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ARRANGING HEAT SOURCES IN FIXED LOCATION FOR RESTRICTING A TEMPERATURE FIELD

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ABSTRACT

A methodology of arranging the heat sources in fixed location is presented in the paper. The sources are arranged so that the generated temperature field satisfies given limitations and minimizes a criterion functional. The superposition principle and the usage of different discretization of the domain are the tools of constructing the minimizing sequences. Test case and results illustrate suggested algorithm.

INTRODUCTION

A trend of electronic equipment consists of concentrating a lot of electronic elements in small spaces. At the same time the heat generation of the latest has a high rate. This can cause a high temperature of some units and provoke a failure of device. Thus the problem of guaranteeing an acceptable thermal behavior is a significant part of designing an electronic apparatus as a whole. This problem is especially urgent when vehicular equipment is developed, since in this case there are strong mass-dimension limitations. As is well known reducing the device dimensions causes a rise in temperature if the rate of internal heat generation is fixed.

As a rule to guarantee a certain thermal reliability it is necessary to arrange the heat sources (microchips, micromodules, microunits) in the device body or its component parts so as to answer some limitations on a temperature field that is generated by these sources. For example such a limitation can be the requirement that the temperature of chips does not exceed the critical value, which can cause its failure.

Similar problems appear when heating devices are designed. In that case the temperature limitations are defined by operating thermal characteristics which the heating device have to provide.

According to the statement this problem is a geometrical inverse heat conduction problem, as it is necessary to find geometrical parameters from known ones of temperature fields.

Consider the particular steady-state problem in which the heat sources have the equal geometrical form and dimensions and can occupy only fixed locations. A problem of such kind is the one of arranging the chips on a printed circuit board when the chips have the same dimensions as a result of unification.

NOMENCLATURE

$E(T)$	criterion functional (4)
P	output heat rating
Q	specific heat generation rating
$R(T)$	functional of temperature limitation (3)
T	temperature
x, y, z	space coordinates
α	heat transfer coefficient
λ	thermal conductivity
π	permutation (arrangement)
Π	minimizing sequence

STATEMENT OF PROBLEM

From the mathematical point of view this problem can be formulated as following. There are n sources with output heat rating P_i (i = 1, 2, …, n). In the considered domain Ω there are m locations, which are described by regions S_k (k = 1, 2, …, m). At each location it is possible to arrange one source. All regions S_k and heat sources have an equal geometrical form and dimensions. It is necessary to find such correspondence between serial numbers of sources and locations at which the temperature field $T(x, y, z)$ described by a heat conduction equation

$$\frac{\partial}{\partial x}\left(\lambda_x \frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda_y \frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\lambda_z \frac{\partial T}{\partial z}\right)- \\ -\alpha(T-T_f)+Q=0 \quad (1)$$

with boundary conditions

$$BT|_{\partial\Omega}=\varphi, \quad (2)$$

satisfies the limitations

$$R_l(T)\le 0, \qquad l=1, 2, \ldots, r. \quad (3)$$

Here λ_x, λ_y, λ_z are thermal conductivities in corresponding directions; the term $\alpha(T-T_f)$ describes a heat outflow distributed in volume; Q is a specific heat generation rating; B is an operator of boundary conditions; φ is a function defined at a border of domain Ω; R_l are some functionals. For example $R_l = T(x_l, y_l, z_l) - T_l^*$ describe the condition of non-exceeding the specified temperature T_l^* in check points (x_l, y_l, z_l).

In region S_k specific heat generation rating Q is equal $Q_k = P_{ik}/V$, where P_{ik} is heat generation rating of the sources arranged at the k^{th} location, V is the volume of the region (it is equal for all S_k according to the conditions of the problem). Outside the locations $Q \equiv 0$. Introduce unit functions $\eta_k(x, y, z)$ which are identically equal to unity in region S_k and zero outside S_k. Then $Q(x,y,z)=\sum_{k=1}^{m} Q_k \eta_k(x,y,z)$. When $n > m$ some sources are not used, and when $n < m$ some locations are free (for them $Q_k = 0$).

