COMPUTATIONAL AND EXPERIMENTAL ESTIMATION OF BOUNDARY CONDITIONS FOR A FLAT SPECIMEN

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ABSTRACT

The objective of the proposed study is to analyze numerically and experimentally transient heating of a flat specimen. The analysis is based on the one-dimensional heat conduction model. The main problem of the analysis is to estimate the heat flux absorbed by a specimen. Temperature evolutions measured inside the specimen are used to solve the inverse heat conduction problem, two methods are used : the iterative regularization method (IRM) and the sequential function specification method (FSM). The finite difference method is utilized for solving the direct problem. These two methods are first verified and compared for one layer specimen by using simulated numerically data. The influence of the dimensionless time step (ΔFo number) is analyzed and comparison results are demonstrated. Then the methods are applied to analyze experimental data obtained with the use of a thermal cycling device.

NOMENCLATURE

C	specific heat
M	number of approximation parameters
NT	number of time step
N	number of temperature sensors
T , T_a , T_0	temperature, ambient, initial temperature
Q_i^j	sensitivity coefficient at time t_j and position X_i
e	thickness of the specimen
f_i	measured temperature at position X_i
h	heat transfer coefficient
t , t_f	time, final time
q , q_{exact}	heat flux density, exact heat flux
ψ	adjoint variable
ϑ	temperature variation
ρ	density
λ	thermal conductivity
α	thermal diffusivity
δ^2	estimated error or criterion
Δt	time step
ΔFo	dimensionless time step number
Δq	heat flux step
WIR	with iterative regularization

KEYWORDS

Inverse problem, regularization, heat flux, experimental analysis, numerical simulation, estimation.

INTRODUCTION

The ill-posed inverse problem of estimating the surface heat flux from transient temperature histories measured in a heat conducting solid is constantly of a great interest during three last decades. A literature review and a presentation of different methods is given, for example, in Tikhonov and Arsenin (1977), Beck et al. (1985), Hensel (1991), Murio (1993), Alifanov et al. (1995). Different applications of various methods are presented, in particular, in Zabaras et al. (1993) et Delaunay et al. (1996).

In this paper, results of an experimental and numerical analysis are reported, the goal of which is to estimate the heat flux absorbed by a flat specimen cooled at the back side and insulated at its lateral surface.

We use the iterative regularization method (IRM) (Alivanov et al., 1995) and the function specification method (FSM) (Beck et al., 1985) to solve this inverse heat conduction problem. The first numerical algorithm is based on the minimization of the residual functional which is the integrated difference between temperature histories measured and those calculated by solving the direct problem. The conjugate gradient method is used to solve the inverse problem. The residual functional gradient is computed by solving the adjoint problem and the optimal descent parameter is calculated by solving the problem for temperature variations. The heat flux evolution is approximated by cubic B-splines (Alivanov et al 1987). The second numerical algorithm is based on the minimization of the discrete least square criterion by taking into account a few future time steps. The finite difference method is utilized for solving the direct problem. These two methods are first verified and compared between them for one layer specimen by using simulated numerically data. The influence of the dimensionless time step ($\Delta Fo = \alpha\Delta t / e^2$) is analyzed.

Then the methods are applied to analyze experimental data obtained with the use of a thermal cycling device. A description of experimental setup and first results of the experimental data processing are reported.

INVERSE PROBLEM FORMULATION

The specimen is heated by a heat flux of unknown density at the active surface and cooled by a forced convection flow at the opposite surface.

The following hypotheses have been taken into account:

- thermophysical properties are supposed to be constant,
- heat transfer is one-dimensional.
- heat transfer coefficient is constant at the cooled surface.

Under these conditions, the heat transfer process in the specimen can be described by the following system of equations :

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}, 0 < x < e, \quad 0 < t \le t_f \qquad (1)$$

$$-\lambda \frac{\partial T(0,t)}{\partial x} = q(t) \qquad (2)$$

$$-\lambda \frac{\partial T(e,t)}{\partial x} = h[T(e,t) - T_a] \qquad (3)$$

$$T(x,0) = T_0, \quad 0 \le x \le e \qquad (4)$$

In the model (1)-(4), the heat flux density $q(t)$ is unknown. To get additional information about the temperature distribution in the specimen, temperature histories are measured in the specimen at a certain number of points N with coordinates $x = X_n$, $n = 1,2,..., N$:

$$T_{meas}(X_n, t) = f_n(t), \; n = 1,2,..., N \qquad (5)$$

This information, together with the model (1)-(4), is used to solve the inverse problem.

