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FLASH EXPERIMENT ON A SEMITRANSPARENT MATERIAL : INTEREST OF A REDUCED MODEL

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ABSTRACT

The subject deals with a reduced model of the conductive-radiative transient transfer in a participating medium. The accuracy of the analytical solution based on the well-known two-flux approximation and expressed with global radiative coefficients is tested in the case of sharp thermal excitation by a heat pulse on the front face of anisotropically scattering media. A very good agreement is achieved compared with numerical and analytical simulations involving high forward scattering, linear anisotropic scattering or Rayleigh scattering. This reduced model is then used in the inverse approach in order to determine the intrinsic diffusivity of semitransparent media.

INTRODUCTION

The increasing use of glasses, insulated foams, polymers in new technological products forced gradually the heat transfer community to take an interest in the study of combined radiative and conductive transfer in semitransparent media. Nevertheless only a limited amount of work is available on the inverse problem in this field [1, 2, 3].

Concerning the thermal diffusivity measurement, a lot of authors tried to extend the use of the flash method, first developed for opaque materials [4], to semitransparent media [5]. In those participating media, the measured diffusivity is an apparent one. It takes both conductive and radiative transfers into account. In order to obtain the phonic diffusivity, which is an intrinsic parameter, it is necessary to build a model including both radiative and conductive effects. As a consequence, the number of parameters increases. The aim of this study does not consist in evaluating all the parameters: it will be utopian with only one information namely the evolution of the rear face temperature. We rather try to show that a reduced model is able to reproduce the thermal behavior of absorbing, emitting and anisotropically scattering in a realistic way. More specifically, it allows to reach the phonic diffusivity which is a significant parameter for the engineering calculations.

NOMENCLATURE

- a : phonic diffusivity (m^2s^{-1})
- ρc_p : volumetric heat capacity ($J.m^{-3}K^{-1}$)
- e : slab thickness (m)
- k : intrinsic thermal conductivity
- L^* : intensity $\pi L / 4n^2 \breve{\sigma} T_0^4$
- M : global radiative parameter $M = (2\chi + \sigma)e$
- N : global radiative parameter $N = \sigma e$
- N_{pl} : Planck number $k\beta / 4n^2 \breve{\sigma} T_0^3$
- n : refractive index
- p^* : dimensionless Laplace variable $e^2 p / a$
- q_r^* : dimensionless radiative heat flux $q_r / 4n^2 \breve{\sigma} T_0^4$
- T : temperature (K)
- t^* : dimensionless time at / e^2

Greek symbols

- β : parameters vector
- χ : absorption coefficient (m^{-1})
- ϕ : heat flux density ($W.m^{-2}$)
- σ : scattering coefficient (m^{-1})
- $\breve{\sigma}$: Stefan Boltzmannn constant
- $\sigma^*_{\beta i}$: normalised standard deviation on β_i
- τ_0 : gray optical thickness $(\chi + \sigma) e$
- θ : dimensionless temperature $T - T_0 / (Q / \rho c_p e)$

Superscripts

- + : refers to the forward direction
- - : refers to the backward direction
- * : refers to the dimensionless quantities
- ^ : refers to an estimated quantity

1. The reduced model

Let consider a semi-infinite slab of a semi-transparent absorbing, emitting and scattering material (thickness *e*) (See Fig. 1). The case of opaque black boundaries is considered as it emphasizes the radiative effects within the material. The medium is initially at uniform temperature T_0 and receives a heat pulse (*Q*: energy of the Flash) on the front face, at dimensionless time $t^*=0$. We then consider the temperature rise of the back face, which is the solution of a coupled conductive-radiative problem.

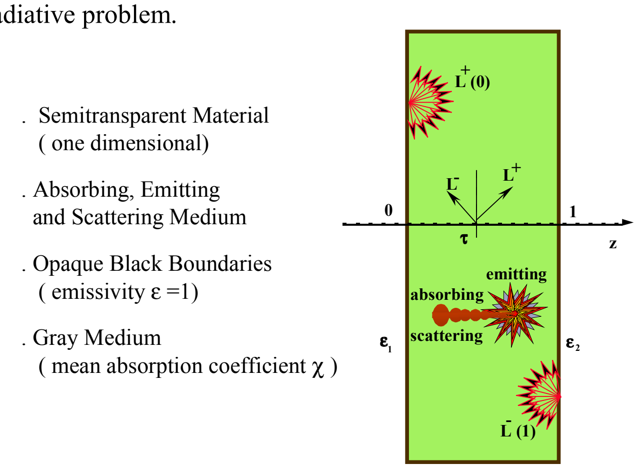

Figure 1 : Scheme of the medium

The mathematical formulation of the problem considered in dimensionless form is given by:

