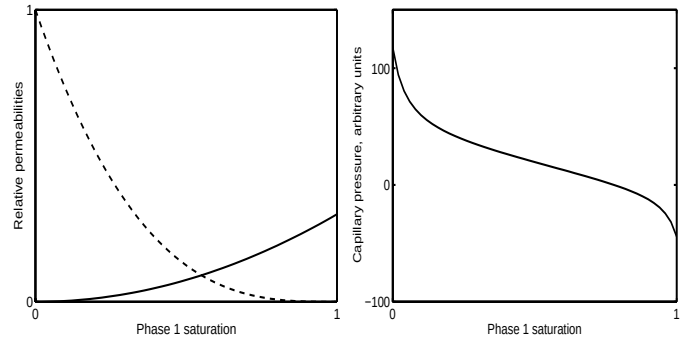

Figure 1. Example of relative permeability (upper) and capillary pressure (lower) functions.

$$\frac{\partial}{\partial s}\left(A\alpha_w \frac{\rho_{wSC}}{B_w} u_w\right) = \frac{\rho_{wSC}}{B_w} q_w.$$

Here, A is cross sectional area of the well, u_i and α_i are the phase velocities and fractions, respectively ($i = w$ (water), g (gas), and o (oil)). ρ_{iSC} and B_i are the fluid density at standard condition and the formation volume factors, respectively. R_v and R_l controls the relative volume of gas and oil when brough to surface, and s is a coordinate along the well trajectory. This way of representing the fluids is referred to as the black-oil model (Alves et al., 1992). q_i is a space dependent source term in these equations – and the desired quantity to be determined from measurements in the well and at surface.

Based on PVT data (e.g, densities, viscosities), the superficial velocities (i.e., $\alpha_i u_i$) can be calculated in all locations in the well by solving Eqs. 7. This will define the flow regime at all locations. Then, from the conservation of momentum, the pressure loss in the well can be calculated (Alves et al., 1992). Thus, the measurable quantities such as pressure drop, flow rates, and phase fractions may then be calculated for all s. If desired, conservation of energy yields the temperature distribution. For further details on formulation and solution techniques, we refer to Alves et al. (Alves et al., 1992).

3 EXPERIMENTAL DESIGN

When designing experiments to meet certain objectives (such as high accuracy, low costs, or combination thereof), it is of great importance to have quantitative criterions to guide the selection of the designs. We here utilize three different measures to help selection of quantitative criterions – confidence intervals, D-optimal design, and contribution of information – all arising from a linearization of the model function around a point of interest in the parameter space. For the porous media model, the nonlinearity assumption has been investigated by Grimstad et al. (Grimstad et al, 1999), indicating that in many cases the linear-

ization results in accurate and conservative confidence intervals for those parts of the flow functions that are well determined by the data. A similar analysis is not available for the well flow model, although our experience indicate a similar behavior. The three measures are briefly discussed in this section. The considerations for selection of specific criterions are discussed in the context of the examples in the next section.

The linearized covariance analysis (LCA) is the basis for confidence interval calculations and for D-optimal design. In the LCA, it is assumed that the mathematical model is capable of describing the physical process (i.e., negligible modeling error) and that the selected functional representation of the desired quantity (here; flow functions or inflow profiles) is adequate (i.e., negligible bias error (Kerig and Watson, 1986)). Also, it is assumed that the model function $\mathbf{F}(\mathbf{p})$ may be reasonably well approximated by a function linear with respect to the parameters $\mathbf{p}$ near the point of investigation in parameter space. Finally, the measurement errors are assumed additive ($\mathbf{Y} = \mathbf{F} + \mathbf{e}$) for some parameter vector $\mathbf{p}$ and error vector $\mathbf{e}$, with error elements e_i having a normal distribution with zero mean and covariance matrix Σ. The covariance matrix of the estimated parameters P is then given by (Kerig and Watson, 1986)

$$P_{n\times n} = (A^T \Sigma^{-1} A)^{-1}, \tag{8}$$

where n is the number of parameters in the representation of the desired quantities, and the matrix A is given by the sensitivity of the simulated data $\mathbf{F}(\mathbf{p})$ to the parameters. Note that, since the vector of data values ($\mathbf{Y}$) does not appear in this expression, no experimental data are required to compute P. Thus, provided that a reasonable value of $\mathbf{p}$ can be selected, a statistical analysis of the estimation problem for a specific experimental design can be performed before actually conducting any experiment (Nordtvedt et al., 1992).

