Inverse Problems in Engineering: Theory and Practice
3rd Int. Conference on Inverse Problems in Engineering
June 13-18, 1999, Port Ludlow, WA, USA

EXP07

DESIGN OF EXPERIMENTS TO ESTIMATE TEMPERATURE DEPENDENT THERMAL PROPERTIES

K. J. Dowding and B. F. Blackwell
Sandia National Laboratories
P.O. Box 5800 Mail Stop 0835
Albuquerque, NM 87185
(505) 844-9699, FAX (505) 844-8251
e-mail: kjdowdi@sandia.gov and bfblack@sandia.gov

ABSTRACT

Experimental conditions are studied to optimize transient experiments for estimating temperature dependent thermal conductivity and volumetric heat capacity. Thermal properties are assumed to vary linearly with temperature; a total of four parameters describe linearly varying thermal conductivity and volumetric heat capacity. A numerical model of experimental configurations is studied to determine the optimum conditions to conduct the experiment. The criterion D-optimality is used to study the sensor locations, heating duration and magnitude, and experiment duration for finite and semi-infinite configurations. Results indicate that D-optimality is an order of magnitude larger for the finite configuration and hence will provide estimates with a smaller confidence region.

NOMENCLATURE

c specific heat, $J/kg\text{-}^oC$

C volumetric heat capacity, $J/m^3\text{-}^oC$

k thermal conductivity, $W/m\text{-}^oC$

L thickness, m

N_s number of sensors

N_t number of discrete time measurements

N_p number of parameters

$\dot{q}''$ heat flux, W/m^2

t time, sec

T temperature, ^oC

T_{max} maximum temperature, ^oC

T_0 boundary temperature, ^oC

T_i initial temperature, ^oC

T_β scaled sensitivity coefficient for parameter β, ^oC

$\boldsymbol{T}$ temperature vector, ^oC

$\boldsymbol{X}$ sensitivity matrix

x_0 characteristic dimension, m

Greek

β parameter

Δ optimality criterion

Δt time step, sec

ρ density, kg/m^3

Superscript

$+$ dimensionless

INTRODUCTION

It is becoming common practice to combine transient experiments with parameter estimation techniques to estimate thermal properties of solids. A sample of investigations using this approach are Jin et al., (1998); Maddren et al., (1998); Dowding et al., (1998); Dowding et al., (1996); Dowding et al., (1995); Scott and Beck (1992a and 1992b); Beck and Osman (1991). Properly designing the experiment strongly influences the accuracy of the parameters estimated from it. Considering the duration of the transient experiment, boundary conditions, and the impact of ancillary materials in the model are issues that can have a significant impact on the accuracy of the estimated parameters. Forethought, in particular using analysis to investigate and design the experiment, is paramount to the final success of the experiment. This statement applies for many experimental endeavors, not only those intended for parameter

estimation.

Conditions to estimate one or more thermal properties have been studied for several one dimensional configurations; Beck and Arnold (1977), Taktak et al., (1993), and Emery and Nenarokomov (1998) are examples. In cases where the goal is to estimate k and ρc simultaneously, a known heat flux boundary conditions is required (Beck and Arnold, 1977). Many of the configurations addressed by Beck and Arnold (1977) and Taktak et al. (1993) have a known heat flux on one boundary. Emery and Nenarokomov (1998) studied other boundary conditions, such as convection. Two dimensional geometries have been studied by Emery and Nenarokomov (1998) and Taktak (1992). Taktak (1992) investigated conditions to estimate orthotropic thermal conductivity. Emery and Fadale (1996) studied the design of experiments while including the uncertainty of model parameters that are not estimated; in all previous studies additional parameters in the model are assumed to be exactly known. In all cited investigations, however, the thermal properties do not depend on temperature (constant properties).

Several investigators have presented formulations to estimate variable (temperature or spatially dependent) thermal properties; see Lesnic et al. (1996); Woodbury and Boohaker (1996); Sawaf et al. (1995); Huang and Yuan (1995); Huang and Ozisik (1991); Flach and Ozisik (1989). Experimental design to maximize the information was not addressed in these studies. Temperature dependent properties can also be obtained by conducting several experiments at different initial temperatures that are analyzed independently assuming constant properties for each independent experiment; this approach requires that the individual experiments cover a small temperature range. By combining the estimated properties from the individual experiments temperature dependent properties are obtained. Several investigations have used this approach: Dowding et al. (1995); Dowding, et al. (1996); and Loh and Beck (1991). Experiments can also be combined in a sequential fashion during the analysis to estimate temperature dependent properties, Beck and Osman (1991) and Dowding et al. (1998).

