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# HT03

# NUMERICAL AND EXPERIMENTAL SIMULATION FOR CUTTING TEMPERATURE ESTIMATION USING THREE-DIMENSIONAL INVERSE HEAT CONDUCTION TECHNIQUE

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## ABSTRACT

In this work the heat flux generated in a cutting process is simulated using the inverse heat conduction technique based on conjugate gradient method. A three-dimensional formulation is used both to describe the physical phenomenon and to solve the inverse problem. The machining process (turning) is simulated and instrumented with nine thermocouples at the bottom face of the tool, opposite to its main rake face. The signals are automatically received and processed using a data acquisition system and a PC-Pentium. The direct solution, adjoint equation and sensitivity problem are numerically solved using finite volumes method. With this technique the cutting temperatures are estimated for various simulated cutting conditions. In order to validate the method the IHCP is applied in a well-controlled experiment where the heat flux input is known. An uncertainty analysis is also presented.

## INTRODUCTION

During machining, high temperatures are generated in the region of the tool cutting edge, and these temperatures have a controlling influence on the rate of wear of the cutting tool and on the friction between the chip and the tool (Trent, 1984). The use of efficient cooling methods may reduce wear and increase tool life. However, direct measurement of temperature by using contact type sensors at the tool-work contact surface is difficult to implement due to the rotating movement of the workpiece and the presence of the chip. Conventional methods such as of infrared pyrometer (Lin et al., 1992), embedded thermocouple (Lin, 1995) and tool-work thermocouple (Eu-Gene, 1995 and Trent, 1984) usually present problems. The infrared pyrometer can represent a good solution since some limitations, like resolution of the sensor and interference of the chip near the cutting zone, are alleviated (Lin et al, 1990). Some tool materials such as ceramics, the high brittleness and electric resistance usually make it difficult to implement contact type sensor and impair the use of tool-workpiece thermocouples for measuring temperatures at the chip-tool interface. In addition, the tool-workpiece thermocouple does not measure the temperature at a specific point, but average temperature at the heat affected zone between the tool and workpieces. Therefore, the use of inverse heat conduction techniques can be a good alternative since this technique takes into account temperatures measured from accessible positions, e.g., the surface opposite to the rake surface of the tool.

The success of any experimental technique depends on the physical model used. In this case, in spite of the application of one-dimensional model in ellipsoidal coordinates (Alves et al., 1998), a three dimensional model is more appropriated to describe heat transfer in machining.

Inverse problem techniques are very well established for one-dimensional case as presented by the sequential function specified method presented by Beck et al. (1985), and the gradient conjugated method presented by Özisik (1993). Recently, an increase interest in appearing in the solution of multidimensional problems is observed, as the two-dimensional thermal problems presented by Alifanov et al. (1985), Beck, et al. (1985), Alencar Jr. et al. (1997) and Lima and Guimarães (1998), among others, and the three-dimensional problems presented by Jarny et al. (1991) using the conjugated gradient method and Colaço and Orlande (1998).

One of the inherent difficulties of the inverse techniques, besides the great sensibility to measurement errors, is the establishment of ideal condition for the variables involved such as measurement time, experiment time and spatial dimensions. In practical conditions the possibilities of controlling these variables are reduced. This fact turns the result analysis extremely important. This work represents an initial step in the direction of solving a three dimensional thermal problem in machining. That is, an immediately previous step to determining the cutting temperature in a real machining process. The cutting tool, its geometry and the zone of heat generation are simulated here. The heating and measurement times are chosen in a way to reproduce conditions similar a machining process. A practical experiment, in conditions similar to the simulated problem is carried out for validation of the present methodology. The gradient conjugated method applied to three-dimensional problem presented in a generic form by Jarny et al. (1991) is used here to derive the model of the machining process. This model is an unknown heat flux imposed through its frontal surface sample. The sample is also subject to the heat convection on its remaining faces. This procedure drives to an analysis of the physical limitations of the inverse methodology and to the study of the design parameters such as the sample cutting tool thickness, time of heat diffusion, and the thermal diffusivity of the tool. The knowledge a priori of the combination of those parameters is fundamental for the analysis of the confidence of results, since the heat flux generated have unknown behavior and magnitude.