Of greater interest than the covariance matrix P for the parameters, is the covariance of the desired quantities themselves at specified values of the independent variable in the representation (here; saturation for the flow functions, and spatial coordinate for the inflow profiles). The covariance, C, of the desired quantities can be written as

$$C = D^T P D, \tag{9}$$

where the matrix D is the sensitivity of the desired quantities with respect to the parameters $\mathbf{p}$ at a specified value of the independent variable in the representation. Using the diagonal of C, pointwise confidence intervals can be constructed

$$\mathbf{r}^* = \mathbf{r} \pm q\sqrt{c_{ii}}, \tag{10}$$

where $\mathbf{r}$ is the collection of desired quantities and q is the appropriate quantile for the given confidence level and distribution.

3.1 D-optimal design

One of the most widely used functions for evaluating an experimental design is the D-criterion (Pukelsheim, 1993; Fedorov and Hackl, 1997). Using the D-criterion our measure on the performance of an experimental design will emerge from the computation of the determinant of the inverse of the covariance matrix for the parameters, i.e.,

$$\det P^{-1} = \det A^T \Sigma^{-1} A. \tag{11}$$

Large values of $\det P^{-1}$ secures small volumes of the confidence ellipsoid around $\mathbf{p}$. By introducing this measure we are able to compare – through a single value – how well different experimental designs are suited for accurate determination of the desired quantities. With our assumption on normal distribution on the measurement errors the D-criterion for experimental design can be interpreted as a measure of the information contents in the data (see (Bard, 1974) or (Pukelsheim, 1993)).

3.2 Contribution of information

When solving the inverse problem, the model parameters are generally allowed to be different from the true parameters within a certain range, provided that the required accuracy of model application is assured. Let η_j be the range, or resolution, of parameter p_j. The parameter is interval identifiable (Sun and Yeh, 1990a) if the estimated value $\hat{p}_j$ of the parameter satisfies $\left|\hat{p}_j - p_{0,j}\right| < \eta_j$ where $p_{0,j}$ is the true value of parameter p_j. The problem of whether a parameter p_j is interval identifiable for a given η_j depends upon the quantity and quality of the observations. To evaluate the data requirements of a system in connection with parameter identification, the concept of "contribution of observation F_i in the identification of parameter p_j," denoted $CTB(F_i, p_j)$, was introduced by Sun and Yeh (Sun and Yeh, 1990a). The contribution of observation F_i for identification of parameter p_j in the experimental design can be represented as:

$$CTB(F_i, p_j) = \frac{\eta_j}{\varepsilon_i} \left| \int_{\Omega_j} \frac{\partial F_i}{\partial p_j} d\Omega \right|. \tag{12}$$

Here, F_i is a component of the model response, p_j is a component of the parameter vector, η_j is a given admissible error of the identified parameter p_j, and ε_i is the upper bound of observation noise associated with F_i. Finally, $\partial F_i / \partial p_j$ is the sensitivity coefficient of observation $\mathbf{F}$ with respect to parameter $\mathbf{p}$ in a region Ω_j associated with parameter p_j (i.e., an element in the sensitivity matrix, A; see Eq. 8). A necessary condition for interval identifiability is that there is at least one observation for each component

for the unknown parameters whose contribution is larger than 1 (Sun and Yeh, 1990b). This can be used as a criterion for parameter identifiability.