Remarkably, the only known investigations to design an experiment to estimate temperature dependent properties are attributed to Beck (1964, 1966 and 1969) over thirty years ago. Beck represented the thermal properties as varying linearly with temperature

$$k(T) = k_1\left(1 - \frac{T - T_1}{T_2 - T_1}\right) + k_2\left(\frac{T - T_1}{T_2 - T_1}\right) \qquad (1)$$

where k_1 and k_2 are the values of thermal conductivity at temperatures T_1 and T_2, respectively. Volumetric heat capacity would have a similar temperature dependence with parameters C_1 and C_2. For a semi-infinite geometry with a prescribed surface temperature history, Beck (1969) proved that the four parameters k_1, k_2, C_1, C_2 cannot be simultaneously estimated for certain conditions. He suggested that some parameters should be set while others are estimated. We address different conditions

than Beck and show later in this study that linearly varying k and C can be simultaneously estimated for the configuration studied in this paper.

The objectives of this study are to investigate the optimal experimental conditions to estimate temperature dependent thermal properties. We are presently interested in an experimental design to estimate temperature dependent properties of relatively low conductivity materials, e.g., polyurethane foams. However, results are presented in dimensionless form and are not restricted to only low conductivity materials. Experiments are considered that utilize electric heaters providing a known heat flux on one surface of a specimen. The opposite surface is isothermal for a finite body, or the body approximates a semi-infinite domain,[1] both of which can be realized in the laboratory for low conductivity materials. The two cases (finite or semi-infinite) are studied to determine which is optimum. A single experiment is assumed for estimating all parameter simultaneously to compare these two cases. All references to experimental conditions are in the context of conditions in a numerical model of the experimental configuration; no experimental data are discussed. However, the results presented here are being used to guide the design of an experiment to estimate the thermal properties of polyurethane foam.

ANALYSIS

Optimality Criterion

A criterion is needed to identify an optimum design. The criterion selected for this study is D-optimality, Beck and Arnold (1977). It is selected because it relates to the volume of the confidence region for the estimated parameters. Emery and Nenarokomov (1998) discuss several other optimality criteria. All cited optimality criteria are related to the sensitivity coefficient matrix X

$$X^T = \left[\frac{\partial T}{\partial \beta_1}\ \frac{\partial T}{\partial \beta_2}\ \cdots\ \frac{\partial T}{\partial \beta_{N_p}}\right], \qquad (2)$$

which elements are the partial derivative of temperature response with respect to the parameter β_i. T in Eq. (2) is a vector (length $N_t N_s$) of temperatures for each time and sensor location. Assuming there are N_t discrete measurements in time, N_s measurement locations, and N_p parameters, the dimensions of X are $N_s N_t \times N_p$.

D-optimality maximizes the determinant of the information matrix

$$\Delta \equiv |X^T X| . \qquad (3)$$

In maximizing this determinant, it can be shown that this

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1. Semi-infinite means that the specimen is sufficiently thick such that the face opposite the heated surface does not experience any temperature rise for the duration of the experiment.

minimizes the hypervolume of the confidence region under certain assumptions. These assumptions are summarized as additive uncorrelated normal errors with zero mean and constant variance, with errorless independent variables, and no prior information, Beck and Arnold (1977). Additionally, we impose constraints on the criterion. It is subject to a *maximum* temperature rise and *fixed* number of measurements. The maximum temperature rise of the experiment is specified as T_{max}. To introduce these constraints the optimality criterion is modified as

$$\Delta^+ \equiv \left| \frac{(\mathbf{X}^+)^T \mathbf{X}^+}{(T_{max}^+)^2 N_t N_s} \right|. \tag{4}$$

Each entry in the $(\mathbf{X}^+)^T \mathbf{X}^+$ matrix is normalized as shown in Eq. (4). The dimensionless sensitivity matrix is

$$(\mathbf{X}^+)^T = \left[\frac{\beta_1}{(\dot{q}_0"x_0/k_1)}\frac{\partial \mathbf{T}}{\partial \beta_1} \quad \frac{\beta_2}{(\dot{q}_0"x_0/k_1)}\frac{\partial \mathbf{T}}{\partial \beta_2} \quad \cdots \quad \frac{\beta_p}{(\dot{q}_0"x_0/k_1)}\frac{\partial \mathbf{T}}{\partial \beta_P} \right], \tag{5}$$

and the maximum dimensionless temperature rise is

$$T_{max}^+ = \frac{T_{max} - T_i}{(\dot{q}_0"x_0/k_1)}. \tag{6}$$

In Eq. (5) and Eq. (6), T_i is the initial temperature, $\dot{q}_0"$ is a constant heat flux, x_0 is a characteristic dimension, and k_1 is a characteristic thermal conductivity value (taken as the value at the low temperature in this case). D-optimality used in this study is essentially a discrete version of the criterion applied in Taktak et al. (1993).

Prescribing a maximum temperature constraint provides consistency when comparing experiments that may otherwise have different temperature ranges. Normalizing by the maximum temperature gives all experiments a maximum value of unity. A large, fixed number of measurements, $N_t N_s$, are assumed to be available from an experiment. Normalizing in Eq. (4) by $N_t N_s$ means the criterion is not influenced by the specific number of measurements. Consequently, the criterion is not increased by taking smaller time steps (increasing N_t) or having multiple sensors at the same location (increasing N_s). Both situations should not reasonably provide more information or produce a better experiment.