- the energy balance :

$$\frac{\partial\theta(z^*,t^*)}{\partial t^*}=\frac{\partial^2\theta(z^*,t^*)}{\partial z^{*2}}-\frac{\tau_0 T_0}{N_{pl}}\frac{\partial q_r^*(z^*,t^*)}{\partial z^*} \qquad (1)$$

where θ is the temperature rise with respect to the initial temperature normalized with respect to the adiabatic temperature, τ_0 the gray optical thickness, N_{pl} the Planck number and q_r^* the radiative flux defined as the difference between the forward and backward intensities $L^{*+}(z^*,t^*)$ and $L^{*-}(z^*,t^*)$ respectively.

- the radiative transfer equation where the two-flux approximation is used :

$$\frac{\partial L^{*+}}{\partial z^*}+ML^{*+}=NL^{*-}+\frac{P}{4}\left(1+\frac{\theta(z^*,t^*)}{T_0}\right)^4 \qquad (2)$$

$$\frac{\partial L^{*-}}{\partial z}-ML^{*-}=-NL^{*+}-\frac{P}{4}\left(1+\frac{\theta(z^*,t^*)}{T_0}\right)^4 \qquad (3)$$

where $M=(2\chi+\sigma)e$, $N=\sigma\, e$ and $P=2\chi\, e$.

The reduced radiative heat flux is then given by:

$$\begin{aligned}q^*{}_r(z^*)&=-2\beta\, C_1 e^{\nu z^*}+2\beta\, C_2 e^{-\nu z^*}+\frac{P}{4\nu}\left(e^{\nu z^*}-e^{-\nu z^*}\right)\\&+\frac{P}{T_0}\left[\int_0^{z^*}\theta(z^{*\prime})e^{\nu\left(z^*-z^{*\prime}\right)}dz^{*\prime}+\int_0^{z^*}\theta(z^{*\prime})e^{\nu\left(z^{*\prime}-z^*\right)}dz^{*\prime}\right]\end{aligned} \qquad (4)$$

with :

$$\beta=\sqrt{\frac{M-N}{M+N}} \qquad \text{and} \qquad \nu^2=M^2-N^2$$

C_1 and C_1 are determined by the radiative boundaries conditions.

The assumption of a linear transfer allows to express the second derivative of the radiative flux as a linear combination of the radiative flux itself and of the derivative of the temperature rise (differential approximation [6]):

$$\frac{\partial^2 q^*{}_r}{\partial z^{*2}}=\nu^2 q^*{}_r+\frac{2P}{T_0}\frac{\partial\theta}{\partial z^*} \qquad (5)$$

Applying then the Laplace transform $\overline{\theta}=\int_0^{+\infty}\theta(z^*,t^*)\exp(-p^*t^*)\,dt^*$, the temperature rise is obtained as the solution of the following fourth order temperature differential equation :

$$\frac{d^4\overline{\theta}}{dz^{*4}}-b\frac{d^2\overline{\theta}}{dz^{*2}}+c\overline{\theta}=0 \qquad (6)$$

$$b=\left(p^*+\frac{2P\tau_0}{Npl}+(M^2-N^2)\right)$$

$$c=p^*(M^2-N^2)$$

Here the quadrupole formulation [6, 7] is used to solve this ordinary differential equation in the Laplace domain. It provides a transfer matrix for the semitransparent layer that links linearly the input temperature-heat flux column vector at the front side (z=0) and the output vector at the rear side (z=1) :

$$\begin{pmatrix}\overline{\theta}(0)\\ \overline{\phi}(0)\end{pmatrix}=\begin{pmatrix}A & B\\ C & D\end{pmatrix}\begin{pmatrix}\overline{\theta}(1)\\ \overline{\phi}(1)\end{pmatrix}$$

2. The "exact" model

Numerical simulations give the thermal behavior of absorbing, emitting and anisotropically scattering media such as foams. D. Doermann and al [8] developed a precise numerical model based on a discrete ordinates method with a 24 directions Gauss quadrature and Henyey-Greenstein phase function approximation. Moreover we have built an analytical model based on an efficient kernel substitution in order to take the effects of a linear anisotropic scattering or a Rayleigh scattering into account.