4 DESIGN EXAMPLES

In this section, examples of design of experiments are considered for the two cases.

4.1 Determination of flow functions

A variety of experiments, flow scenarios (i.e., injection strategies and rates / rate fractions), and data types may be utilized for determining two-phase flow functions. Typical experiments include flooding experiments (unsteady state (Watson et al., 1988) or steady state (Urkedal et al., 1999)), centrifuge (Nordtvedt et al., 1993), or porous plate / micro membrane (Guo and Hammervold, 1993)). Examples of data are time series of produced volumes and pressure drop as well as saturation profiles.

The many options for experimental design presents a dilemma. Wheras one generally accepts that more data can provide for more accurate estimates, the relative costs for some types of data or experimental scenarios can be quite high. For example, while measurements of saturation profiles within the sample would seem to be very desirable (Kulkarni et al., 1998), the corresponding equipment can be quite expensive. Thus, one aims at experimental designs that balances the desire for accurate estimates of the flow functions and the experimental effort (e.g., time and costs) required.

In this work, we assume the absolute permeability and porosity to be given quantities (note, however, that they may be varying spatially), and that the fluid characteristics (viscosity, densities) are constant and known. We represent the relative permeability and capillary pressure by a linear expansion of basis function, and evaluate the accuracy for one or several points in the parameter space spanned by this representation. This can be done for a series of competing experimental strategies, such as flooding experiments, centrifuge, or porous plate / micro membrane. For the selected strategy, we need to specify the type of data, and the location (in time and/or space) where a data point will be taken. We consider here flooding experiments. For these type of experiments, one or several fluid phases are injected into a core sample having some initial saturation. Constant injection rates for one phase or constant pressure drop experiments are commonly utilized, although varying injection rates (and rate fractions between two phases being injected) have proven to be useful for simultaneous determination of the flow functions (Urkedal et al., 1999). Here, we investigate two flow scenarios, involving single or multiple constant rates, one of them with simultaneous injection of two phases – see next section for details. Examples of data types for a flooding experiments are differential pressure over the entire core sample as a function of time, volume produced from the outlet end as a function of time, saturation profiles, and *in situ* phase pressures or saturation as a function of time for one or several positions within the porous media.

Once the flooding scenario and types of data with given accuracy and time and position have been selected (i.e., the design), the covariance analysis can be used to determine the expected accuracy. Selecting several designs, one can investigate into which of them would be preferable for a given application. In summary, the evaluation of the experimental designs comprises the following steps:

1. Select core and fluid properties, and select relative permeability and capillary pressure functions (i.e., select a point in parameter space);
2. Select an experimental design (i.e., flow scenario, types and number of data and the accuracy of these data);
3. Perform the covariance analysis (i.e., calculate the confidence intervals around the selected flow functions); and
4. Analyze the confidence intervals with respect to desired accuracy in the flow functions.

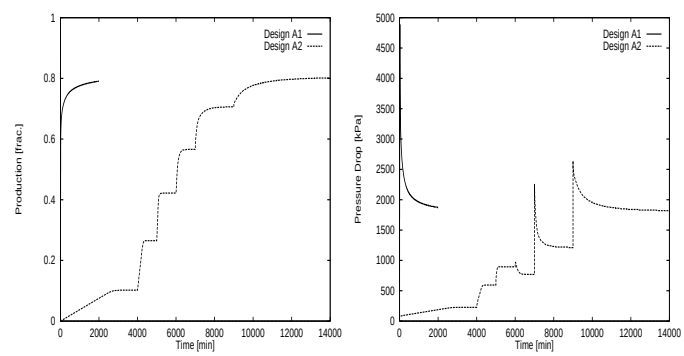

Figure 2. Pressure and production data for the two designs. Left: Production data; Right: Pressure drop data.