## NOMENCLATURE

| | | |
|---|---|---|
| $a,b,c$ | = | Geometric parameters of the sample |
| $k$ | = | Thermal conductivity |
| $h_{1,\ldots,6}$ | = | Mean surface heat transfer |
| $q$ | = | Heat flux |
| $t$ | = | Time |
| $t_f$ | = | Total experiment time |
| $x,y,z$ | = | Plane coordinates |
| $J$ | = | Functional, Eq. (19) |
| $J'$ | = | Gradient of the functional, Eq. (29) |
| $P$ | = | Direction of descent, Eq. (33) |
| $T$ | = | Temperature |
| $Y$ | = | Measured temperature |

### Greck Letters

| | | |
|---|---|---|
| $\alpha$ | = | Thermal diffusivity |
| $\beta$ | = | Search step size, Eq. (31) |
| $\varepsilon$ | = | Standard deviation of measurement errors |
| $\gamma$ | = | Conjugation coefficient, Eq. (34) |
| $\lambda$ | = | Lagrange Multiplier |
| $\tau$ | = | New time coordinate |
| $\xi$ | = | Uncertainty |
| $\zeta$ | = | Relative small number |
| $\Delta F_0$ | = | Fourier number |
| $\Delta T$ | = | Sensitivity function, Eqs. (10-18) |

## NUMERICAL FORMULATION

The general procedure described by Jarny et al. (1991) for a three dimensional generic problem for both heat flux and parameter estimation is used here to derive the specific problem similar to those appear in machining processes.

### Direct Problem

The model of heat flux in a cutting tool is presented in the Fig. 1. The physical problem is a three dimensional transient heat conduction that occurs in a sample (simulated tool) due to an unknown heat flux imposed at one of its upper edge. All sample faces are subjected to heat loss by the heat transfer coefficient, $h_{1\text{-}6}$ = 20 W/m² K.

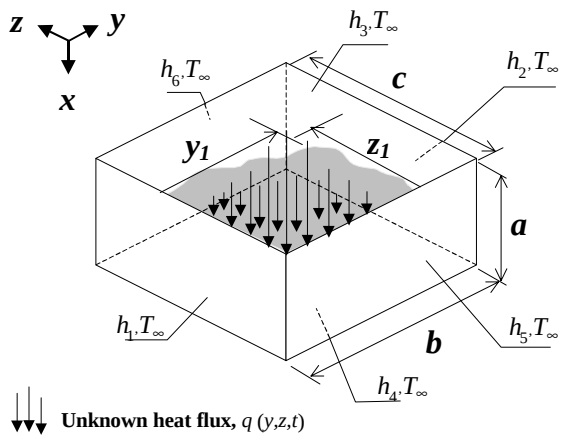


**Figure 1: Three dimensional simulation of heat flux in a cutting tool.**

The heat conduction problem for this sample can be described by

$$\frac{\partial^2 T(x,y,z,t)}{\partial x^2}+\frac{\partial^2 T(x,y,z,t)}{\partial y^2}+\frac{\partial^2 T(x,y,z,t)}{\partial z^2}=\frac{1}{\alpha}\frac{\partial T(x,y,z,t)}{\partial t}$$
$$\text{at } 0<x<a,\ 0<y<b,\ 0<z<c,\ t>0 \qquad (1)$$

$$-k\frac{\partial T(0,y,z,t)}{\partial x}=q(y,z,t) \text{ at } 0\le y\le y_1,\ 0\le z\le z_1 \qquad (2)$$

$$-k\frac{\partial T(0,y,z,t)}{\partial x}=h_3\left[T_\infty-T(0,y,z,t)\right] \text{ at } z_1<z<c,\ y_1<y<b \qquad (3)$$

$$-k\frac{\partial T(a,y,z,t)}{\partial x}=h_4\left[T(a,y,z,t)-T_\infty\right] \qquad (4)$$

$$-k\frac{\partial T(x,0,z,t)}{\partial y}=h_1\left[T_\infty-T(x,0,z,t)\right] \qquad (5)$$

$$-k\frac{\partial T(x,b,z,t)}{\partial y}=h_2\left[T(x,b,z,t)-T_\infty\right] \quad (6)$$

$$-k\frac{\partial T(x,y,0,t)}{\partial z}=h_5\left[T_\infty-T(x,y,0,t)\right] \quad (7)$$

$$-k\frac{\partial T(x,y,c,t)}{\partial y}=h_6\left[T(x,y,c,t)-T_\infty\right] \quad (8)$$

$$T(x,y,z,0)=T_0 \text{ for } t=0\text{, in the region} \quad (9)$$

If the value of the heat flux, $q(y,z,t)$, is known, the Eqs. 1-9 represents the direct problem related to the inverse problem studied.