The solution of problem (1)–(3) is the ordered collection of numbers $\pi = (i_1, i_2, \ldots, i_m)$, where i_j is the serial number of source arranged at the j^{th} location. If $n < m$, π is an arrangement from the set of arrangements A_n^m; if $n = m$, π is a permutation from the set of permutations P_n; and if $n > m$, π is an arrangement from the set of arrangements A_m^n. From now on denote by $T(\pi)$ the temperature field generated by the sources placed according to the π. Let permutation (arrangement) π be called the permissible solution of the problem, if $T(\pi)$ satisfies limitations (3). In general the problem (1)–(3) has several solutions, as a few permutations (arrangements) can satisfy inequalities (3).

If some extremum condition

$$E(T)\to\text{extr} \quad (4)$$

(for example the requirement of minimizing the temperature in check points) is set in addition, it is possible to choose an optimal solution from the permissible ones. This solution is unique as a rule.

One formulation variant is the problem (1)–(2), (4), in which limitations (3) are missing. In this case all permutations (arrangements) are permissible.

The problem can also be generalized to the case when there are other heat sources in the domain Ω, the location and heat generation of which are fixed. Let such sources be called fixed ones. In this case specific heat generation rating, which appears in equation (1), can be represented in the form $Q(x,y,z)=Q_0(x,y,z)+\sum_{k=1}^{m} Q_k \eta_k(x,y,z)$, where $Q_0(x, y, z)$ is known specific rating of fixed sources. In general $Q_0 \neq 0$ in regions S_k, i.e. the fixed sources can be situated in the locations.

PROCEDURE OF ARRANGING THE SOURCES

The problem of finding an extremum of the functional E in Equation (4) on the set of permutations (arrangements) relates to the problems of discrete optimizations (Plane and McMilland, 1971; Philips et al., 1976). During the solution of such problems some permutations (arrangements) are successively considered and the value of optimized functional is calculated for each of them. In our case such a calculation is connected with solving the direct problem (1)–(2). To check limitations (3) it is also necessary to find temperature distribution.

In real problems the number of heat sources and locations amounts to some tens. So the exhaustive search is unacceptable because of it is impossible to check all permutations (arrangements) during the reasonable time.

When the controlled search of permutations (arrangements) is used the main time is spent in solving the direct heat conduction problem. It is therefore reasonable to try to reduce these time expenses.

One of the ways of reducing these expenses is based on the superposition principle in the case of linear problems. It consists of the presenting the temperature field in the form of a linear combinations of fields, which do not depend on a concrete permutation (arrangement). In this case it is necessary to find this distributions in advance, and during search for each investigated permutation (arrangement) to find the coefficients of the linear combinations and then to calculate the temperature.

As a rule problem (1)–(2) can only be solved by a numerical method, when the temperature is considered in the discrete set of nodal points (Richtmyer and Morton, 1967; Patankar, 1980). A large number of nodes results in best accuracy of the solution, but at that the expense of large computational times. Therefore as an alternative we have suggested using coarse discretization of the domain Ω to investigate most of the permutations (arrangements), using a large number of nodes for some small quantity of permutations (arrangements).

Superposition Principle

If the coefficients appearing in equation (1) are constants and operator B is linear, problem (1)–(2) is linear and the superposition principle (Rightmayer, 1987) may be applied. That is the temperature is representable in the form

$$T = T_0 + \sum_{k=1}^{m} Q_k u_k \quad (5)$$

where T_0 is the solution of the problem

$$\frac{\partial}{\partial x}\left(\lambda_x \frac{\partial T_0}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda_y \frac{\partial T_0}{\partial y}\right)+\frac{\partial}{\partial z}\left(\lambda_z \frac{\partial T_0}{\partial z}\right)- \\ -\alpha(T_0-T_f)+Q_0=0,$$

$$BT_0|_{\partial\Omega} = \varphi,$$

and the u_k are the solutions of the problem

$$\frac{\partial}{\partial x}\left(\lambda_x \frac{\partial u_k}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_y \frac{\partial u_k}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda_z \frac{\partial u_k}{\partial z}\right) - \alpha u_k + \eta_k = 0,$$

$$Bu_k|_{\partial\Omega} = 0.$$

It is obvious that T_0 and u_k do not depend on a placement of heat sources. So, one can first solve $m + 1$ boundary value problems, store the distribution T_0 and u_k, $k = 1, \ldots, m$, and then search the permutations (arrangements) using (5) to obtain temperature fields generated by the concrete arrangement of heat sources. As a result the geometric inverse problem (1)–(4) is reduced to the problem of discrete optimization (3)–(5).