EXPERIMENTAL DEVICE

The experimental study was realized on a thermal cycling device with a one-layer specimen (gray cast iron). The heating device uses a heat source of a relatively weak heat flux (a hotwind). The cooling fluid used is the pressurized water circulating in channels. Two sensors have been incorporated in the specimen to record the temperature evolutions when the specimen is subjected to different types of thermal cycling. In order to limit lateral heat losses, the sample is coated with a ceramic resin. A fluxmeter (heat flux sensor), placed at the back surface of the specimen, allows one to estimate the flux density applied on the former.

The experimental device is composed of the following elements (figure 1) :

- heat source system : hot air with the temperature measured by the thermocouple TC_0,
- specimen in witch the temperatures histories are measured by TC_1 (X_1= 1.6 mm) and TC_2 (X_2= 7.8 mm) respectively,

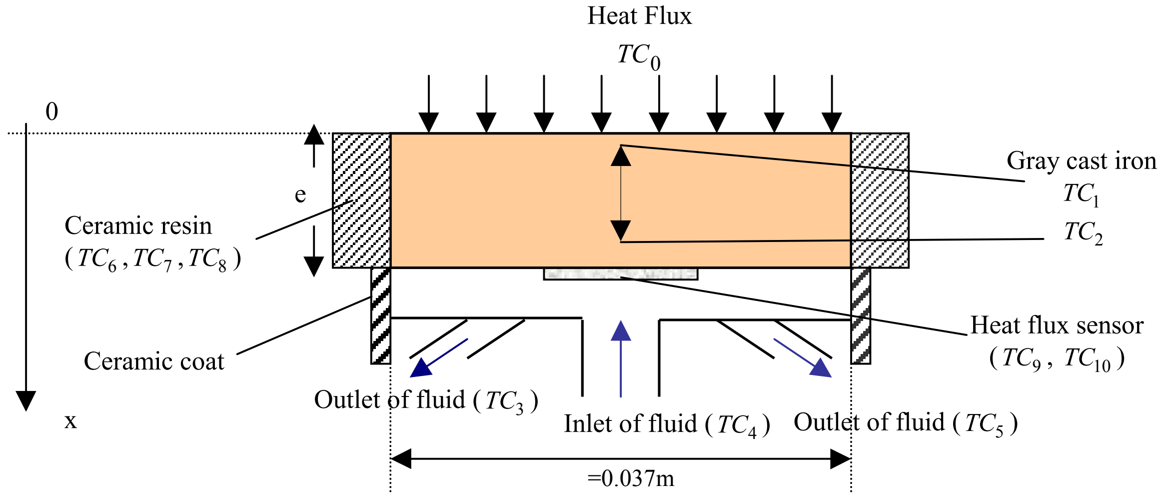

Figure 1. Simplified scheme of experimental device

- water cooling system : one input (TC_4), two outputs (TC_3, TC_5).
- Insulating ceramic ring with three thermocouples (TC_6, TC_7, TC_8) located at different positions. The purpose of this ring is to limit thermal losses in radial

direction in order to consider the heat transfer as monodimensional.

- heat flux sensor consisted of in alumina coating (1 mm thickness and 5mm radius) deposited on the back face of the sample. Temperatures (TC_9, TC_{10}) inside this coating are measured at two locations separated by 0.5 ± 0.1 mm. Numerical simulations show that the time response of the heat flux sensor is very weak and the temperature profile inside the system is linear.

The type of the thermocouples TC_i $(i = 0,...,8)$ is K and E for $i = 9,10$.

ITERATIVE REGULARIZATION METHOD (IRM)

To build a computational algorithm, we use the variational formulation of the inverse problem of interest. The problem is to find such unknown function $q(t)$ for which temperature histories computed from the mathematical model (1) to (4) at the sensor locations are close to measured histories. That leads to the problem of minimizing the residual functional :

$$J(q) = \sum_{n=1}^{N} \int_0^{t_f} \left[T(X_n, t; q) - f_n(t)\right]^2 dt \qquad (6)$$

where $T(X_n, t; q)$, $n = 1,2,...,N$, are temperature histories computed at the sensor locations with the heat flux density $q(t)$ given.

Unknown function is parametrized in the form of a cubic B-spline

$$q(t) = \sum_{i=1}^{M} p_m \varphi_m(t) \qquad (7)$$

where p_m, $m = 1,2,...,M$, are unknown parameters, $\varphi_m(t)$, $m = 1,2,...,M$, are given basis cubic B-splines. The number of approximation parameters M is usually fixed a priori. As a result, the inverse problem is reduced to the estimation of a vector of parameters $p = [p_1, p_2,..., p_M]^T$.