## Sensitivity Problem

The sensitivity problem is obtained assuming that when $q(y,z,t)$ undergoes an increment $\Delta q(y,z,t)$, the temperature $T(x,y,z,t)$ changes by an amount $\Delta T(x,y,z,t)$ (Ozisik, 1993). Then, to get the sensitivity problem that satisfies the function $\Delta T(x,y,z,t)$, $T(x,y,z,t)$ is replaced by $T(x,y,z,t) + \Delta T(x,y,z,t)$ and $q(y,z,t)$ by $q(y,z,t) + \Delta q(y,z,t)$ in the direct problem (Eqs.1-9) and subtract form the original problem. The sensitivity problem, is then, obtained

$$\frac{\partial^2\Delta T(x,y,z,t)}{\partial x^2}+\frac{\partial^2\Delta T(x,y,z,t)}{\partial y^2}+\frac{\partial^2\Delta T(x,y,z,t)}{\partial z^2}=\frac{1}{\alpha}\frac{\partial\Delta T(x,y,z,t)}{\partial t} \text{ at } 0<x<a,\,0<y<b,\,0<z<c,\,t>0 \quad (10)$$

$$-k\frac{\partial\Delta T(0,y,z,t)}{\partial x}=\Delta q(y,z,t) \text{ at } 0\le y\le y_1,\,0\le z<z_1 \quad (11)$$

$$k\frac{\partial\Delta T(0,y,z,t)}{\partial x}=h_3\Delta T(0,y,z,t) \text{ at } y_1<y<b,\,z_1<z<c \quad (12)$$

$$-k\frac{\partial\Delta T(a,y,z,t)}{\partial x}=h_4\Delta T(a,y,z,t) \quad (13)$$

$$k\frac{\partial\Delta T(x,0,z,t)}{\partial y}=h_1\Delta T(x,0,z,t) \quad (14)$$

$$-k\frac{\partial\Delta T(x,b,z,t)}{\partial y}=h_2\Delta T(x,b,z,t) \quad (15)$$

$$k\frac{\partial\Delta T(x,y,0,t)}{\partial z}=h_5\Delta T(x,y,0,t) \quad (16)$$

$$-k\frac{\partial\Delta T(x,y,c,t)}{\partial z}=h_6\Delta T(x,y,c,t) \quad (17)$$

$$\Delta T(x,y,z,0)=0 \text{ for } t=0\text{, in the region} \quad (18)$$

## Adjoint Problem

The inverse heat conduction problem is solved as an optimum control problem of finding the unknown control function $q(y,z,t)$ (Jarny et al., 1991),

$$J(q)=\int_{t=0}^{t=t_f}\int_{y=0}^{y=b}\int_{z=0}^{z=c}\left[T(a,y,z,t,q)-Y(a,y,z,t)\right]^2 dz\,dy\,dt \quad (19)$$

where $T(a,y,z,t)$ is the computed temperature at $x = a$ from the solution of the direct problem, Eqs. (1-9), and $Y(a,y,z,t)$ is the measured temperature at $x = a$.

The adjoint problem is obtained by introducing a new function $\lambda(x,y,z,t)$, called the Lagrange multiplier (Alifanov, 1974). The Eqs. 1-9 is then multiply by $\lambda(x,y,z,t)$and the resulting expression is integrated over the spatial domain and then over the time domain. This mathematics manipulation leaves to the adjoint problem

$$\frac{\partial^2\lambda(x,y,z,t)}{\partial x^2}+\frac{\partial^2\lambda(x,y,z,t)}{\partial y^2}+\frac{\partial^2\lambda(x,y,z,t)}{\partial z^2}=-\frac{1}{\alpha}\frac{\partial\lambda(x,y,z,t)}{\partial t} \text{ at } 0<x<a,\,0<y<b,\,0<z<c \quad (20)$$

$$\frac{\partial\lambda(0,y,z,t)}{\partial x}=0 \quad \text{at } 0\le y\le y_1,\,0\le z\le z_1 \quad (21)$$

$$\frac{\partial\lambda(0,y,z,t)}{\partial x}=\frac{h_3}{k}\lambda(0,y,z,t) \text{ at } y_1<y<b,\,z_1<z<c \quad (22)$$

$$\frac{\partial\lambda(a,y,z,t)}{\partial x}=-\frac{h_4}{k}\lambda(a,y,z,t)+2\left[T(a,y,z,t,q)-Y(a,y,z,t)\right] \quad (23)$$

$$\frac{\partial\lambda(x,0,z,t)}{\partial y}=\frac{h_1}{k}\lambda(x,0,z,t) \quad (24)$$

$$\frac{\partial\lambda(x,b,z,t)}{\partial y}=-\frac{h_2}{k}\lambda(x,b,z,t) \quad (25)$$

$$\frac{\partial\lambda(x,y,0,t)}{\partial z}=\frac{h_5}{k}\lambda(x,y,0,t) \quad (26)$$

$$\frac{\partial\lambda(x,y,c,t)}{\partial z}=-\frac{h_6}{k}\lambda(x,y,c,t) \quad (27)$$

$$\lambda(x,y,z,t_f)=0 \text{ for } t=t_f\text{, in the region} \quad (28)$$

## Gradient Equation

As in the procedure of Alifanov (1974) the gradient equation for the functional $J(q)$ can be given by