4.1.1 Some results

We consider here the design of a drainage experiment (oil flood). We have investigated two different injection schemes, referred to as A-1 and A-2. Simulated data for A-1 (one injection step) and A-2 (six injection steps) are plotted in Figure 2; here only pressure drop and volume produced data are considered. The impact of measurement error on estimated flow functions is represented by confidence intervals. Figure 3 shows the confidence intervals around the true relative permeability and capillary pressure curves resulting from utilization data from the two scenarios. For A-1, the generally wide

confidence intervals show that the data contain little information about the relative permeability and capillary pressure functions for water saturation from 1.0 down to 0.5. In A-2 the confidence intervals are more narrow, showing that this design is preferred for accurate detremination of the flow functions. This is due to the multi rate and fraction injection scheme in design A-2, provides for data better reflecting all saturation values.

For both cases, the confidence intervals become very large for $S_w \leq 0.2$ since saturation does not take on values in that range during the experiments. The narrow confidence intervals in Figure 3 for A-2 show that production and pressure drop data from an injection strategy with six rate fractions will provide accurate estimates of the relative permeability and capillary pressure functions.

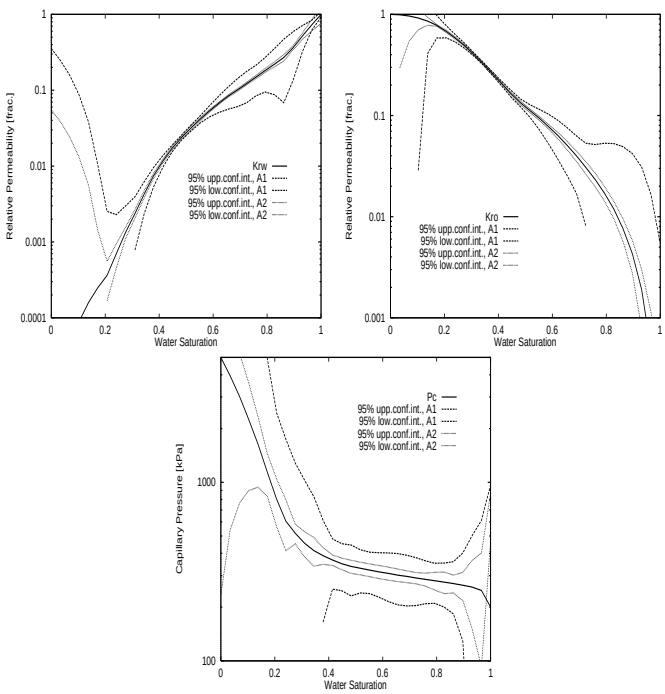

Figure 3. 95% confidence intervals for the two experimental designs considered. Upper left: Water relative permeability; Upper right: Oil relative permeability; Lower: Capillary pressure.

The contribution of observations for determining each parameter is also addressed for cases A-1 and A-2. Figure 4 shows the contribution of pressure drop data on the water relative permeability parameters. The left figure shows the results on pressure drop data from experimental design A-1 (where only one injection step has been used). Here, all the contributions are below 1. This means that the computed observations $\mathbf{F}(\mathbf{u},\mathbf{p})$ are unable to distinguish two parameters within the given resolution η_j and accuracy $\bar{\varepsilon_i}$ on the observations. The right figure shows that the A-2 design (with six injection steps) gives data where $CTB(F_i, p_j) \geq 1$ for several the parameters and high for the others. Hence, the parameters are identifiable with the suggested experimental design, resolution and measurement error.

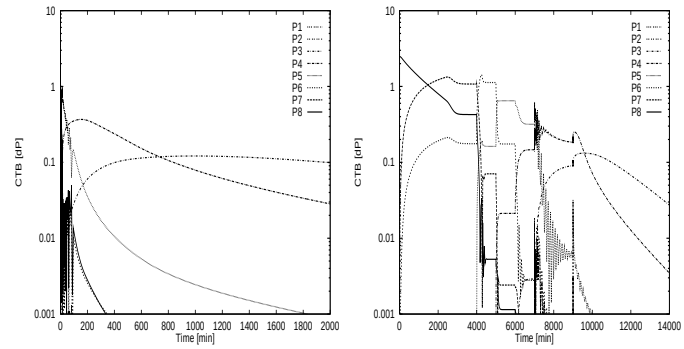

Figure 4. Contribution of information (pressure drop) versus time for water relative permeability. Left: Design A-1; Right: Design A-2.