Unknown function is considered as an element of the function space $L_2[0, t_f]$ of parametrized functions. We use the unconstrained conjugate gradient method of optimization. The residual functional gradient as well as the descent direction in L_2 space have the form :

$$J'_q = \sum_{m=1}^{M} g_m \varphi_m(t) \qquad (8)$$

$$D(t) = \sum_{m=1}^{M} d_m \varphi_m(t) \qquad (9)$$

So, the gradient is characterized by the vector $g = [g_1, g_2,..., g_M]^T$ and the descent direction by the vector $d = [d_1, d_2,..., d_M]^T$. It is easy to show that the residual functional minimization with respect to desired parametrized functions is reduced to those with respect to unknown parameters. The successive improvements of desired parameters are constructed as follows :

$$p^{s+1} = p^s + \gamma^s d^s, \quad s = 0,1,... \qquad (10)$$

where s is the number of the iteration under way, γ^s is the descent parameter, p^0 is an initial guess for the vector of unknowns parameters given a priori. The vector d^s is computed as follows :

$$d^s = -g^s + \beta^s d^{s-1}, \quad s = 0,1,..., \qquad (11)$$

where $\beta^0 = 0$, $\beta^s = \dfrac{\left(g^{(s)} - g^{(s-1)}, g^{(s)}\right)}{\left\| g^{(s)} \right\|} \qquad (12)$

The realization of the iterative procedure (10) is based on computing the vector g at each iteration. This vector is determined by the relationship for the residual functional variation :

$$\delta J(q, \delta q) = (J'_p, \delta p)_{R^M} = (J'_q, \delta q)_{L_2} \qquad (13)$$

where J'_p is the residual functional gradient in R^M space of approximation parameters and J'_q is the gradient in L_2 space of parametrized functions, (,) is the scalar product.

By using the parametric form (7), it can be shown that (Alivanov et al., 1987):

$$J'_p = Gg \qquad (14)$$

where G is the Gram's matrix for basis functions :

$$G = \left\{G_{j,m} = (\varphi_j, \varphi_m)_{L_2}, j, m = 1,2,..., M\right\} \qquad (15)$$

This matrix is symmetric and positive definite. The Cholesky decomposition is used to solve the system (14).

The most effective method for calculating the gradient J'_p in R^M space is based on introducing an adjoint problem. The following expression for the gradient components can be derived :

$$J'_{p_m} = \int_0^{t_f} \psi(0, t) \varphi_m(t) dt, \quad m = 1,2,..., M \qquad (16)$$

where $\psi(x, t)$ is the solution of the following adjoint problem (Alivanov et al., 1995):

$$-\frac{\partial \psi}{\partial t} = \alpha \frac{\partial^2 \psi}{\partial x^2}, \; 0 < x < e, \quad 0 < t \le t_f \qquad (17)$$

$$-\lambda\frac{\partial\psi(0,t)}{\partial x}=0 \tag{18}$$

$$\lambda\left[\frac{\partial\psi_n(X_n,t)}{\partial x}-\frac{\partial\psi_{n+1}(X_n,t)}{\partial x}\right] =2\left[T(X_n,t)-f_n(t)\right],\quad n=1,2,...,N-1 \tag{19}$$

$$\partial\psi_n(X_n,t)=\partial\psi_{n+1}(X_n,t),\quad n=1,2,...,N \tag{20}$$

$$-\lambda\frac{\partial\psi}{\partial x}=h\,\psi(e,t) \tag{21}$$

$$\psi(x,t_f)=0,\ 0<x<e \tag{22}$$

A linear approximation is used to estimate the parameter γ^s (Alivanov et al., 1995) :

$$\gamma^s=-\frac{\sum_{n=1}^{N}\int_0^{t_f}\left[T(X_n,t)-f_n(X_n,t)\right]\vartheta(X_n,t)dt}{\sum_{n=1}^{N}\int_0^{t_f}\left[\vartheta(X_n,t)\right]^2dt} \tag{23}$$

where $\vartheta(x,t)$ is the solution of the following boundary-value problem :

$$\frac{\partial\vartheta}{\partial t}=\alpha\frac{\partial^2\vartheta}{\partial x^2},\ 0<x<e,\ \ 0<t\le t_f \tag{24}$$

$$-\lambda\frac{\partial\vartheta(0,t)}{\partial x}=D(t) \tag{25}$$

$$-\lambda\frac{\partial\vartheta(e,t)}{\partial x}=h\,\vartheta(e,t) \tag{26}$$

$$\theta(x,0)=0,\ 0<x<e \tag{27}$$

where $D(t)$ is the descent direction in L_2 space of parametrized functions (9).