The structure of functionals R_l and E, which appear in real problems, and a representation of temperature fields in form (5) allow using the branch and bound method to solve problem (3)–(5).

As an example consider the problem of minimizing the maximum (in domain Ω) temperature under the condition of non-exceeding the specified temperature in given check points. That is $E(T) = \max\limits_{(x,y,z)\ \Omega} T(x, y, z) \to \min$, $R_l = T(x_l, y_l, z_l) - T_l^* \le 0$. Let the heat sources be numbered in the order of decreasing heat generation rate.

It is obvious that the function u_k describes the temperature field generated by the unit source at the k^{th} location under zero boundary conditions. Let the temperature be measured in Kelvin. So, the value of functions T_0 and u_k in any point of domain Ω can be only positive.

Specify some initial placement of sources π_1 and find $T_1 = T(\pi_1)$ and $E_1 = E(T_1)$.

Start to search the sources from the first one with the higher heat generation rate. Place it sequentially at all locations. For each placement find the temperature distribution generated by this sources along with fixed sources and boundary conditions: $T^{(1,k)} = T_0 + Q_k u_k = T_0 + P_1 u_k / V$. For this temperature field calculate the value of the functionals R_l. As u_k and Q_k are positive according to (5), any placement of remaining sources increase (according to (5)) the temperature at any point of the domain Ω. Hence, it also increases the value of the functionals R_l. Therefore, if one of these functionals is positive at temperature $T^{(1,k)}$, any placements of remaining sources is not permissible, as it violates limitations (3). That is all permutations (arrangements), in which first source placed at the k^{th} location, can be excluded from the search. Reasoning similarly we can exclude these placements if there is a point in which $T^{(1,k)}$ is not less than E_1 (i.e. $E(T^{(1,k)}) \ge E_1$), as the value of functional E the field generated by any placement of remaining sources is greater than $E(T^{(1,k)})$.

If placing the first sources does not result in the violation of inequalities (3) and condition $E(T^{(1,k)}) < E_1$ holds, it is necessary to fix the first source at the k^{th} location and proceed to the second source. Similarly to the first sources the second one is sequentially placed at each free location. For each case the temperature distribution, which is generated by these two sources, is calculated as $T^{(1,k,2,s)} = T^{(1,k)} + Q_s u_s$ $(s \ne k)$. Then condition $T^{(1,k,2,s)} < E_1$ is examined in each point of the domain Ω. Limitations (3) are also verified for the temperature distribution $T^{(1,k,2,s)}$. If one of these inequalities is violated, it may be concluded that any permutation (arrangement) in which the first source placed at the k^{th} location and the second source placed at s^{th} location, cannot be a solution of the problem. That is, one can skip searching the remaining sources in this case.

If the condition $E(T^{(1,k,2,s)}) < E_1$ and limitations (3) are valid for the field $T^{(1,k,2,s)}$, we proceed to the third source and so on. If this procedure results in an exhaustion of all sources or locations and inequalities (3) and $E(T_2) < E_1$ (T_2 is the temperature field generated by all sources), we obtain a permutation (arrangement) π_2, which is permissible and better (in the sence of (4)) than π_1. It is necessary to store π_2 as a new approximation of the solution and replace E_1 by $E_2 = E(T_2)$ in the following calculations. Such a procedure is continued until all permutations (arrangements) are examined.

If after fixing the next sources at some location investigate all variants of placing the remaining sources, it is necessary to store $\min(n, m)$ temperature field ($T^{(1,k)}$, $T^{(1,k,2,s)}$ etc.) in the computer memory at the same time to realize this procedure.