4.2 Determination of inflow profile

The linear covariance analysis outlined in Section 3 allows for evaluation of the accuracy of the estimated inflow profile for a given design. If the accuracy is not satisfying, additional measurements need to be included. If the accuracy is satisfying, the issue of reducing the number of instruments used can be addressed.

To search for the best instrumentation of the well it is necessary to be able to compare different designs of instrumentation. A quantitative measure on the performance of each design of instrumentation is needed. We here utilize the D-measure discussed in the previous Section.

To design the instrumentation of the well, we need a list of potential measurements. This list must include the type, location, and measurement errors for each sensor providing the measurements. We must ensure that by using all the possible measurements, we are able to achieve the required accuracy.

A common assumption in the design process is that some measurements should be included in the design, either because they are already implemented in the well, or because of the fact that their relatively low cost makes them a good starting choice for designing the instrumentation. Then we need to search for extra measurements to complement the set of measurements which are already included in the design.

In the further discussion a *data point* is the information obtained by measuring a physical quantity at a specified position in the well with a certain accuracy. To obtain a data point one need an instrument at the specified position which are able to measure the quantity in question with the required accuracy.

Let us give a short mathematical description of the optimization problem we need to solve. Each data point corresponds to a row, $\mathbf{a}_i$, in the sensitivity matrix (A) and a corresponding diagonal entry σ_i^2 in the covariance matrix Σ. Since we assume that the measurement errors are uncorrelated, Σ is diagonal, and

$$A^T\Sigma^{-1}A = \sum_{i=1}^{M} \mathbf{a}_i^T \sigma_i^{-2} \mathbf{a}_i \tag{13}$$

where the sum runs over the measurements included in the simulation.

Since we have made a decision that some instruments should be used in the design, there is a set Q of data points that we have access to, and we want to augment this set with N extra measurements from M candidate measurements. The optimization problem is then formulated as

$$\max_S \det\left(\sum_Q \mathbf{a}_j^T \sigma_j^{-2} \mathbf{a}_j + \sum_{i\in S} \mathbf{a}_i^T \sigma_i^{-2} \mathbf{a}_i\right). \tag{14}$$

Here, S runs over all sets with N elements. Unfortunately, the above optimization problem is very hard to solve. This force us to look for solutions which are acceptable and fast to compute, but not necessarily optimal. Acceptable solutions are obtained utilizing a greedy algorithm which ranks the additional instruments. For simplicity we assume that

$$\det\left(\sum_Q \mathbf{a}_i^T \sigma_i^{-2} \mathbf{a}_i\right) > 0. \tag{15}$$

To rank the M candidate instruments, we use the following algorithm:

1. Begin at iteration 1 with S_0 as the full set of M data points.
2. At iteration i, find the element $k \in S_{i-1}$ which solves the optimization problem

$$\max_{k\in S_{i-1}} \det\left(\sum_{j\in S_{i-1}\setminus\{k\}} \mathbf{a}_j^T \sigma_j^{-2} \mathbf{a}_j + \sum_Q \mathbf{a}_j^T \sigma_j^{-2} \mathbf{a}_j\right) \tag{16}$$

   Let the new set $S_i = S_{i-1}\setminus\{k\}$.
3. The algorithm terminates when the set S_i contains N elements.

Since the data points are thrown away in the same order independently of N we can run the algorithm with $N = 1$ and make a ranking of the instruments. With this ranking the search for a suitable design reduces to select the number of (additional) measurements. This decission will be a trade off between the reduction of the confidence intervals versus the cost of introducing extra measurements.