To obtain stable solutions of the inverse problem under consideration, the iterative regularization is used (Alivanov et al 1995). The main idea is to terminate the iterative procedure with the residual criterion :

$$J(q^{s^*})\approx\delta^2 \tag{28}$$

where δ^2 is the total (integrated) measurement error defined by :

$$\delta^2=\sum_{n=1}^{N}\int_0^{t_f}\sigma_n^2(t)dt \tag{29}$$

$\sigma_n^2(t)$ is an estimate of the time-dependent standard deviation for the *n*th temperature history measured. This procedure gives the most stable solution. The number s^* of the last iteration is the regularization parameter of the method.

It is necessary to note that the number of approximation parameters M should be correctly chosen for the desired function. This number has to be chosen so that the residual criterion would be verified. In this case, the flexibility of the solution is good enough for using the iterative regularization method.

One iteration of the numerical algorithm is composed by the following steps :

- solution of the direct problem and computation of the residual functional,
- verification of the residual criterion,
- solution of the adjoint problem and computation of the residual functional gradient in L_2 space,
- computation of the descent direction,
- solution of the problem for temperature variations and computation of the optimal descent parameter,
- calculation of the heat flux approximation.

FUNCTION SPECIFICATION METHOD (FSM)

To estimate transient heat flux, the sum of squares function (Beck et al. 1985) :

$$J(q^{n+1})=\sum_{i=1}^{N}\sum_{j=1}^{r}[T(X_i,t_{n+j};q^{n+1})-f_i(t_{n+j})]^2 \tag{30}$$

is minimized with respect q^{n+1}.

The first subscript refers to space where N is the sensor number and the second to time where r is the future times.

The temporal stabilization, for the function specification method, is obtained through the temporary assymption that flux is constant over the r futures temperatures :

$$q^{n+1}=q^{n+2}=...=q^{n+r}=q^n+\Delta q^{n+1} \tag{31}$$

then the minimization of the sum $J(q^{n+1})$:

$$\frac{\partial J(q^{n+1})}{\partial\Delta q^{n+1}}=0 \tag{32}$$

leads to the expression of Δq^{n+1} :

$$\Delta q^{n+1}=\frac{\sum_{i=1}^{N}\sum_{j=1}^{r}Q_i^{n+j}\left[T(X_i,t_{n+j};q^{n+1})-f_i(t_{n+j})\right]}{\sum_{i=1}^{N}\sum_{j=1}^{r}\left[Q_i^{n+j}\right]^2} \tag{33}$$

where $Q_i^{n+j}=\partial T_i^{n+j}/\partial q^{n+1}$ is the sensitivity coefficient at time t_{n+j} and sensor locations X_i. The sensitivity coefficients are computed numerically by using the following relation

$$Q_i^{n+j}=\frac{[T_i^{n+j}(q^n+\varepsilon q^n)-T_i^{n+j}(q^n)]}{\varepsilon q^{n+1}} \tag{34}$$

where ε is a small coefficient.

One iteration of the estimated algorithm is composed by the following steps :

- computation of temperatures at the sensor locations by solving the direct problem for q^n and $q^n + \varepsilon q^n$ conditions.
- computation of the sensitivity coefficient Q_i^{n+j} ,
- computation of the flux variation Δq^{n+1} ,
- computation and test of heat flux q^{n+1} .

NUMERICAL SIMULATION AND COMPARISON OF THE TWO METHODS

The first experimental study was carried out with relatively low heating rate. But our future goal is to analyze intensive heating with small enough ΔFo numbers. That was a reason to compare the efficiency of the analyzed methods with different ΔFo values. For our purposes, the main criterion was the accuracy of estimated heat flux evolutions. Results of such a comparison are presented in this section.

To simulate the numerical solution and to compare the methods, we have supposed in the problem (1) to (4) that $\lambda = \alpha = e = 1$, $h = T_0 = 0$. Two types of the heat flux evolutions were studied :

- Step heat flux evolution (W/m^2)

$q(t) = 1000$ if $0 < t < t_f/2$ and

$q(t) = 0$ if $t_f/2 < t < t_f$

- Sinusoidal heat flux evolution with decrease amplitude:

$q(t) = q_0[1 + \sin(\quad t)]\exp(\text{-at})$, $0 < t \le t_f$

$q_0 = 1000$ W/m^2, $\omega = 20/t_f$, rd/s ,

a =0,05 for Δt =0,1 to 0,001 and a =2 for Δt =10.