$$J'(y,z,t)=\lambda(0,y,z,t) \quad (29)$$

## Conjugate Gradient Method of Minimization

The unknown function $q(y,z,t)$ can be determined by a process based on the minimization of the functional $J(q)$. If the conjugated gradient method is used (Alifanov, 1974) the recurrence equation for the determination of $q(y,z,t)$ can be given by

$$q^{n+1}(y,z,t)=q^n(y,z,t)-\beta^n P^n(y,z,t) \quad n=0,1,2,\ldots \quad (30)$$

where the $\beta^n$ is the step size given by

$$\beta^n = \frac{\int_0^{t_f}\int_0^{z_1}\int_0^{y_1} \left[T(L,y,z,t,q) - Y(L,y,z,t)\right]\Delta T(L,y,z,t)\, dy\, dz\, dt}{\int_0^{t_f}\int_0^{z_1}\int_0^{y_1} \left[\Delta T(L,y,z,t)\right]^2 dy\, dz\, dt} \tag{31}$$

and $P^n(y,z,t)$, the direction of descendent, given by

$$P^0(y,z,t) = J'^0 \tag{32}$$

$$P^n(y,z,t) = J'^n + \gamma^n P^{n-1}(y,z,t) \tag{33}$$

and the conjugate coefficient given by

$$\gamma^n = \frac{\int_{t=0}^{t=t_f}\int_{z=0}^{z=z_1}\int_{y=0}^{y=y_1} \left[J'^n(y,z,t)\right]^2 dy\, dz\, dt}{\int_{t=0}^{t=t_f}\int_{z=0}^{z=z_1}\int_{y=0}^{y=y_1} \left[J'^{n-1}(y,z,t)\right]^2 dy\, dz\, dt} \tag{34}$$

## Stopping Criterion

The stopping criterion is taken using a small specified number, $\zeta$, as

$$J(q) < \zeta \tag{35}$$

## Computational Algorithm

To start the iterations an initial estimation is made from the function $q(y,z,t)^*$, which may be chosen as a constant, or zero. The direct problem is solved and $T(x,y,z,t)$ is computed based on $q(y,z,t)^*$. The adjoint problem is then solved. The value of $\lambda(0,y,z,t)$ computed is used to calculate the $J'(y,z,t)$, that for its time is used to calculate $\gamma^n$ and $P^n(y,z,t)$. Then the sensitivity problem can be solved setting $\Delta q(y,z,t) = P^n(y,z,t)$. The resulting $\Delta T(a,y,z,t)$ is used to calculate the step size, $\beta^n$, from Eq. (31). The value of $\beta^n$ allows to calculate $q^{n+1}(y,z,t)$ and the new value of $T(a,y,z,t,q)$. The last step is to check the stopping criterion. This procedure is general for any different functional as a thermal property like thermal diffusivity or conductivity or an unknown source term that can be presented in a heat conduction problem. It can also be found in detail for one-dimensional unknown source term in the work by Ozisik (1993) or for a general three dimensional problem in the work by Jarny et al. (1991).

## SIMULATION OF THE THERMAL PROBLEM

Figure 2 shows a sample that simulates a tool used in machining. The dimensions used are normally found in an orthogonal insert. Two materials are considered in this simulation. For both, cemented carbide (WC + Co) and ceramic ($Si_3N_4$) the dimensions are $a$ = 0,0049m, $b$ = 0.0127m and $c$ = 0.0127m.

Table 1 presents the thermal properties used. They are considered constant and independent of the temperature. The initial, $T_0$, and room temperature, $T_\infty$, are equal to 300K. The heat transfer coefficient is also considered constant and equal to 20 W/m² K in the reminiscent faces.

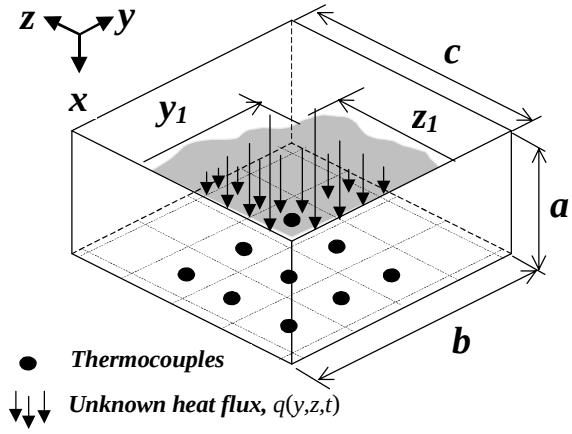


**Figure 2: Thermal problem.**

**Table 1: Samples thermal properties (Trent, 1984).**

| Sample Material | Thermal conductivity (W/m K) | Thermal diffusivity (m²/s) |
|---|---|---|
| ceramic ($Si_3N_4$) | 25.0 | $7.2\times10^{-06}$ |
| cemented carbide | 100.0 | $2.7\times10^{-05}$ |

Nine temperature history are simulated to be measured at $x = a$ as shown in the Fig. 3.