If the problem is formulated without condition (3) or (4), the suggested procedure is also applicable. In the first case ((3) is absence and (4) is available) all permutations (arrangements) are permissible and it is necessary examine only inequality $T^{(1,k,2,s\ldots)} < E_j$ for each point of the domain Ω. In the second case ((4) is absence and (3) is available) it is not possible to specify initial placement. If we obtain a permutation (arrangement) during the search, which do not violate limitations (3), the calculation process can be stopped and this combination is the solution of problem.

Investigations have shown that the time consumption of the branch and bound method is less than that of exhaustive search when heat generation rates of sources are significantly different and heat removal within the domain and from its boundaries are non-uniform. Otherwise time savings are negligible and can also turn negative, as at the exhaustive search conditions (3) and inequality $T < E_j$ are not verified for intermediate temperature fields.

If both the exhaustive search and the branch and bound method are not applicable due to giant time expenses, one should use another methods of discrete optimization to solve problem (3)–(5). Such methods are described in detail in (Plane and McMilland, 1971; Philips et al., 1976). In this case formula (5) should be used to calculate temperature field and the functionals R_l and E for concrete permutation (arrangement).

If the extremal condition (4) is absent in the problem statement and it is impossible to use neither the exhaustive search nor the branch and bound method, the problem (3)–(5) is not a discrete optimization one. It can be solved by the random search. Another way consists of reducing it to the problem of

discrete optimization. It can be made by introducing a formal extremal condition. If during the solution process some permutation (arrangement), which satisfies limitation (3), is obtained, we may stop the calculations and assume this permutation (arrangement) as a final result. As a formal functional $E(T)$, we can set some combination of $R_l(T)$ (for example, $E(T) = \min R_l(T)$ or $E(T) = \sum_{l=1}^{r} R_l(T)$) and then minimize it. Condition (4) can be formulated on the basis of some additional thermal reasoning.

Different Discretization of Domain

If problem (1)–(2) is non-linear, the superposition principle is not applicable. And when an inverse problem is solved it is necessary to solve problem (1)–(2) for each investigated permutation (arrangement). As stated above, the heat conduction problem must be solved by a numerical method (Richtmyer and Morton, 1967; Patankar, 1980) and temperature is examined in the nodal points.

To reduce the time expended on direct problems, it is suggested to introduce two sets of nodes in the domain Ω. The number of points in the first set is small and the solution of problem (1)–(2), which is obtained with the help of this discretization, is crude. The number of points in the second set is large to provide the necessary accuracy of the solution.

From now assume that condition (4) is written in the form of $E(T) \rightarrow \min$. Assume that both limitation (3) and condition (4) are specified in the problem. Otherwise the procedure stated below should be slightly modified, as it was made at the end of the previous paragraph.

Chose one of the discrete optimization methods (Plane and McMilland, 1971; Philips et al., 1976), allowing to find a sequence of permutations (arrangements) $\Pi = \{\pi_1, \pi_2, \ldots, \pi^*\}$, which is minimizing the functional E. Here π^* is the approximate solution of the problem. As a rule such methods introduce a distance function in the set of permutations (arrangements), and the solution process consists of examining the functional E in the points of some neighborhood of the current approximation of solution. If one of these points improves the value of functionals, the neighborhood center is placed at this point and it becomes a new element of minimizing sequence. This procedure implies solving the boundary value problem (1)–(2) for each examined permutation (arrangement).

The first stage of the suggested procedure consists of constructing the minimizing sequence, using the first set of nodal points to discretize the domain Ω. Since in this case the solutions of the direct problems are crude, the values of functionals E and R_l are calculated with certain error. It is quite natural to expect that the real temperature fields generated by some elements of the minimizing sequence can violate conditions (3), or the functional E is not monotone decreasing at the Π.

Therefore at the second stage it is necessary to take some errors $\delta_0, \delta_1, \ldots \delta_r$, the value of which can be calculated from the solution accuracy of the direct problem on the basis of the forms of E and R_l. Then the continuous subsequence Π_2 of last elements of Π is selected. This subsequence includes the permutations (arrangements), which satisfy one of the following inequalities

$$E(T(\pi_j)) \le E(T(\pi^*)) + \delta_0, \qquad R_l(T(\pi_j)) \ge -\delta_l. \tag{6}$$

After that the second more detailed discretization of domain Ω is used, and the temperature field and values of functionals E and R_l are recalculated for all elements of this subsequence. Using these improved values of functionals the best permutation (arrangement) π^+ can be obtained.