In these conditions, the ΔFo number is equal to the time step that we have varied from 0.001 to 10.

In the above test cases the measured temperatures evolutions was simulated numerically at the back surface of the specimen with a random noise of 10% of the maximal value of the temperature.

For the IRM, we have used M =51 parameters for to estimate the unknown heat flux evolution. For the FSM, the sensitivity coefficients were computed by using ε =0.001.

For each particular case (ΔFo), the number of future steps r was chosen by realizing numerical experiments individually. This number was increased until fluctuations in the solution would be minimum, that is there would be no further stabilization.

In figures 2a and 2b, we show an example of temperature evolutions at x =0 and $x = e$ =1 for both test cases analyzed.

In figures 3a, b, c and d for the first type of the heat flux evolution and in figures 4 a, b, c and d for the second one respectively, we present a comparison between the two methods for ΔFo numbers varying from 0.001 to 10. These results of comparison show that for ΔFo >0.1 the estimated an exact heat flux evolutions obtained by the both methods are in a good agreement.

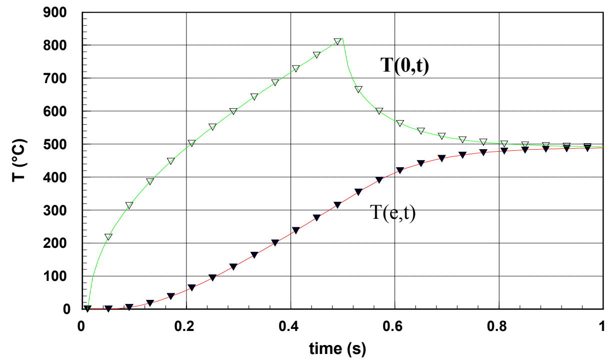

Figure 2a. Temperature evolution at $x = 0$ and $x = e$ for the first test case.

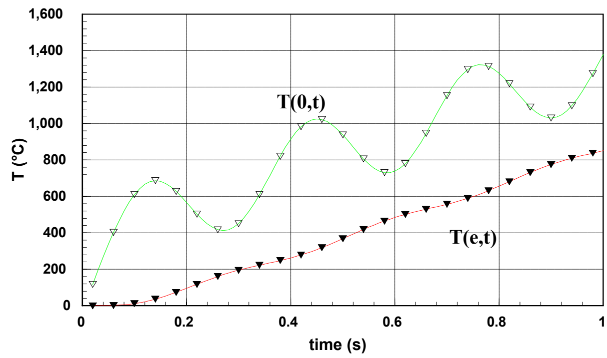

Figure 2a. Temperature evolution at $x = 0$ and $x = e$ for the second test case.

For numbers ΔFo = 0.1, results estimated by FSM remain close to exact heat flux but, as the ΔFo number decreases, the difference between exact and estimated values is increased.

For the IRM, the difference between exact and estimated heat flux evolutions is also increased but less than that obtained by the FSM. These results show that, for the test cases analyzed in this paper, heat flux evolutions estimated by the IRM are more accurate that those obtained by the FSM. This is a direct consequence of the use of the regularizing residual criterion (28). For all considered cases, the residual values δ^2 were computed by using the formula (29). These values are:

ΔFo	δ^2 $(\times 10^6)$
10	1.33
0.1	1.83
0.01	2.34
0.005	4.03
0.001	4.1

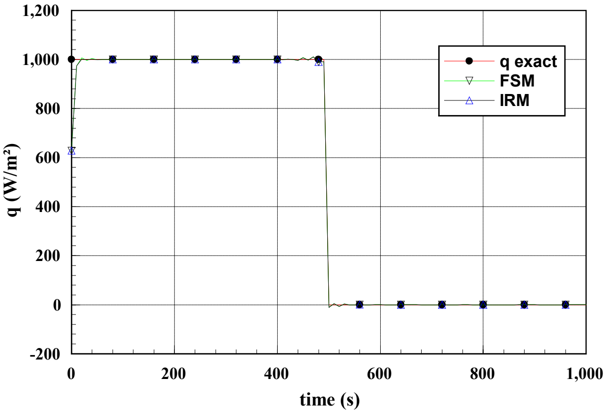

Figure 3a. $\Delta Fo = 10$

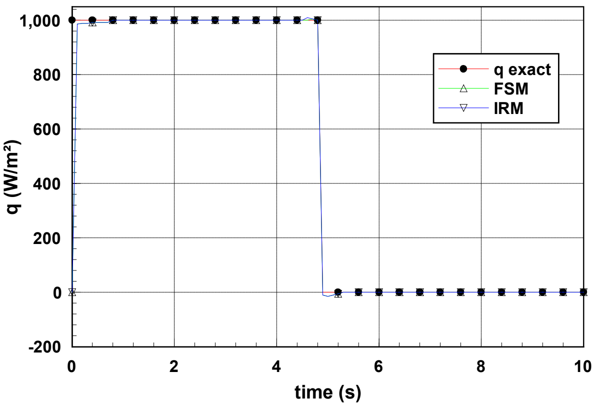

Figure 3b. $\Delta Fo = 0.1$

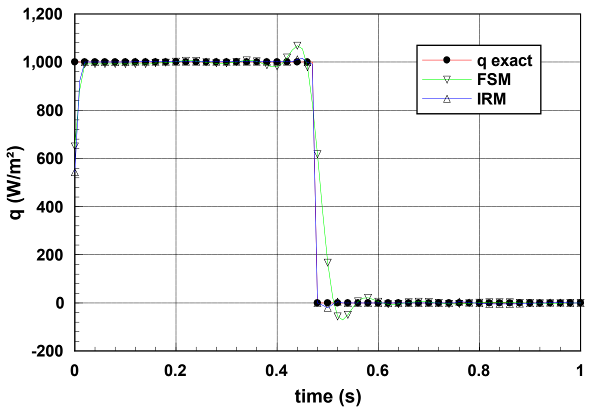

Figure 3c. $\Delta Fo = 0.01$

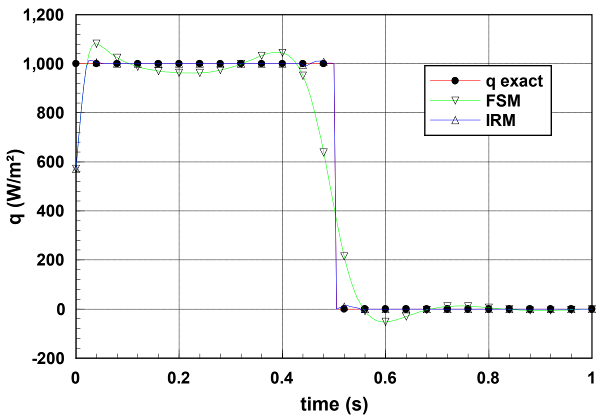

Figure 3d. $\Delta Fo = 0.005$

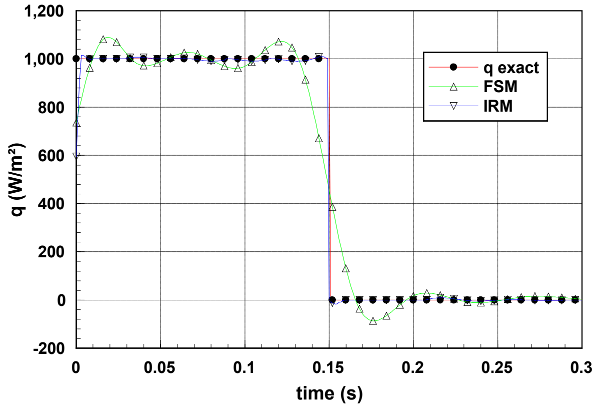

Figure 3e. $\Delta Fo = 0.001$

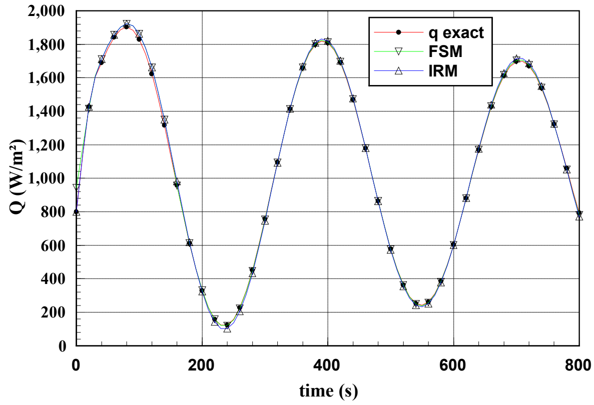

Figure 4a. $\Delta Fo = 10$

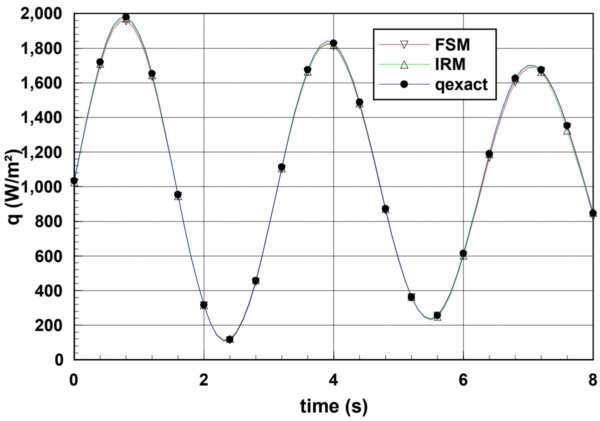

Figure 4b. $\Delta Fo = 0.1$

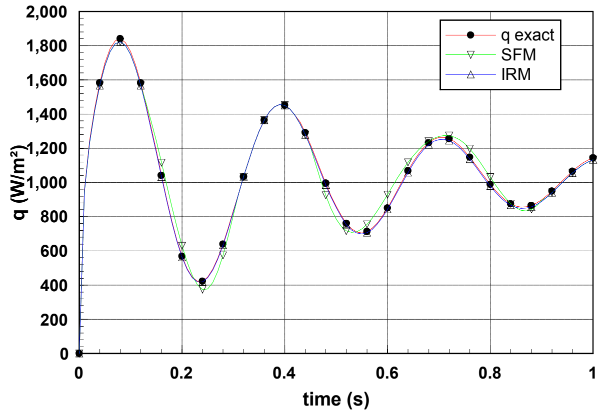

Figure 4c. $\Delta Fo = 0.01$

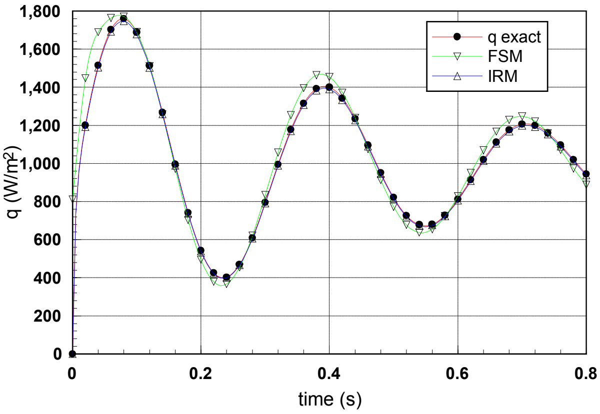

Figure 4d. ΔFo **=0.005**

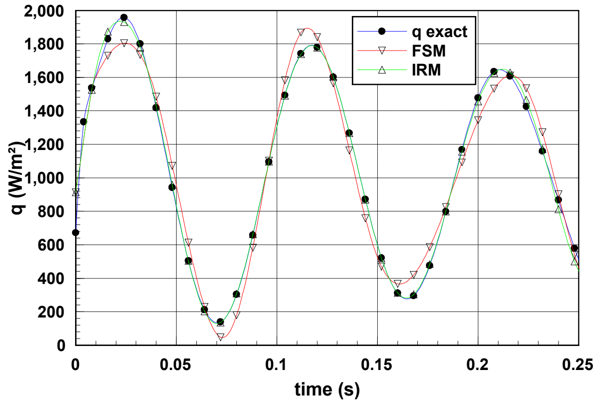

Figure 4e. ΔFo =0.001

We note that the comparison results obtained in this paper differ from those presented for example, in Beck, 1993. An explanation is that we use another realization of the IRM based on a spline-approximation of the unknown heat flux evolution. In particular, the residual functional gradient is not equal to zero at the final time with this approximation.