The configuration studied in this paper is shown in Figure 1. Two variations of the configuration are addressed. The first variation is a finite slab of thickness L heated with a constant heat flux $(\dot{q}_0")$ on one surface while the opposite surface is isothermal (T_0). Although Figure 1 represents an idealized case, the configuration can be closely approximated in the laboratory for relatively low thermal conductivity materials. For low conductivity materials the isothermal boundary condition is achieved by placing a high conductivity material in intimate contact with the surface. The second variation approximates a semi-infinite domain; the length L is taken to be large enough such that thermal effects do not reach $x = L$ for the
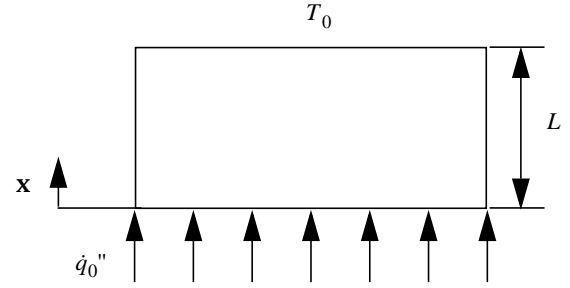

Figure 1   One dimensional thermal model

experimental duration, i.e., the boundary temperature remains at its initial value, $T_0 = T_i$. Again, for low conductivity materials this case can be closely approximated in the laboratory. The applied heat flux can be quantified by using an electric heater and symmetric design of the experiment, Dowding et al. (1995). For the most accurate estimated properties, the thermal effects of the heater should be included in the model. However, for generality in this study the thermal effects of the heater are not included. Because the heater's effect should be secondary compared to the specimen for a well-designed experiment, neglecting the heater should not significantly change the optimum design. It may be necessary to run secondary experiments to characterize thermal properties of the heater, as discussed in Dowding, et al. (1995).

Constant Properties. Optimal experimental conditions to estimate constant thermal properties for the configuration in Figure 1 are well known and given in Table 1. Two cases are listed. The two cases are for a semi-infinite and finite geometry, as specified in the second column which refers to the thickness of the body. For the semi-infinite case the sensors (column three) are located at the surface of the applied heat flux and below the surface; the finite case is optimal with sensors only at the heated surface. It is impossible to estimate both k and C with measurements only at the surface of a semi-infinite body. The optimum (dimensionless) heating duration, t_h^+ in column four, is the time that the applied heat flux is equal to $\dot{q}_0"$, after which the applied heat flux is zero. The value of D-optimality with constraints is given in column five for the dimensionless experimental duration listed in the sixth column. Dimensionless optimal experiment duration t_f^+ is the time that the maximum value of D-optimality is achieved. The dimensionless times are defined as

$$t_h^+ = \frac{(k/C)t_h}{x_0^2} \tag{7}$$

$$t_f^+ = \frac{(k/C)t_f}{x_0^2}, \tag{8}$$

Case	Geom.	Sensor Locations x/x_0	Heating Duration t_h^+	Max Δ^+	Optimal time $t_f^+(\Delta^+)$
1	Semi-infinite[a]	$0, x/x_0 > 0$	1.5	0.00263	1.5
	Semi-infinite[b]	$0, x/x_0 > 0$	1.5	0.0055	1.72
2	Finite[b]	0	7	0.012	7
	Finite[b]	0	2.25	0.020	2.98

a. Beck and Arnold (1977)
b. Taktak et al. (1993)

where t_h is the duration that the heating is applied, t_f is the duration of the experiment (to achieve the maximum in Δ^+), (k, C) are the constant thermal properties, and x_0 is a characteristic dimension. For the configuration shown in Figure 1 the characteristic dimension is the thickness for the finite body $x_0 = L$ and the depth of the sensor from the heated surface for the semi-infinite body. Temperature can be made dimensionless by a group containing the heat flux $\dot{q}_0'' x_0/k$ to remove dependence on the heat flux magnitude. Consequently, the optimal conditions depend only on the sensor location, dimensionless heating time, and experiment duration.

Two variations of each case (semi-infinite and finite) are shown in Table 1. In the first variation, the heating duration continues for the entire experiment duration $(t_h^+ = t_f^+)$. The second variation has a heating duration that is shorter than the experiment duration $(t_f^+ > t_h^+)$. By comparing the two variations we see that continuing the experiment after the heat flux ends $(t_f^+ > t_h^+)$ significantly increases the value of Δ^+ for both cases. A greater value of Δ^+ indicates that the finite case is better than the semi-infinite for constant properties. The ratio of Δ^+ for the two cases is approximately four. This means the volume of the confidence region is four times larger for a semi-infinite body than it is for a finite body; more accurate estimates of the properties are expected with a finite body.

Temperature Dependent Properties. Optimum conditions to estimate temperature dependent properties are less clear than those for constant properties. In this study we will address estimating linearly varying temperature dependent properties without this restriction. We assume the properties are described as shown in Eq. (1). Hence there are two parameters describing the temperature dependence of thermal conductivity (k_1, k_2) and two for the volumetric heat capacity (C_1, C_2); where (k_i, C_i) are values of the properties at the prescribed temperature T_i. Linear variation is the simplest description of temperature variable

properties, but is commonly used in general purpose thermal analysis codes. Two cases (semi-infinite and finite) discussed previously for constant properties are studied.