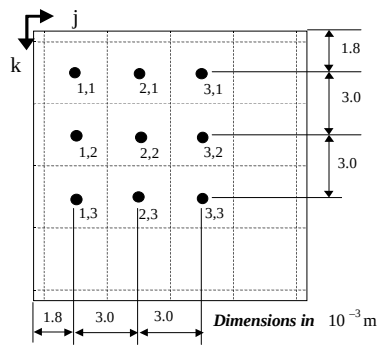


**Figure 3: Identification of experimental temperature *Y(a,y,z,t)* at opposite surface *x = a*.**

At $x = 0$, an unknown heat flux $q(y,z,t)$ with spatial and temporal variation is imposed. The value of the heat flux simulated is of the same order of magnitude of those that occur in a metal cutting process. Once known the heat flux value, the Eqs. (1-9) can then be solved (direct problem). In this case, the inverse problem is applied through the computational algorithm already described, using the measured temperature at $x = a$. The procedure used here simulates these measured temperature introducing random errors $\varepsilon$ to the exact temperatures as

$$Y(y,z,t) = T_{\text{exact}}(a,y,z,t) + \varepsilon \tag{36}$$

where the exact temperature $T_{exact}$ is determined from the solution of the direct problem, Eqs. (1-9), by using the exact values of the heat flux imposed. The value of $\varepsilon$ is in within $\pm$ 1K.

The sensitivity, the direct and the adjoint problem are solved by numerical algorithms based on the finite volumes in Visual C++, (Patankar, 1991). The numerical integrations are performed using the Simpson method, (Press et al., 1989).

## Sensitivity Analysis-1D Test Case

An analysis of some physical characteristics should be done before the application of the inverse method. Some design parameters $a$, $t_f$ and $\alpha$ should then be studied in order to give more confidence to the results. One relation of these parameters, the Fourier number, can be defined by

$$\Delta F_0 = \frac{\alpha t_f}{a^2} \tag{37}$$

This dimensionless number gives the diffusion time and represents one good indicator to the sensibility of the measured temperature at the opposite surface in relation to any changes in the heat flux imposed at frontal surface of the tool. To verify its influence an one-dimensional case is simulated. Due to its simplicity the results can be shown with more clarity.

The geometry of the samples is the same to that presented in Fig. 3, while the heat flux is imposed at the intire frontal surface. In addition, a sample of stainless steel is also tested and analyzed.

One characteristic of the conjugate gradient method is the zero value of the heat flux at the final time. This fact is due to the problem of final value represented by the adjoint problem, Eqs. (20-28). Therefore, the last component of the heat flux estimated should not be considered. In order to avoid this problem the component of the heat flux is considered only during 70% of the value correspondent to total time. Since, the inverse problem has a great sensitivity to measurement errors, the estimation in this analysis is considered successfully if the value estimated is closed to the value imposed with a uncertainty by difference between estimated and calculated values, $\xi$, of 8 %.

This tolerance is acceptable considering the practical results from metal cutting processes. Figure 4 presents the behavior of the uncertainty, $\xi$, against $\Delta F_0$ for several materials. Initially, for small values of $\Delta F_0$ the uncertainty, $\xi$, is high. It decreases up to $\Delta F_0$ = 3.5and becomes practically constant (at about 8%) for higher of $\Delta F_0$.

Table 2 presents various tests considering different experiment time ($t_f$), sample interval $\Delta T$ and thickness, $a$, for a sample of stainless steel AISI 304.

The experimental test in one-dimensional case will give some directions for the three dimensional case to be shown in the next section.

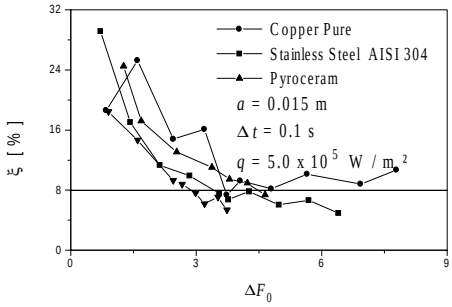


**Figure 4: Uncertainty between calculated and estimated heat fluxes for different materials.**

**Table 2: Parameters *a*, $t_f$ e $\alpha$ used for testing 1D.**

| Stainless steel AISI 304 | | $\alpha$= 4.0×10$^{-6}$ m² /s | $k$ =15.1W/m °C |
|---|---|---|---|
| test | $a$ (m) | $\Delta T$ (s) | $t_f$ (s) |
| A | 0.015 | 0.1 | 40.0 to 360.0 |
| B | 0.015 | 0.5 | 40.0 to 320.0 |
| C | 0.011 to 0.030 | 0.5 | 212.0 |

Figure 5 presents the results for tests A, B and C. The behavior of the curves are very similar to those presented in Fig. 4 for different (cemented metal and ceramic). It can be concluded that the way of changing of $\Delta F_0$ does not affect the behavior of $\xi$. Note that $t_f$, $\Delta T$ and $a$ are changed while $\alpha$ is kept constant (tests A,B, C). It suggests that Fourier Number represents an important rule for success of the inverse problem using the gradient conjugated and that for $\Delta F_0$ smaller than 3.5 poor results can be obtained.