In terms of computing time (we used a PC of 260 MHz) , the FSM is more effective that the IRM, that is the FSM takes less time that the IRM. For the one dimensional test cases considered in this paper the computing time is rather small. So, the computing time is not the best criterion to compare the methods in this case. For two and three dimensional problems this criterion may be much more important. This question should be carefully analyzed.

It should be underlined that the realized comparison of the two methods was rather restricted. It is necessary to continue this analysis to establish the domains where each of the methods is more effective.

EXPERIMENTAL DATA ANALYSIS

The experimental tests were realized with a relatively low heat flux with a specimen of the gray cast iron (e=0.0097 m, λ =50 W(m.K), ρ =7200 kg/m^3, C=670 J/(kg.K) to obtain a significant difference between the temperature histories measured. The heat transfer coefficient is determined, in steady state regime, by a heat balance equation at the interface specimen-fluid.

$$h = \frac{q}{T_s - T_f} = \frac{31700}{45-9} = 880\,W/(m^2.°C)$$

Δh =142 W/(m^2°C), ΔT =0.5 °C, Δq =4.7 kW

where T_s is the temperature of the back surface of the sample, T_f is the mean fluid temperature and q is the steady heat flux estimated by the Fourier law.

$$q = \lambda \frac{\Delta T}{e} = 5.32 \frac{2.98}{0.5} = 31.7\,kW/m^2$$

ΔT =0.5 °C, $\Delta\lambda$ =3 W(/m°C), Δe=0.1 mm,
Δq =6.45 kW.