Note the use of crude discretizations. When the neighborhood of permutations (arrangements) is examined, the best points in this neighborhood may not be obtained. Therefore suggested procedure should be modified in the following way to take this aspect into consideration. All examined permutations (arrangements), which satisfy one of inequalities (6), should also be added to Π_2. For that it is necessary to store each permutation (arrangement), which was encountered during calculation process, along with the values E and R_l for it.

The another way of modification consists of the following. After the recalculation and obtaining the permutation (arrangement) π^+ the new minimizing sequence is found. At that the values of functionals E and R_l are calculated using more detailed discretization of the domain Ω, and π^+ is taken as the first element of the new minimizing sequence Π^+.

As a rule the first set of nodal points can be constructed so that only one node is put in each location and the number of points between location is minimal.

If the number of elements in Π_2 is too large for exhaustive search, the additional sets of node points may be introduced so that the number of nodal points (and hence the solution accuracy) is increased when one set changes another. This allows gradually to narrow a collection of permutations (arrangements), which were selected after the first stage, by the means of the passage from the more crude discretization to a more accurate one. After recalculation of temperature field it is necessary to sort the remaining permutations (arrangements) in decreasing order (for E), select new π^*, reduce errors $\delta_0, \delta_1, \ldots \delta_r$, and execute new choice of supposed solution using (6).

The solution process may be changed by modifying the number of sets of nodal points, values of errors $\delta_0, \delta_1, \ldots \delta_r$, and discrete optimization method used at the first stage.

If the number of heat sources and locations is small, it is possible to use exhaustive search along with sorting at the first stage, when the coarser discretization is used to construct the minimizing sequence Π.

Some methods of discrete optimization allow finding the several local minimums, i.e. construct the several sequences of Π-kind, and each sequence is finished at a local minimum. In this case it is necessary to choose best (in the sense of (4)) π^* among all local minima, and select permutations (arrangements) from all sequences using (6). After that we can use a more

accurate discretization of the domain Ω and continue the calculations as described above.

If coefficients appearing in equation (1) and boundary conditions (2) depend slightly on the temperature, the suggested procedure can also be modified in the following way. At the beginning of the first stage, problem (1)–(2) is linearized and the direct heat conduction problem for each examined permutation (arrangement) is solved, using principles described in previous paragraph. As the superposition principle significantly reduces computational times, it is possible to try using the exhaustive search or branch and bound methods like it was described above.

Selection of Initial Placement of Sources

In most methods of discrete optimization the search is started from some initial permutation (arrangement). It can be obtained, using a random search or taking in to account a form of optimized functional E. As an example, consider a case when condition (4) consists of minimizing the maximum value of the temperature in the domain Ω. Let the heat sources be arranged in the order of decreasing heat generation rate.

In this case an initial permutation (arrangement) can be chosen so that a source with higher rate of heat generation occupies a location with more intensive heat removal in the case of uniform heat generation rate. For that it is necessary to find a temperature field specifying $Q_k = \sum_{i=1}^{n} P_i \Big/ nV$ for all k. If the problem is linear, this temperature distribution can be calculated according to (5). Otherwise it is necessary to solve problem (1)–(2) with above-mentioned Q_k. After that locations are enumerated in the order of increasing their temperature. By the temperature of location we understand the temperature of its center or the volume averaging temperature. Then first source associates with the first location, the second source – the second location and so on. If the number of sources is greater than number of locations, the arranging of sources is started from the $(n - m + 1)^{\text{th}}$ source in order that high-power sources are idle. This placement provides quite even distribution of temperature, and hence functional E takes quite good approximation of its optimal value.

TEST CASES AND RESULTS

To illustrate the feasibility of suggested procedure, we consider two cases of arranging the heat sources within the 2D rectangular domains. So $\frac{\partial}{\partial z}\left(\lambda_z \frac{\partial T}{\partial z}\right) \equiv 0$ in equation (1).