Those fine simulations reproduce the thermal behavior of most of anisotropic scattering media in a very precise way. These references will be used in sections 3.2 and 3.3.

3. The inverse problem of parameters estimation

3.1 Estimation method: criteria and optimizer

The Levenberg-Marquardt algorithm uses the Gauss-Newton's nonlinear least squares method to estimate the values of unknown parameters [9]. It consists in finding suitable values of the β parameters by an iterative process to fit the theoretical curve $\theta(\beta)$ obtained with the two-flux model with the simulated reference points $\Theta(\underline{\beta})$ (subscript j refers to time t_j).

Let J be this sum: $J = \sum_j \left(\Theta_j(\underline{\beta}) - \theta_j(\beta)\right)^2$. It is also possible to add a noise on $\Theta(\underline{\beta})$ in order to test the stability of our model reduction. Minimizing J with respect to β_i is equivalent to make its derivatives equal to zero. The sensitivity coefficient X_i naturally appears in this minimization. It is defined by the following relation:

$$X_i(\mathrm{t_j}, \beta) = \frac{\partial\, \theta(\mathrm{t_j}, \beta)}{\partial \beta_i} \qquad (7)$$

By a linear expansion of the model around the solution, one can obtain an analytical relation between the estimated values $\hat{\beta}$ of the parameters and their real values β :

$$\hat{\beta} = \beta + (X'X)^{-1} X' \varepsilon(t) \qquad (8)$$

($\boldsymbol{\varepsilon}(t)$ being the noise at time t). This relation shows that :

- $E(\hat{\beta}) = \beta$: expected values of parameters
- $V(\hat{\beta}) = \sigma_\varepsilon^{\,2} (X'X)^{-1}$: variance of parameters

(σ_ε : Standard deviation of noise)

The previous relation is very important because il allows to evaluate the errors on the estimated parameters. It also clearly shows that if the signal is not corrupted by the measurement noise, one can expect to estimate the parameters with a high accuracy, even if their effects on the signal are strongly coupled. In contrast, in the case of a noisy signal, errors on the values of the estimated parameters directly depend on the noise level, particularly if they are strongly coupled.

In order to compare the sensitivities, one can use the reduced sensitivities, which are independent on the values of the parameters and on their units:

$$X_i^*(t) = \beta_i X_i(t_j) = \beta_i \frac{\partial \theta(t_j, \beta)}{\partial \beta_i} \qquad (9)$$

3.2 The parameters

The four parameters of the reduced model are :

- The thermal intrinsic diffusivity a
- The global coefficients M and N for the radiative transfer
- The Planck number N_{pl} , expressing the relative importance of the two modes of heat transfer.

In a non-scattering medium the coefficient $N=0$ and in the purely scattering medium $M=N$. Moreover, the parameter P representing the hemispherical absorption is related to M and N: $M=N+P$. This relationship appears in the two-flux formulation but is intrinsic to this approximation in that way that it expresses the conservation of radiant energy. Therefore P is not a new parameter and relation $M=N+P$ can not be broken. In other words, anisotropically scattering medium can be dealt with the reduced model but only the two radiative parameters M and N equivalent to optical thicknesses must be considered.

Figure 2 shows that the reduced model is able to reproduce the thermal behavior of participating medium in the case of the flash method, even if the phase function corresponds to highly anisotropic conditions.