Thermal Model

One can write the describing mathematical equations for the temperature of the model in Figure 1, assuming thermal properties vary linearly with temperature, as

$$\frac{\partial}{\partial x^+}\left\{\left[1 + \frac{k_2 - k_1}{k_1}\left(\frac{T - T_1}{T_2 - T_1}\right)\right]\frac{\partial T}{\partial x^+}\right\} = \left[1 + \frac{C_2 - C_1}{C_1}\left(\frac{T - T_1}{T_2 - T_1}\right)\right]\frac{\partial T}{\partial t^+} \tag{9}$$

$$-\left[1 + \frac{k_2 - k_1}{k_1}\left(\frac{T - T_1}{T_2 - T_1}\right)\right]\frac{\partial T}{\partial x^+}\bigg|_{x^+ = 0} = \begin{cases} \dfrac{\dot{q}_0'' x_0}{k_1} & (0 < t^+ < t_h^+) \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

$$T|_{x^+ = 1} = T_0 \text{ (Finite)}, \quad T|_{x^+ \to \infty} = T_0 \text{ (Semi-infinite)} \tag{11}$$

$$T|_{t^+ = 0} = T_i. \tag{12}$$

The boundary condition in Eq. (11) on the surface opposite the applied heat flux depends on whether the body is finite or semi-infinite. Distance has been made dimensionless with a characteristic length

$$x^+ = (x/x_0), \tag{13}$$

which is taken as the thickness of the body for the finite case or depth of a sensor below the heated surface for the semi-infinite case. Dimensionless time is defined as

$$t^+ = \frac{(k_1/C_1)t}{x_0^2}. \tag{14}$$

Property values at the lower temperature (T_1) are used in this definition.

For thermal properties varying linearly with temperature the mathematical formulation shows the following three parameter groups describe the thermal properties:

$$\frac{k_2 - k_1}{k_1}, \frac{C_2 - C_1}{C_1}, \frac{T - T_1}{T_2 - T_1}. \tag{15}$$

The properties depend on the normalized variation over the linear segment. We also see a (normalized) temperature dependence; the group contains the temperature change of the experiment, $(T - T_1)$, divided by the temperature variation of the properties, $(T_2 - T_1)$.

Because the properties depend on the temperature variation, there is a dependence on the heating magnitude

$$\frac{\dot{q}_0'' x_0}{k_1}. \tag{16}$$

This group is indicative of the temperature magnitude. Ideally,

we would like to explicitly specify all parameter groups describing the properties in Eq. (15) and study sensor locations and durations (heating and experimental) that are optimum. But the last parameter group in Eq. (15) can not be explicitly set. Consequently, by varying the group in Eq. (16), we can adjust the temperature magnitude so that the last parameter group in Eq. (15) can be implicitly set.

The optimum conditions to estimate temperature dependent properties for the configuration in Figure 1 will be characterized by the following parameter groups

$$\frac{k_2 - k_1}{k_1}, \frac{C_2 - C_1}{C_1}, x^+, t^+, \frac{\dot{q}_0'' x_0}{k_1}. \qquad (17)$$

The relative magnitudes of the property variation, the sensor location, duration, *and* magnitude of the heating are all needed to describe conditions when properties vary linearly with temperature. For convenience the initial temperature and boundary temperature are set to be equal to the lower temperature at which properties are defined $T_i = T_0 = T_1$. Consequently, the optimum design depends on the five parameter groups in Eq. (17). There are actually six design parameters because two dimensionless times are of interest. The first is the duration of the heat flux, t_h^+ in Eq. (10). The second is the duration of the experiment. We noted previously that for constant thermal properties we can describe the optimal conditions independent of the properties and heating magnitude; the design only depends on dimensionless sensor location and (two) dimensionless times.

In the results that follow, property variation is specified as $(k_2 - k_1)/k_1 = 0.66$ and $(C_2 - C_1)/C_1 = 1.2$. The variation is typical of polyurethane foam from room temperature up to $150°C$, Jin et al. (1998). Specifying the property variation assumes a given temperature range of the properties, $T_2 - T_1$. Groups representing the sensor locations (x^+), heating time (t_h^+) and magnitude $(\dot{q}_0'' x_0/k_1)$, and experiment duration (t_f^+) are studied to maximize D-optimality for estimating the four parameters describing temperature dependent conductivity and volumetric heat capacity.

Because the ultimate objective of the experiment is to estimate thermal properties, the design process must proceed with approximate values of the properties. Consequently, if the approximation is poor the design may not be appropriate. Fortunately, the design does not depend on the specific magnitudes of the thermal properties. Instead, it depends on relative change in the properties over the linear segment as shown in Eq. (17).

Solution Procedures. Calculations are performed with a control-volume finite-element based code. In addition to solving the heat conduction equation for the temperature, sensitivity calculations have been incorporated directly into the code, Blackwell et al. (1998) and Dowding and Blackwell (1999). Sensitivity equations are derived by differentiating the describing heat conduction equation. These sensitivity equations are numerically solved with the control-volume, finite-element procedure. The temperature and sensitivity calculations are used to calculate the optimality criterion in Eq. (4).