Case#1

It is necessary to arrange 6 sources at the rectangular plate so that the functional $E(T) = \max_{(x,y)\ \Omega} T(x, y) \to \min$. Limitation (3) are absent. The dimensions of plate are 0.18×0.12 m, its thickness is 0.002 m. The sizes of sources and locations are 0.03×0.03 m. The coordinates of centers of locations and the power of sources are given in table 1. The thermal conductivity of the plate is 30 W/(K m). The boundary of the plate is heat-insulated. The external temperature $T_f = 0$ °C, the convective heat transfer coefficient at the bottom side of the plate $\alpha_1 = 10$ W/(K m^2) and at the top side it depends on coordinates: $\alpha_1 = 10 + 100x + 50y$ W/(K m^2). So the term $\alpha(T - T_f)$ in equation (1) has the form of $\alpha_1(T - T_f) + \alpha_2(T - T_f) = (\alpha_1 + \alpha_2)(T - T_f)$. Consider two discretization of the domain. In the first coarse discreatization the size of node is 0.03 m, so only one node is put in each location and there is one nodal point between the locations. And in the second more accurate discretization the size of node is 1/3 cm, so 81 nodes are put in each location and there are 9 nodal point between location in each direction.

Table 1

Location	x, m	y, m	Source	P, W
1st	0.03	0.03	1st	6
2nd	0.09	0.03	2nd	5
3rd	0.15	0.03	3rd	4
4th	0.03	0.09	4th	3
5th	0.09	0.09	5th	2
6th	0.15	0.09	6th	1

The number of all possible permutation is equal to 6! = 720. It is not too large, so it is possible to use the exhaustive search when the minimizing sequences Π is constructed.

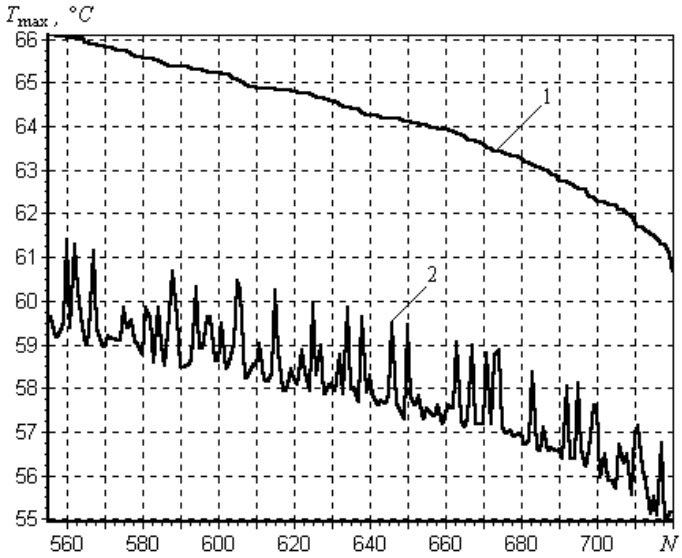

Figure 1. Maximal temperature versus permutation number (case#1).

Figure 1 illustrates the necessity of recalculation of value of functional E using the more accurate discretization. N is the permutation number. All permutations are numbered in the order of decreasing maximal temperature, which is calculated using the first discretization. Line 1 illustrates it. Line 2 is the values of functional E, which are calculated using the second discretization. One can see that line 2 is not monotone

decreasing, and the best permutation π^*, which is obtained at the first stage, differs from the improved one π^+.

Carried out calculations yielded the following results. $\pi^* = (5, 2, 4, 3, 6, 1)$. After the recalculation of functional E for last 166 permutations ($\delta_0 = 5.6$), the new permutation $\pi^+ = (6, 2, 5, 3, 4, 1)$ was obtained.

Case#2

This case differs from case 1 in the following. The plate sizes are 0.18×0.18 m. The number of location and sources is 9. The coordinates of centers of locations and the power of sources are given in table 2.

Table 2

Location	x, m	y, m	Source	P, W
1st	0.03	0.03	1st	9
2nd	0.09	0.03	2nd	8
3rd	0.15	0.03	3rd	7
4th	0.03	0.09	4th	6
5th	0.09	0.09	5th	5
6th	0.15	0.09	6th	4
7th	0.03	0.15	7th	3
8th	0.09	0.15	8th	2
9th	0.15	0.15	9th	1

The number of all possible permutation is equal to 9! = 362880. It is too large, so it is not possible to use the exhaustive search. All permutations, which differ from a given permutation only in two elements, were considered as its neighborhood when the minimizing sequences was constructed.