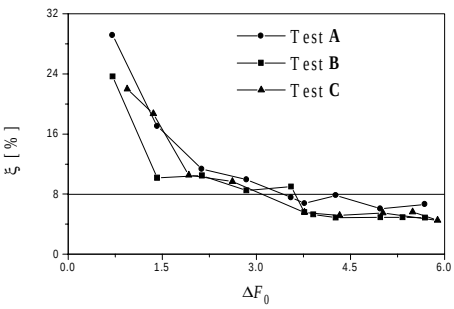


**Figure 5: Percent difference between calculated and estimated heat fluxes for different parameters *a*, $t_f$ e $\alpha$.**

To check this behavior experimentally an one-dimensional test for a 50 x 50 mm$^2$ sample of AISI 304 stainless steel with thickness of 9 mm was carried out. The heat flux imposed is generated and measured by a transducer/electrical resistance and compared with the estimated component. The tests for two different values of $\Delta F_0$ 1.2 and 3.2 are shown in Figs. 6 and 7 respectively. The heating time and total experiment time in test A were 30s and 55s respectively. In this case a great deviation among the estimated and measured values is observed. However, once the value proposed for $\Delta F_0$ is of the order of 3.0 the experiment was modified to attend it. In the test b, the

heating time and the total time of the experiment were altered respectively for 80s and 105 s.

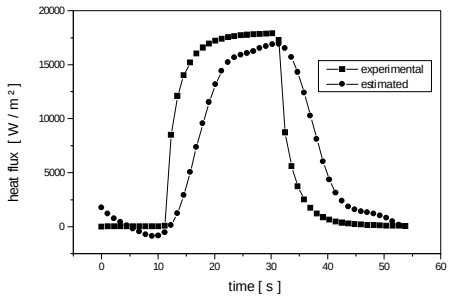


**Figure 6: Experimental test, 1D, $\Delta F_0$ = 1.2.**

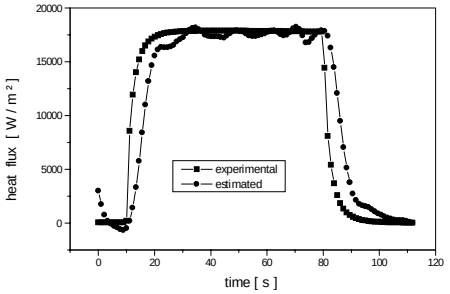


**Figure 7: Experimental test,1D, $\Delta F_0$ = 3.2.**

It is observed in the Fig. 7 a reasonable improvement between the estimated and measured of the heat fluxes. Thus, as verified in the Figs. 4-7, the number of Fourier is really an important parameter for good estimations and the value of $\Delta F_0$ = 3.0 a good reference value for three-dimensional. This test will be considered in the next section.

## RESULTS AND DISCUSSIONS – 3D ANALYSIS

Figures 8 to 10 show the simulated results for a ceramic. In these illustrations, the real and estimated evolutions of the heat flux can be compared for different positions along the direction *y* (Fig. 2). The results are presented in this way to facilitate the visualization of the temporary and the space behavior in discrete plans of the simulated tool. The Fig. 8, presents the temporary variation and the space variation along the axis *y* for the position *z = 1* that can be identified in the Fig. 3. A good agreement is verified among the real and estimated values for positions at *y* = 1 and *y* = 2. For the position *y* = 3 the results presents a slight discrepancy in relation to the real curve of heat flux. This discrepancy can be attributed to position its below the interface among the area that is subjected to heat flux and the face that is subjected to the convection heat loss, Fig. (3) position (1,3). Another fact that can deviate the estimated results is the necessary minimum time of diffusion in the directions *y* and *z*.

It can be seen in the Fig. 9, position *z* = 2, that the results are overestimated for the position *y* = 1 and *y* = 2, and it presents a good agreement for *y* = 3. In this case, it can be considered that the experimental temperatures in the positions (1,2) and (2,2) have a great influence of the components of heat flux in the adjacent positions. For the position (3,2) as the position of the thermocouple is near to an area without components of heat flux in a normal direction, the diffusion of heat in the directions *y* and *z* does not affect the values. Therefore, these results can be overestimated or subestimated depending on the characteristic of the minimization method. The conjugated coefficient, $\gamma$ and step size, $\beta$ are calculated by triple integrals or represent average values. This effect can then appear in multidimensional cases since each direction can affect this coefficient in different ways. If this happens they will be discarded when calculating the average value. An analogous results can be observed in the Fig. 10 for position *z* = 3.