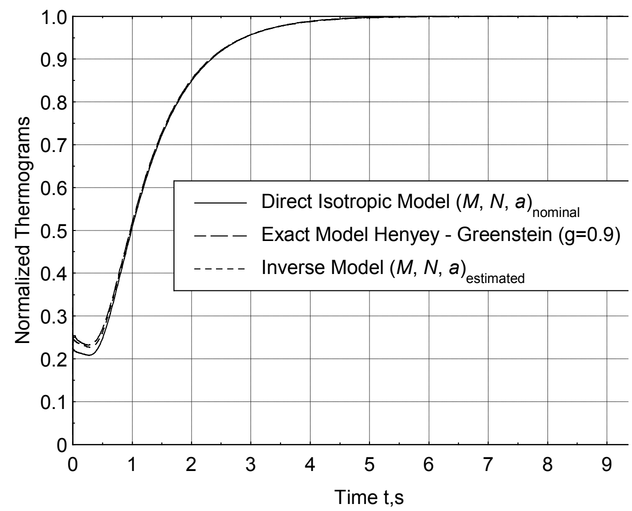

Figure 2 : Two Flux model / Anisotropic Model

In fact, the case with the parameter g=0.9 for the Henyey-Greenstein phase function represents a highly anisotropic forward scattering. This explains a most accentuated rear face temperature jump at time t=0 (black boundaries) compared to the direct modeling of the isotropic scattering case (exact two-flux formulation). In most unfavorable conditions (high radiative effects, highly anisotropic scattering), the reduced model has been proved to be able to reproduce exact behaviors.

Table 1 Input data for all the simulations

Pulse of energy Q	8600 J/ m^2
Heat capacity $\rho\, c_P$	2 10^6 J/ m^3 K
Thermal conductivity k	1 W/mK
Reference Temperature T_0	800 K
Refractive index n	1
Thickness of the slab e	2.1552 10^{-3} or 7 10^{-4} m
Radiative parameter M	0.1108 or 1.2
Radiative parameter N	0.0892 or 1

Our attention will now be focused on the problem of possible parameters estimation. The sensitivity and the correlation coefficients analysis reveals that the Planck Number has a low sensibility (Figure 3) or is correlated with the other parameters.

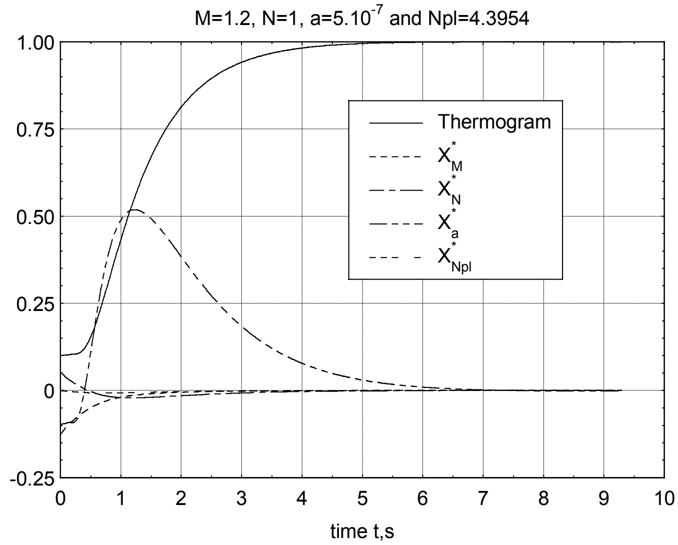

Figure 3 : Example of Reduced Sensitivities to the Parameters

It is possible to consider three parameters now on since, assuming the heat capacity of the sample known, the Planck number is then expressed as a function of the three parameters:

$$Npl = f(M, N, a) = \frac{a\,\rho\,c_p}{4\,n^2 \breve{\sigma}\, T_0^3}\,\frac{(M+N)}{2\,e} \tag{8}$$

Moreover it has been proved that any linear bi-univoque reparametrisation of a problem leads to same results in terms of normal values and standard deviation. Therefore the estimation of the parameters (M, N, a) allows to evaluate the values of the absorption coefficient, the scattering coefficient, the optical thickness or the albedo with the same accuracy (standard deviation) as it had been made with the same reduced model written with $(\chi\, e, \sigma\, e, a)$ or (τ_0, ω_0, Npl).

3.3 Parameters estimation

3.3.1 Isotropic scattering:

Here the temperature Θ is simulated with the two-flux model. To simulate the measured data Θ_ε containing measurement errors, random errors of normal distribution and of standard deviation σ_ε are added to the exact quantities. Several test cases were run using input data with and without measurement errors. For the cases with no measurement errors (i. e. σ=0) the exact values for the parameters were recovered. A run with M=1.2, N=1 and a=5 10^{-7} presented in Figure 4 is obtained using a reduced standard deviation of noise σ^*=0.005. The residuals between the thermogram with noise Θ_ε and the estimated thermogram θ are very small.