M =21, NT =350, ΔFo =40,5

The experimental data analysis confirms that heat transfer coefficient h remains practically constant.

In figures 5a and 5b, we present measured and estimated temperatures at sensor locations X_1=1.6 mm, X_2=7.8 mm respectively and, in figure 6, we present the estimated heat flux evolutions. Measured and estimated temperature histories show a good agreement and the difference between them remain less 4%.

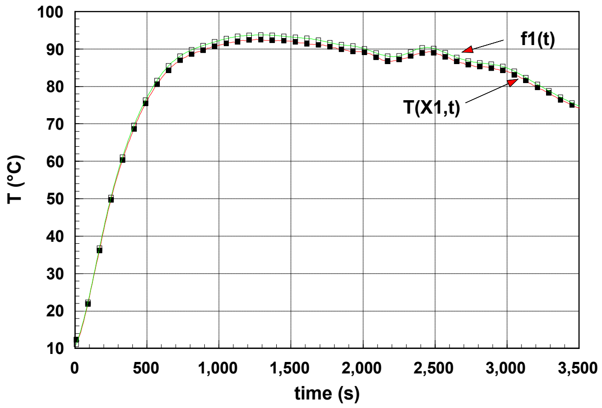

Figure 5a.Temperature histories measured and estimated at location X_1=1.6 mm,

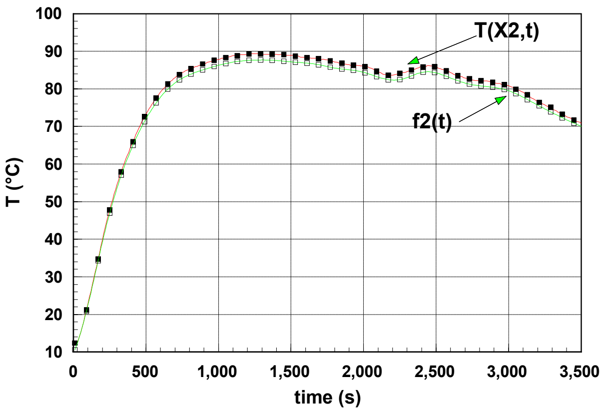

Figure 5b. Temperature histories measured and estimated at locations X_2 =7.8 mm

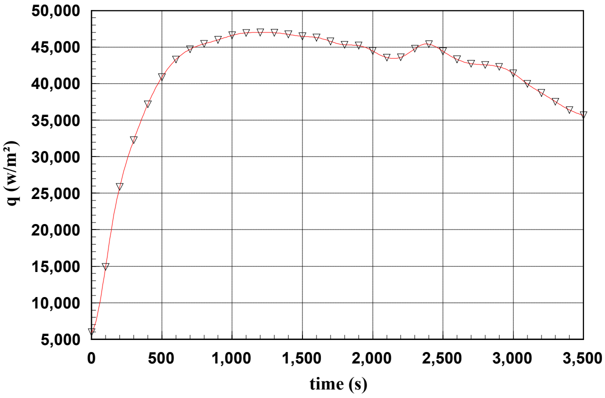

Figure 6. Estimated heat flux density.

These results were obtained with the use of the both methods under consideration. Practically, there is no difference between the both solutions. This fact confirms that there are domains of applications where different methods give practically the same results (Beck et al., 1996).

CONCLUSIONS

To estimate the heat flux evolution from measured internal temperature histories, the IRM and FSM was studied. A comparison between these two methods was realized for different dimensionless time steps (ΔFo numbers) by analyzing two test cases. This study shows that for ΔFo >0.1, the both methods gives practically the same results. For ΔFo <0.1, the IRM gives more accurate results and the errors in the estimated solution are increased much faster for the FSM that for the IRM in the test cases considered.

The use of the first experimental results allows to obtain the similar estimated heat flux evolutions for the two methods.

The first experimental study was realized by using a thermal cycling device with a one-layer specimen (gray cast iron). This study was carried out with relatively low heat flux. Two temperatures histories were measured inside the specimen. The heat flux evolution at the active surface was estimated with the use of the IRM and FSM. The results obtained by both methods are practically identical.

The present study is a preparation for an experimental study of thermal regimes of multilayer specimens wish are typical thermal barriers heated intensively during short period of time.

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