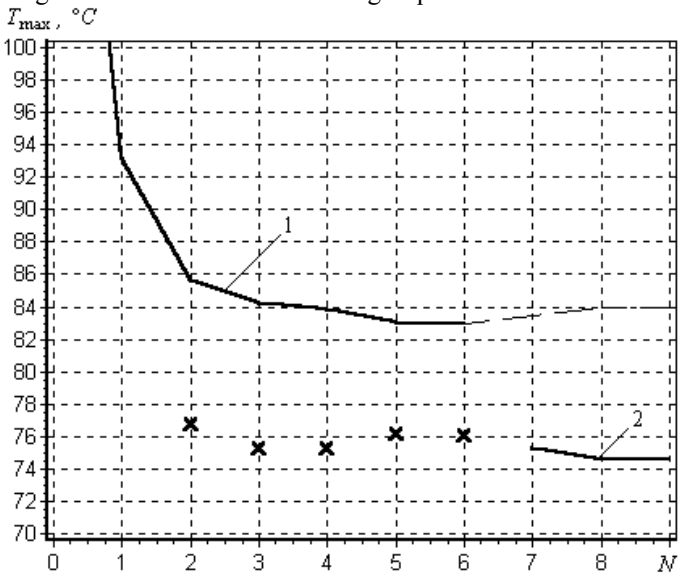

Figure 2. Maximal temperature versus iteration number (case#2).

Figure 2 illustrate the iterative process of determination of the sequences Π and Π^+. N is an iteration number, line 1 corresponds to Π and line 2 corresponds to Π^+. The crosses under line 1 are recalculated values of maximal temperature with the usage of the second discretization ($\delta_0 = 10$). Dotted continuation of line 1 illustrates the increase of functional E calculated for the permutations from Π^+ with the usage of the first coarse discretization.

The sequences $\Pi = (\pi_0, \pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6)$ and $\Pi^+ = (\pi_7, \pi_8, \pi_9)$ along with the values of maximal temperature are given in table 3. In this case $\pi_7 = \pi_3$.

Table 3

Iteration number	Permutation	T_{max}, °C, calculated using the first discretization	T_{max}, °C, calculated using the second discretization
0	(1, 2, 3, 4, 5, 6, 7, 8, 9)	133.98	–
1	(9, 2, 3, 4, 5, 6, 7, 8, 1)	93.27	–
2	(9, 5, 3, 4, 2, 6, 7, 8, 1)	85.66	76.76
3	(9, 5, 3, 7, 2, 6, 4, 8, 1)	84.30	75.28
4	(8, 5, 3, 7, 2, 6, 4, 9, 1)	83.88	75.30
5	(5, 8, 3, 7, 2, 6, 4, 9, 1)	83.04	76.22
6	(5, 7, 3, 8, 2, 6, 4, 9, 1)	82.94	76.10
7	(9, 5, 3, 7, 2, 6, 4, 8, 1)	–	75.28
8	(5, 9, 3, 7, 2, 6, 4, 8, 1)	–	74.61
9	(5, 7, 3, 9, 2, 6, 4, 8, 1)	–	74.54

Note the both case#1 and case#2 are liner problems and the superposition principle was used to calculate the temperature fields.

CONCLUSION

The procedure of arranging the heat sources in the fixed locations is presented. It is based on the minimization of a criterion functional and takes in to account the temperature limitations. Usage of different discretizations allows reducing computational times. In the linear cases it is proposed to use the superposition principle to calculate the temperature fields. The numerical examples illustrated utility of this procedure as it is allowed to reduce computational times without coarsening the solution.

Note, suggested procedures reduce only time expended on the solving the direct heat conduction problems, which arise during examination of inverse problem. The choice of concrete discrete optimization method has also an influence on the efficiency of solution process. A detailed discussion of this problem is found in the literature on the subject (Plane and McMilland, 1971; Philips et al., 1976; Stoyan and Putyatin, 1988).

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