RESULTS AND DISCUSSION

Temperature and Sensitivity Coefficients

Prior to studying an optimality criterion it is instructive to observe sensitivity coefficients for the two cases. Transient temperature and heating magnitude are shown in Figure 2 for experiments representative of a finite and semi-infinite body. Temperature rise at the surface of the applied heat flux is comparable to the temperature range of the property variation; the group, $(T_{max} - T_1)/(T_2 - T_1)$, is near unity for both cases.

Figure 3 shows the sensitivity coefficients for the temperature in Figure 2 at the surface of the applied heat flux $(x^+ = 0)$. Sensitivity will have the largest magnitude at $x^+ = 0$; in general, sensitivity decreases as we move inside the body. The heating conditions selected for the simulations shown in Figure 3 and Figure 2 are shown later in this paper to provide the most information about the thermal properties for each case.

There are two notable differences between the sensitivities for the finite and semi-infinite cases. Overall, the finite body has
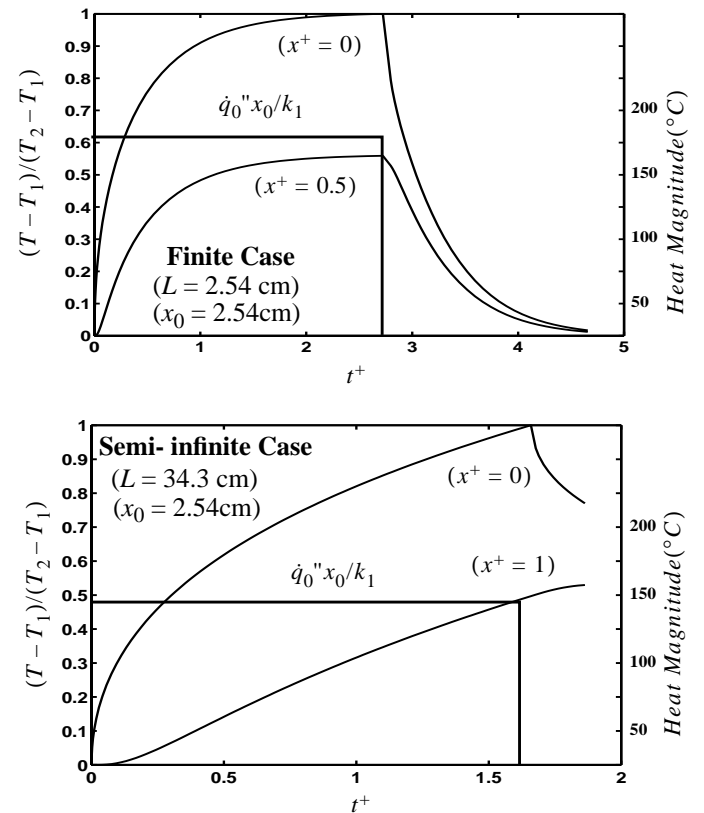

Figure 2   Normalized temperature and heating magnitude for the finite and semi-infinite cases
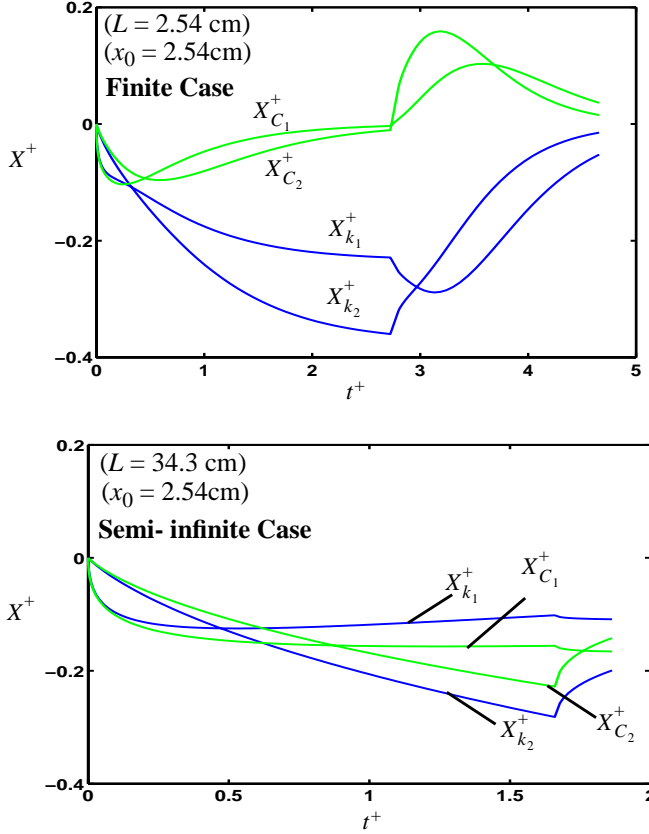
Figure 3   Dimensionless sensitivity coefficients at $x^+ = 0$ for the finite and semi-infinite cases

sensitivities which are larger in magnitude than the semi-infinite case. The second difference is that the shapes of the sensitivity coefficient curves for the four parameters are more different for the finite case than for the semi-infinite case. Larger sensitivity means there is more information about the thermal properties in the measurement. Different shapes mean the effect of each parameter on the temperature is independent. The more different the effect of each parameter, the more easily we can estimate multiple parameters. In Figure 3 for the semi-infinite case the sensitivity for pairs k_1, C_1 and k_2, C_2 demonstrate very similar shapes. This indicates that the parameters are correlated to some degree.