As a reference on the use of greedy algorithms in constructing D-optimal designs we refer to (Robertazzi and Schwartz, 1989), but we must remark that our problem formulation is different from theirs. In the next section, the use of these algorithms are exemplified.

4.2.1 Some results

Using the algorithms above one can assess the instrumentation in the well. Since the results refer to a fixed inflow profile, these algorithms must be run for every inflow profile of interest separately.

Before designing the instrumentation we need information about the geometry of the well, a list of instruments (type and position) we want to consider, the measurement errors of the instruments, the resolution of the well inflow, and a list of actual inflow profiles.

Then, for each inflow profile on the list we compute the sensitivity matrix and use this to make a ranking of the instruments for this inflow profile. After obtaining the rankings for all the inflow profiles in the list, the rankings can be used as a guideline for designing instrumentation for the well. The behavior of a selected design should be tested on each of the inflow profiles, to check that the confidence intervals of the inflow profiles are satisfactory.

We have utilized this procedure to study how well the fluid inflow profile may be determined from pressure, temperature, fluid velocities, phase fraction, and total production of each phase. A water, oil, gas case is considered. We consider how 809 data points may be utilized to determine the inflow of water to the well. The well considered is a 2000m inclined well, divided into four sections. Figure 5 shows the results. In the upper left figure, we have plotted the true inflow profile along with the pointwise confidence intervals when all data are utilized. In the upper right figure, only 401 temperature data are utilized along with the total production of each of the phases. As can be seen, the accuracy is lowered for most of the inflow area. We then utilize the above procedure to identify an appropriate number of instruments to add to the temperature data. Four additional instruments are included, giving an accuracy as illustrated in the lower figure of Figure 5.

5 CONCLUSIONS

We have discussed experimental design strategies for two inverse problems arising within petroleum engineering, namely the estimation of flow functions from displacement experiments and the inflow profile from distributed data in the well. We demonstrate that confidence intervals, D-optimal design, and contribution of information, can be used successfully to obtain experi-

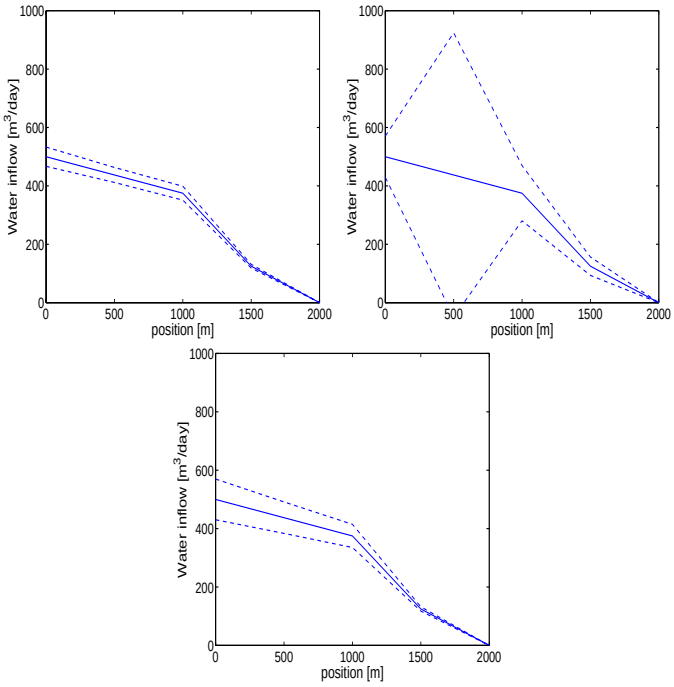

Figure 5. Confidence intervals for different instrumentation strategies. Upper left: All datapoints are included; Upper right: Only temperature and total production data are used; Lower: Temperature and four additional measurements are used.

mental designs yielding accurate estimates of these quantities.

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