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OPTIMUM CONTROL PROBLEMS SOLUTION BY MEANS OF INVERSE METHOD OF ADAPTIVE ITERATIVE FILTER

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ABSTRACT

This presentation centers on the application and modification, if any, of well-known method of Adaptive Iterative Filter to the solution of various thermal control problems. The solution of these problems implemented in terms of the optimum control theory involves preliminary constructions of a quality criterion accounting for the initial mathematical model of the thermal system and control aim. The ultimate goal of this research is to show the universal approach to the inverse and control problems that allows the solution of both problems as problem of parameter identification by means of the comprehensive method of Adaptive Iterative Filter.

Numbers of the problems has been solved. These problems include: the construction of a optimum law for starting a turbomachine, the optimization of procedure of vehicle heat exchanger selection, the identification of optimum thermal mode of post-implantation activation annealing of semiconductor materials, the optimization of slag-granulation process.

INTRODUCTION

The control of a thermal system is aimed at changing the state of an object and it is desirable that the control procedure be dependent on the available information about the system state obtained from its simulation. The latter can be based on the solution of external, internal, or combined Inverse Heat Transfer Problems (IHTP), where some uniqueness conditions, initially unknown, are identified or the mathematical model of the phenomenon under study is refined by the limited and rather approximate data on the temperature field.

NOMENCLATURE

T	temperature
ΔT	temperature differential
τ, t	time
q, $q(T,\tau)$	heat flux*
h, $h(T,\tau)$	heat transfer coefficient*
G	heat transfer
S	frontal surface area of heat exchanger
$\lambda(T)$	thermal conductivity*
C_P	specific heat capacity at constant pressure*
$C_V=C_P*\rho$	specific heat capacity at constant volume*
L	latent heat of crystallization
ρ	density*
$\vec{X}$	state vector
$\vec{U}$	vector of controlling signals
$\vec{W}$	vector of perturbation
$\vec{a}$	vector of constraints
$\Phi_{k,k-1}$, $F_{k,k-1}$, $G_{k,k-1}$	transition matrices
$\hat{\vec{Z}}$	estimated vector of unknown parameters
$\tilde{\vec{Y}}$	measurements vector
$\vec{V}$	vector of stochastic errors of measurements
$I_k^{(i)}$	functionals
P	covariance matrix of estimate errors

R	covariance matrix of measurement errors
K	weight matrix
σ	mean square error of measurements
H	measurement matrix
h[•]	operator of relation between estimated and measured parameters
E[•]	mathematical expectation

*Most of heat transfer parameters denominations correspond to reference Hewitt, et al., (1996).

Subscripts and Superscripts

i, j	iteration numbers
k	number of time steps

Symbols

IHTP	Inverse Heat Transfer Problem
AIF	Adaptive Iterative Filter
HVAC	Heating, Ventilating, and Air-Conditioning.

MATHEMATICAL MODEL OF THERMAL SYSTEM FOR SOLVING CONTROL STOCHASTIC PROBLEMS

The main problem for simulation of an engineering system (including a thermal one) is identification of its state parameters based, as a rule, on incomplete information. Such a problem supposes the definition of the state vector (the temperature field) and a set of parameters (boundary conditions, thermal properties, geometrical parameters), which characterize the system behavior from the viewpoint of formulation and objective of research performed, for example, simulation and further control.

The problems of controlling of heat engineering objects are characterized by complex constraints imposed on the parameters of the system state and control. The solution of these problems implemented in terms of the optimum control theory involves preliminary construction of a quality criterion accounting for the initial mathematical model of the thermal system and control aim.

Since the behavior of thermal or any technical system can be most totally described in terms of probabilistic characteristic of its parameters, and because the information obtained from experiment always bears a random character, the initial model of heat transfer process under study should be written as non-linear stochastic equation

$$\tilde{\vec{Y}}_k = \vec{h}\left[\vec{Z}_k\right] + \vec{V}_k , \qquad (1)$$

where

$h[*]$ is an operator of the relation between estimated, unknown parameters $\vec{Z}_k$ (in particular case $\vec{Z}_k$ is the vector of controlling signals $\vec{U}_k$) and "measured" parameters $\tilde{\vec{Y}}_k$, that can be represented by the measurements as well as by the constraints imposed on the system (mostly, for the control problem), and/or by all known, so-called, rigid parameters of the object under study;

$\vec{V}_k$ is the stochastic (white Gaussian sequence) errors of measurements that include: 1. direct errors of measurements, 2. any other errors or accuracy of determination of constraints imposed on the state or control parameters, 3. accuracy of discreteness of the mathematical model of process under study, etc.