Experimental Design

To investigate whether a finite or semi-infinite geometry is better to estimate linearly varying thermal properties the optimality criterion is studied. In the search for the optimum conditions, sensor location, duration of heating, magnitude of the heating, and experiment duration are varied to determine which combination produces the largest magnitude of the optimality criterion. By presenting the results in dimensionless terms the specific size (thickness) of the specimen and properties are not important. However, the optimum design will depend on the magnitudes of the variation, $(k_2 - k_1)/k_1$ and $(C_2 - C_1)/C_1$;

these values are fixed at 0.66 and 1.2, respectively.

The other design variables (sensor location, duration of heating, magnitude of the heating, and experiment duration) are studied to determine the optimum. The search for a maximum in the optimality criterion is easily done by trial-and-error; we believe additional insight is possible with this approach. An optimization code could be used in the search, but is not used in this study. In the search process we assumed and selected two sensor locations. Then we varied the heating duration and magnitude to determine which combination resulted in the largest magnitude of D-optimality. Because the maximum will occur after the heating ends, we get the optimal duration, t_f^+, from the simulation; the duration of the numerical simulation is set to be long enough such that we get a maximum in the optimality criterion.

Semi-infinite Case

The transient variation of the optimality criterion for the semi-infinite geometry is shown in Figure 4 for different heating durations and magnitudes. Sensors are located at the surface of the applied heat flux and 2.54 cm below the surface. A larger magnitude of heating for a shorter duration produces larger magnitudes of Δ^+. The approach taken to investigate the optimal conditions is to select a heating duration (t_h^+) and magnitude ($\dot{q}_0'' x_0/k_1$) such that the temperature variation during the experiment would cover the temperature range of the property variation as indicated by the last group in Eq. (15), $(T - T_1)/(T_2 - T_1)$. The maximum allowable value of this group is unity; the temperature can not exceed the temperature range for which the properties are defined. The maximum normalized temperature $(T_{max} - T_1)/(T_2 - T_1)$ varied from 0.95 to 1. The magnitude of this group is a result of the heating magnitude selected. In general, the optimality criterion is shown to increase as this group increases. The effect of this parameter group is discussed further in the next section.

The transient plots in Figure 4 demonstrate that the maximum of Δ^+ is attained after the heating ends. At the time when the heat flux ends (t_h^+) the value of Δ^+ increases and the curve changes shape in Figure 4. This increase is due to the sensitivity coefficients changing shape after the heat flux ends; see Figure 3.

The optimum information is shown in the top of Table 2 for the semi-infinite body. Reported in column two of the table are the assumed sensor locations. All cases have a sensor at the surface where the heat flux is applied and internal to the body. We assume there are only two sensors. Conditions describing the applied heat flux are listed in columns three and four. Heating time t_h^+ is the time that the applied heat flux goes to zero. Prior to this time it is equal to the magnitude listed in column four (see Eq. (10)). Conditions in columns two to four are prescribed in the analysis; the remaining information in the table is data resulting from the analysis. The resulting maximum

Table 2: Optimum conditions to estimate linearly varying temperature-dependent thermal properties

x_0 (cm)	Sensor Locations x/x_0	Heating		$\dfrac{T_{max}-T_1}{T_2-T_1}$ (°C)	Max Δ^+	Optimal time $t_f^+(\Delta^+)$
		t_h^+	$\dot{q}_0'' x_0/k_1$ (°C)			
Semi-infinite Body						
2.54	0, 1.0	1.68	120	1.0	4.81E-10	1.86
1.27	0, 1.0	1.68	120	0.99	4.45E-10	1.84
3.81	0, 1.0	1.68	115	0.98	4.87 E-10	1.89
Finite Body						
2.54	0, 0.5	2.79	156	0.98	8.23 E-09	3.87
2.54	0, 0.25	2.33	156	0.97	7.91 E-09	3.55
2.54	0, 0.75	2.79	156	0.98	3.95 E-09	3.87
2.54	0, 0.5	2.79	71.9	0.51	1.73 E-09	3.74

temperature of the experiment is listed in column five. The maximum of the optimality criterion and the dimensionless time when the maximum is achieved are listed in the remaining columns.