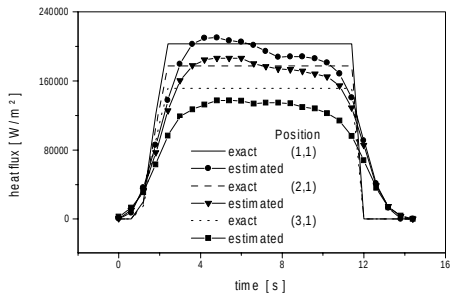


**Figure 8: Results for the ceramic sample in *z* = 2.**

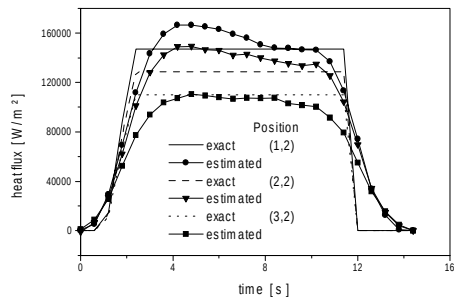


**Figure 9: Results for ceramic sample in *z* = 2.**

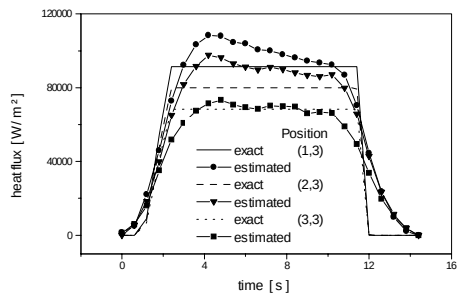


**Figure 10: Results for the ceramic sample in *z* = 3.**

The results for the ceramic tool are also presented by the uncertainty between the calculated and estimated temperature at *x* = 0 as shown in Fig. 11. In all positions, the same behavior was verified but position *y* = 3 showed smaller percent variation. It can also be observed that the peak of deviation is near the time region that there is no changing in the behavior of the heat flux. Once the heat flux becomes constant the error

tends to decrease. The same experiment and heating time were also used for a cemented carbide sample. Figures 12 and 13 present the results.

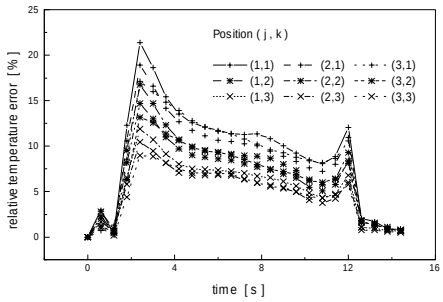


**Figure 11: Uncertainty variation between exact and measured temperatures at *x* = 0 for ceramic.**

Figure 12 shows that the same conclusions can be obtained in relation to the position of the component of heat flux. However the results are worse than those presented by the ceramic sample. The main difference is the Fourier number that are equal to 13.5 for the cemented carbide while for the ceramic sample it was approximately 3.0. While the influence of time diffusion in the *y* and *z* directions can affect the results as previously mentioned the higher conductivity of the cemented carbide can increase this problem. In order to minimize this the Fourier number was diminished up to 3.5 by decreasing the total experiment time. The results for position $z = 2$ are shown in the Fig. 13. A good agreement can be observed between the real and estimated heat fluxes. The position $z = 1$ and $z = 3$ were omitted here but they have presented similar results.

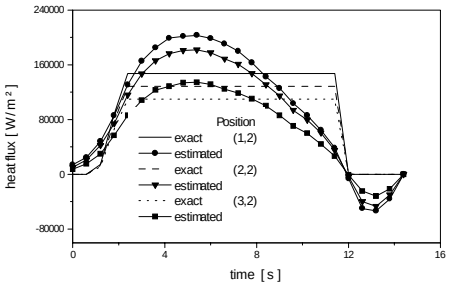


**Figure 12: Heat flux for the cemented carbide sample in *z* = 2, $\Delta F_0$ = 13.5.**

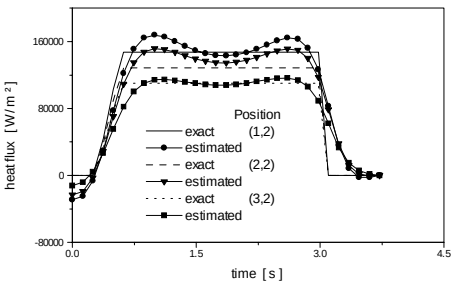


**Figure 13: Heat flux for the cemented carbide sample in *z = 2*, $\Delta F_0$ = 3.5.**

Figure 14 presents the uncertainty between the estimated and calculated temperature at $x = 0$ for the cemented carbide and $\Delta F_0 = 3.5$. The agreement in this case is better than for the ceramic tool. The same behavior is observed in the region of changing of the heat flux.