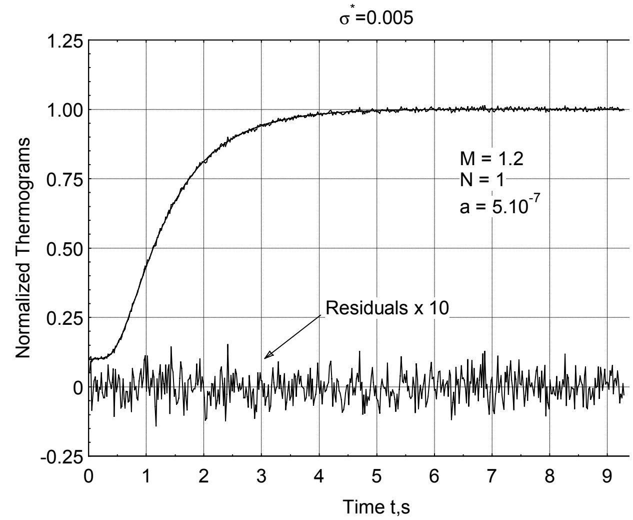

Figure 4 : Simulated and Estimated Thermograms

Table 2 shows the results of 10 runs in the same conditions as the test case (Figure 4), using initial guesses M^0=0.9, N^0=0.75, a^0=3.75 10^{-7}. One can notice that the diffusivity is estimated with a high accuracy (error less than 1%). The errors on the optical parameters are high even if the noise level is relatively small (0.5%).

Table 2 Estimated values (10 simulations with noise)

True values	M=1.2	N=1	a=5. 10^{-7}
Mean $\overline{\beta}_i$	1.20744	1.02657	5.0043 10^{-7}
Standard deviation $\sigma_{\beta i}$	0.02776	0.08717	2.725 10^{-9}
Normalized $\sigma^*_{\beta i} = \sigma_{\beta i} / \overline{\beta}_i$	2.3%	8.5%	0.5%

3.3.2 Anisotropic scattering

Now we want to test the robustness of the reduced model in the case of anisotropic scattering. The rear face exact temperature Θ is simulated with precise numerical or analytical models which take the anisotropic effects into account. No noise is added to the thermogram Θ since the aim is to test whether it is possible or not to evaluate the phonic diffusivity with the reduced model. Several type of anisotropic scattering phase functions have been considered in the exact model.

Table 3 Estimated parameters with the reduced model

True values	M=0.1108	N=0.0892	a=5. 10^{-7}
Type of scattering	$\hat{M}$	$\hat{N}$	$\hat{a}$
Henyey-Greenstein g=0.9	0.00154	0.0005	4.929 10^{-7}
L. A. S. forward x=+1	0.084056	0.074966	5.005 10^{-7}
L. A. S. backward x=-1	0.13308	0.11155	5 10^{-7}
Rayleigh	0.11013	0.08833	4.999 10^{-7}

For the Henyey-Greenstein highly anisotropic phase function, the estimated radiative parameters $\hat{M}$ and $\hat{N}$ are very different from the input exact model values. Indeed they take the anisotropy into account. On the other hand, in the case of a Rayleigh scattering, $\hat{M}$ and $\hat{N}$ are very close to the true values. When the scattering is rather forward, the increase of the temperature jump at the beginning is equivalent to a decrease of the Planck number. As a consequence $\hat{M}$ and $\hat{N}$ are underestimated. Nevertheless the intrinsic diffusivity is relatively well estimated in all cases. As expected, the phonic diffusivity of absorbing, emitting and anisotropically scattering media could be estimated with the reduced model.

3.4 Investigation of the parameters domain

Attention is now paid on maps represented in the radiative parameters space. The analysis of sensitivity maxima to the parameters, of the correlation coefficients and of standard deviations is performed in order to determine (if they exist) domains where the parameters could be estimated. Since M must be higher than N, these radiative parameters do not define a square domain so we rather consider $(M\text{-}N)/2=\chi e$ and $N=\sigma e$ to plot the maps. $(M\text{-}N)/2$ is equal to the absorption coefficient multiplied by the thickness of the medium. It could be viewed as an optical thickness due to absorption. In the same way, N represents the contribution of the scattering to the optical thickness.