Because temperature variable properties depend on $(T-T_1)/(T_2-T_1)$, we cannot completely remove the dependence on the sensor location by nondimensionalizing. Consequently, three combinations of sensor locations are selected and the optimum results are shown in Table 2. All have one sensor located at the surface of the applied heat flux, $x = 0$. The second sensor is located internal to the body at x_0. A sensor closer to the applied heat flux would respond at an earlier time and require a shorter experiment than a sensor farther away. However, when time is made dimensionless using the distance of the sensor from the heated surface (x_0) there is only a small change in the optimal time; optimal experiment duration is $t_f^+ = 1.84, 1.86,$ and 1.89 for internal sensors located at

$x_0 = 1.27, 2.54,$ and $3.81 cm$, respectively. Furthermore, the magnitude of Δ^+ is not sensitive to the sensor location; moving the in-depth sensor from a location of $x_0 = 2.54 cm$ to 1.27 or 3.81cm changes Δ^+ -7.0 and +1.2 percent, respectively.

Finite Case

The transient variation of Δ^+ is shown in Figure 5 for sensors located at the surface of the applied heat flux and at one-half the thickness of a finite body. The heating magnitude is the same for all cases shown because the temperature field essentially approaches steady-state. The maximum temperature for all heating durations is approximately the same and comparable to the range of the temperature for the property variation; $(T_{max}-T_1)/(T_2-T_1)$ is near unity.

We can see a more pronounced difference between an experiment that heats continuously for a time of 5.59 with experiments that heat for a shorter duration but stop the heat flux prior to the end of the experiment. Comparing the curve for a heating duration of 5.59 with the curves using durations from 1.86 to 3.73 demonstrates the significance of continuing the experiment after ending the heat flux $(t_f^+ > t_h^+)$. We get over a 600 percent increase in Δ^+ by terminating the applied heat flux but continuing the experiment. The reason for this significant increase is that the sensitivity coefficients dramatically change shape after the heat flux ends; see Figure 3. We note that the increase in Δ^+ after heat flux ends is greater for the finite case (Figure 5) than it is for the semi-infinite (Figure 4) case. The sensitivity coefficients have a more pronounced change in shape for the finite case.

The optimum conditions for a finite geometry are listed in the bottom of Table 2. The characteristic dimension for the finite body is taken as the thickness of the one dimensional body $x_0 = L$. A sensor is located at the surface of the applied heat flux, $x^+ = 0$, and internal to the body, $x^+ > 0$. Three locations
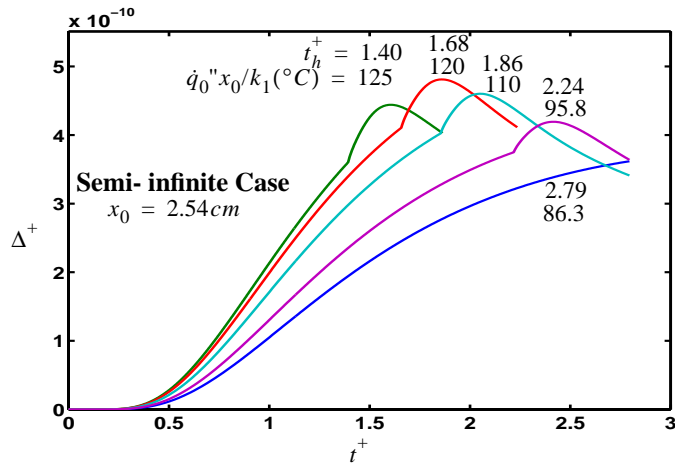
Figure 4 Optimality criterion for estimating linearly varying thermal properties for the semi-infinite case. Sensors are located at $(x = 0, x_0)$
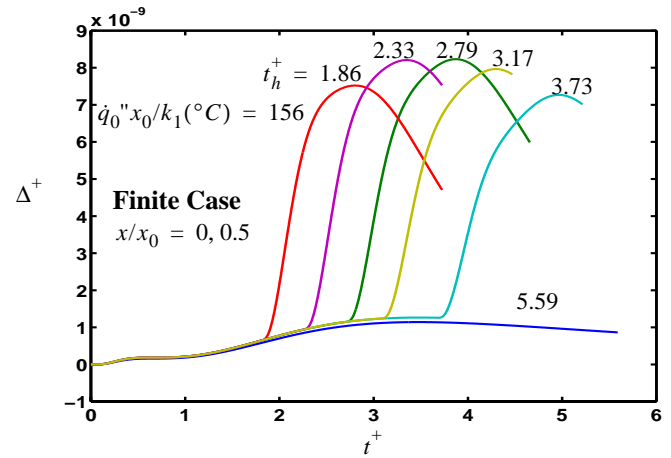
Figure 5 Optimality criterion for estimating linearly varying thermal properties for the finite case

are investigated for the internal sensor: $x/x_0 = 0.25, 0.5, 0.75$.

Moving the internal sensor closer to the heated surface does not significantly change Δ^+. Its value decreases about 4 percent when the sensor is moved from $x/x_0 = 0.5$ to 0.25. We note that the optimal time listed in Table 2 is different for these sensor locations. The maximums for sensors located at $(x/x_0 = 0, 0.5)$ in Figure 5 is quite broad, however. Similar magnitudes of Δ^+ are obtained for heating durations between $t_h^+ = 2.33$ and $t_h^+ = 2.79$ for $(x/L = 0, 0.5)$ with the optimal time being $t_f^+ \approx t_h^+ + 1.05$. With a broad maximum in Δ^+, the slight change in the optimum for sensors located at $x/x_0 = 0, 0.25$ is not significant.