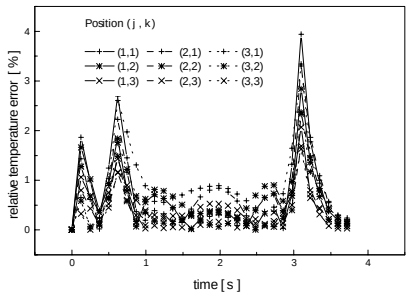


**Figure 14: Uncertainty variation between real and measured temperatures at *x* = 0 for cemented carbide sample, $\Delta F_0$ = 3.5.**

## EXPERIMENTAL TEST

An experimental test was carried out in order to analyze the algorithm efficiency. The Fig. 15 shows the apparatus. In this case, the dimensions *a*, *b*, *c*, $y_1$ and $z_1$ were established for a sample of 1020 steel and a double sensor - transducer/electrical resistance of heat flux. 0.1 x 0.1 x 0.001 $m^3$ were the dimensions of the 1020 steel sample (*a*, *b* and *c*) while the double sensor had lateral dimensions of 0.05 x 0.05 $m^2$ ($y_1 \times z_1$) and 0.001m of thickness. The double sensor is responsible for supplying and measuring the heat flux. The heat flux transducer is based on thermopiles with a time response smaller than 10 ms. The Fig. 15a shows the apparatus.

Nine thermocouples were brazed on the bottom surface of the sample opposite to the surface that receives the heat, at the points shown in Fig. 15b.

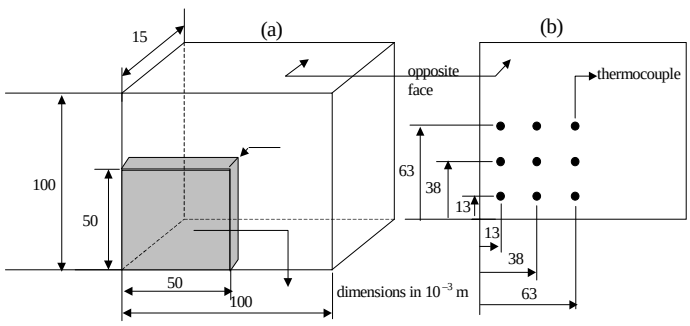


**Figure 15: Experimental apparatus: a) transducer/electrical resistance b) thermocouple attached.**

The Figs. 16 and 17 present the heat flux obtained for $\Delta F_0$ in the direction of 2.2 (test C) and 3.5 (test D) respectively.

It should be mentioned that the comparison among experimental and simulated results for the components of heat flux can only be made in the area covered by the double sensor,

transducer/electrical resistance of heat flux, as indicated in Fig. 2. In areas outside $y_1 \times z_1$ the components of heat flux were not measured therefore it can only be estimated. It is observed that, as found in the one-dimensional experimental test, the results are better for the test D, with the number of Fourier around 3.0.

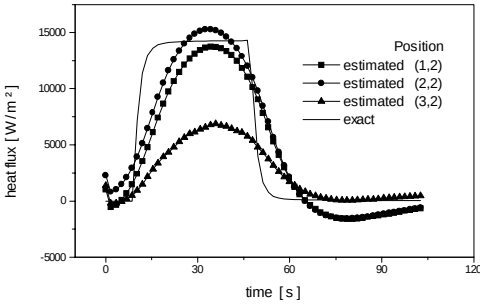


**Figure 16: Experimental test, 3D, $\Delta F_0$ = 2.22.**

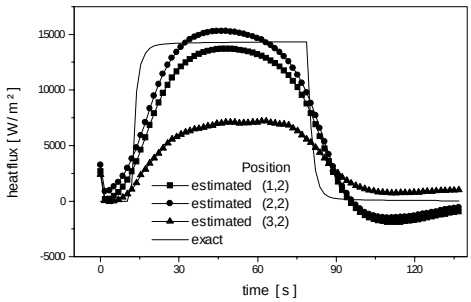


**Figure 17: Experimental test, 3D, $\Delta F_0$ = 3.55.**

## CONCLUSIONS

The conjugate gradient method with adjoint equation for solving inverse problems represents an encouraging alternative for application in three-dimensional problems. However, the analysis of the suitable time of diffusion for the number of Fourier is fundamental to be successful. In a real problem, such as in a machining process the control of the time is extremely difficult and parameters as the final and time of experiment and sampling intervals can strongly affect the heat flux estimation.

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