3.3.1 Sensitivity maxima

Figures 5a 5b 5c show that the sensitivity maxima to the radiative parameters are very small compared with the sensitivity maxima to the phonic diffusivity (a factor larger than 10). As expected, the phonic diffusivity is the more sensitive parameter of the model: it has a great influence on the rear face temperature whereas the radiative parameters appear less influent. The value of the sensitivity maxima to the diffusivity is obtained for large optical thickness values, in other words when the heat transfer is dominated by conduction. The value 0.64 is the maximum of the maxima sensitivity to diffusivity that can be reached for an opaque material. It can only be obtained when the radiative transfer disappears (optical thickness larger than 5). On the contrary, the radiative parameters are sensitive when their values are small (thin optical thickness). It may appears paradoxical that a parameter is more influent when it is small. However, as they expressed the attenuation law within semitransparent materials in a exponential decay, the smaller the absorption and scattering coefficients are, the greater their effects on the thermogram are.

One can appreciate in the figure 5a that above a level of $\chi e=0.5$ the absorption mechanism is preponderant in that way that it becomes relatively insensitive to the value of σe. The analysis of the sensitivity maxima is not enough to know whether a parameter could be well estimated or not.

The correlation coefficients must be considered too.

3.3.2 Correlation coefficients

The correlation coefficients are not showed here for two reasons. First they are always about 0.98 or 0.99. The only region where the parameters are not so correlated corresponds to small optical values, the correlation coefficient is then about 0.94. The second reason is that the analysis of the standard deviation includes both the effects of the sensitivity maxima and the correlation between parameters.

3.3.3 Standard deviation

The standard deviation is obtained from the variance-covariance matrix (see section 3.1). It represents the errors on the estimated parameters. Figures 6a 6b and 6c show respectively the standard deviation for $(M\text{-}N)/2$, N, a. They are proportional to the reduced noise level σ^*, for instance if $\sigma^*=0.01$, this map can be viewed as values of errors expressed in per cent. Figure 6c shows that the errors on the phonic diffusivity are very small compared with the errors on the radiative parameters. It corroborates the results obtained in the foregoing sections 3.3.1. As we can see in the Figure 6a the standard deviation on $(M\text{-}N)/2$ is low when the scattering is weak and vice versa for the scattering.

It could be surprising that the standard deviation on the diffusivity is large for large values of optical thickness. The thermogram is then mainly conductive in this region and the sensitivity to the diffusivity is maximum. This result can be explained by the fact that the diffusivity value is obtained with a non-linear model that takes both conductive and radiative effects into account. Since in this case the diffusivity is strongly correlated with optical parameters, which are not well estimated, the errors on the radiative parameters are then reported on the diffusivity. This explains the artificial isovalues in the map (Figure 6c) for $\chi e \geq 2.5$. The minimum standard deviation on the diffusivity is obtained for small values of absorption, a domain where the correlations between the diffusivity and the optical parameters are small.

The region in the radiative parameter space where an identification of the whole parameter vector could be estimated is very small. On the basis of the foregoing results, the diffusivity can always be well estimated anywhere in the previously defined domain contrary to the radiative parameters.

CONCLUSION

A methodology has been developed and tested to determine thermophysical and radiative properties of a semitransparent material. The algorithm of Levenberg–Marquardt was used with simulated data to determine unknown parameters of anisotropic scattering media. The results obtained for the radiative parameters M and N must be viewed as first estimates. As expected, values of the phonic diffusivity completely reproduce those of the simulated data. This theoretical study will be completed with an experimental one involving glasses and insulating foams excited by a heat pulse in order to determine the true phonic diffusivity.

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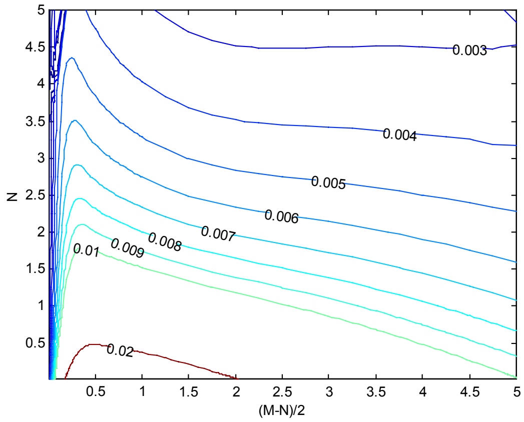