Moving the internal sensor farther away from the heated surface reduces the magnitude of Δ^+ in Table 2. It decreases over 50 percent when the internal sensor is moved from $x/L = 0.5$ to 0.75. The optimum heating time does not change, however. This decrease in Δ^+ is caused by moving the sensor closer to the isothermal boundary where the sensitivity is zero for all parameters. Less information is available from the internal sensor in this case.

As stated earlier, the thermal properties depend on the parameter groups given in Eq. (15) and one of these group depends on the temperature change of the experiment $(T - T_1)/(T_2 - T_1)$. Consequently, the optimal conditions depend on the magnitude of the applied heat flux in addition to the heating and experimental duration. Studying the effect of the normalized temperature indicated that $(T_{max} - T_1)/(T_2 - T_1)$, should be near unity; most cases presented used value. For comparison, the value of this group is reduced to 0.51 for sensors located at $x/x_0 = 0, 0.5$. The results are shown in the bottom of Table 2. Although the magnitude of Δ^+ is considerably smaller than when the parameter group is 0.98, the conditions where the maximum occurs are similar. Consequently, Δ^+ increases as $(T_{max} - T_1)/(T_2 - T_1)$ increases. In other words, selecting heating magnitudes that result in the temperature covering the entire temperature range of the properties produces larger magnitudes of Δ^+ and is desirable. However, the optimal heating times are similar for heating magnitudes that cover different temperature ranges.

Summary. If we compare the magnitudes of Δ^+ for the semi-infinite and finite case in Table 2 we can conclude that the finite case would provide estimates with a smaller volume confidence region. The ratio of Δ^+ for the finite and semi-infinite cases is nearly 20. Furthermore, selecting heating magnitudes so that the temperature covers the entire temperature range of the properties $(T_{max} - T_1)/(T_2 - T_1) \to 1$ produces larger magnitudes of Δ^+. These conclusions are based on linear property variation such that $(k_2 - k_1)/k_1 = 0.66$ and $(C_2 - C_1)/C_1 = 1.2$. We believe that the outcomes (finite body is better than semi-infinite and larger temperature variation is better) will not change for other magnitudes of the parameter variation. The specific optimum conditions would be expected to change for different magnitudes of the parameter variation.

Because results were presented in dimensionless form, the specific magnitudes of the properties and dimensions of the geometry are not important. The laboratory experimental conditions can be selected by computing the dimensional values of the durations (heating and experiment) and heating magnitude. Using the groups in Table 2, the anticipated properties of the material, characteristic size of the geometry, dimensional experimental conditions can be calculated. Of course this requires we know the properties of the material, which we do not. Hence some iteration is require in this process. The point is that the optimum conditions derived are applicable for high, as well as low, conductivity materials. The restriction is that property variation with temperature is such that $(k_2 - k_1)/k_1 = 0.66$ and $(C_2 - C_1)/C_1 = 1.2$.

The outcome and optimum conditions obtained for temperature dependent properties are consistent with those for constant properties. The finite case is better than the semi-infinite for constant properties as well. Furthermore, heating and experiment duration for constant properties in Table 1 are not significantly different from those for temperature variable properties in Table 2. In general, the durations are longer for temperature variable properties.

The fact that an experiment should cover the entire temperature range of the properties has implications concerning the use of a sequential analysis approach. One may decide to conduct several experiments, each addressing a portion of the temperature range of the properties, and combine the experiments during the analysis. Based on the results of this study, such an approach would not be as good as covering the entire temperature range. This outcome, however, may depend on how the temperature dependence of properties are described. If higher order functions represent the temperature dependence the sensitivities may be correlated. In such a case a sequential analysis of experiments covering a portion of the temperature range may be better.

CONCLUSIONS

A study of conditions to optimize transient experiments for estimating linearly varying temperature dependent thermal properties was presented. Four parameters describing linearly varying thermal conductivity and volumetric heat capacity were to be estimated. Thermal property variation was assumed to be $(k_2 - k_1)/k_1 = 0.66$ and $(C_2 - C_1)/C_1 = 1.2$. An optimality criterion was presented and optimal conditions, including sensor location, heating duration, and heating magnitude (or temperature range), were derived for a one dimensional finite and semi-infinite body.

The finite case produced larger values of D-optimality than the semi-infinite case when estimating all four parameters simultaneously; the ratio was nearly 20 for the finite and semi-infinite cases. Sensors were located at the surface of the applied

heat flux and below the heated surface. The semi-infinite case was not sensitive to the location of the sensor below the heated surface. In the finite case, optimal conditions were comparable for sensors located below the heated surface at one-half and one-fourth the specimen thickness; a sensor located at three-quarters of the specimen thickness was less optimal. For both the finite and semi-infinite cases the optimality criterion increases as the temperature range of the experiment increases relative to the temperature range of the property variation.

ACKNOWLEDGMENTS

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