Figure 5a Map of the sensitivity maxima to *(M-N)/2*

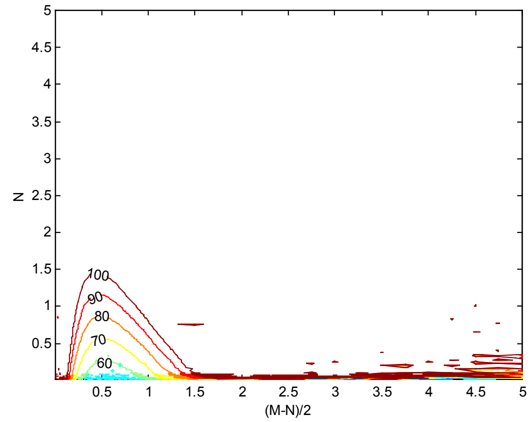

Figure 6a Map of the reduced standard deviation on *(M-N)/2*

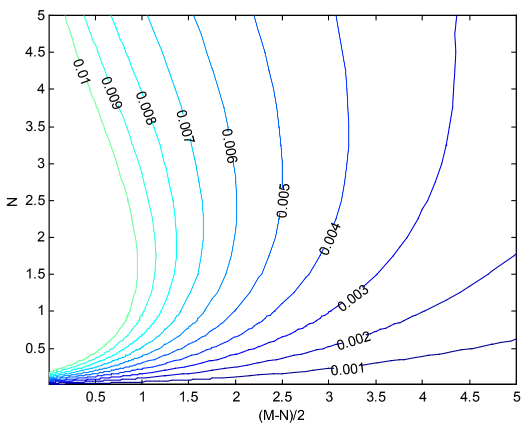

Figure 5b Map of the sensitivity maxima to *N*

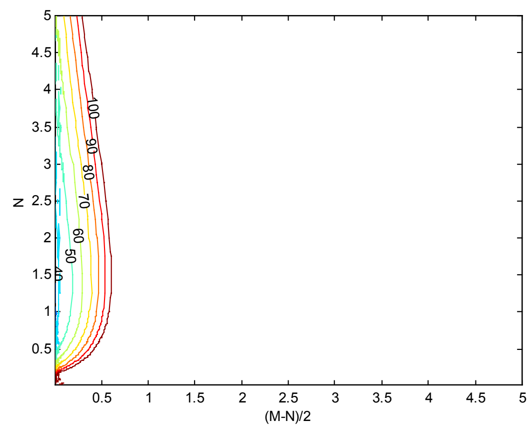

Figure 6b Map of the reduced standard deviation on *N*

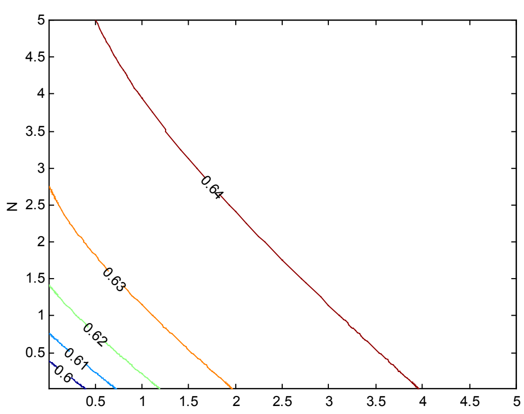

Figure 5c Map of the sensitivity maxima to the phonic diffusivity

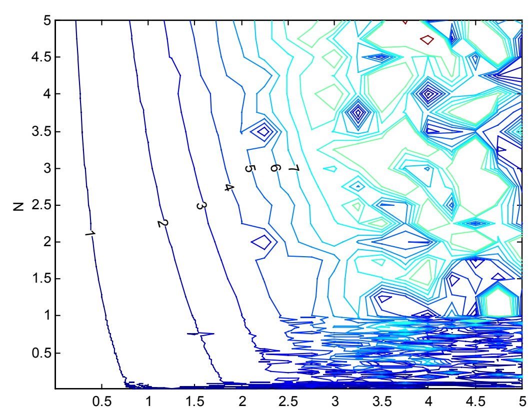

Figure 6c Map of the reduced standard deviation on diffusivity