On this basis, in the real thermal system, the case in point should be the stochastic approach to the solving of the control, identification, or simulation problems.

The thermal system mathematical model formalized in the form of matrix-vector equation relates the system state vector $\vec{X}_{k+1}$ to the vectors of state, control, and perturbation in the previous moment of time describes, actually, the Markov process and represents a non-linear stochastic equation

$$\vec{X}_{k+1} = \vec{f}\left(\vec{X}_k, k+1\right) + j\left(\vec{X}_k, k+1\right)\vec{U}_k + \\ + y\left(\vec{X}_k, k+1\right)\vec{W}_k . \qquad (2)$$

At this, due to probabilistic character of the corresponding parameters $\vec{X}_k$ (stochastic state vector), $\vec{U}_k$ (stochastic vector of controlling actions), and $\vec{W}_k$ (uncorrelated white noise), it is advisable to carry out the identification of the controlling signals from stochastic positions.

One of the specific features of the control problems and their main difference from the identification (estimate) one is the presence of constraints $\left(\vec{a}\right)$ on the phase coordinates (state and control vector). It can be the constraints: on the value of the elements (or their norms) of the vectors in question; on the gradients of state or control vectors $\left(\vec{X} \text{ or } \vec{U}\right)$ in time or in space; on the parameters secondary with respect to $\vec{X}$ or $\vec{U}$; etc. The class problems under study includes both the problems for controlling the thermal processes (for instance, the systems that can function on ultimate loads) and the problems of identification or optimization of thermal and/or technological parameters (in this case, the measurements of certain temperatures of object can act as constraints).

It should be noted, that in the problems of parametric identification the desired parameters always depend on time and space coordinates as well as on the state vector. Reduction of the parametric identification problems to the optimum control ones leads to the search of the identified parameters acting as control actions represented also in the form of functions of the coordinates and state vector.

Such an approach is all the more natural, since these problems (both identification and control ones) are interrelated. Practically always the solution of one of them requires the solution of the other. Let us consider the control problem stated as the problem of constructing the method (the algorithm)

forming control signals shifting the system under control to the required state. The problem of optimum stochastic control will include, as a necessary condition, the extremalization of the selected quality criterion (control cost) by the found control signal. There, according to the theorem of division (Feldbaum, 1966), the problem can be divided into two: system behavior estimate (parameters identification) and construction of the control algorithm (changing this behavior) proper with the use of the obtained estimates of parameters. In a case of designing the optimum control system, it is necessary to have reliable information about the behavior and characteristics of the object, i. e. the problem of identification of the system's parameters arises again. Generally speaking, the statement and solution of the problems of parameter identification are due to the necessity of simulating the process under investigation at the stage preceding the solution of the optimum control problem.

A combination of identification with control was called Dual Control of Feldbaum (Feldbaum, 1966). The division of these two problems (according to the theorem of division) is not always necessary, and often, it is just a trick for simplifying the solution of the problem stated. In fact, one can control efficiently, in the first place, only when sufficient information about the control object is available, and, in second place, with timely action on the object (real time control). As a result, the system can either process the information about the object properties failing to deliver timely the controlling signal, or the controlling action is delivered in time but without sufficient information about the system. In both cases the control function may be unfulfilled. Therefore, it is advisable, (if it is possible) to solve simultaneously both problems: identification of the parameters and control of the object to bring it to the required state.

Let us temporarily get back to the statement of controlling problem. Taking into consideration the stochastic character of the initial models (1) and (2) and all cited arguments that lead to the use of these equations, the definition of the controlling actions will be considered as stochastic optimum control problem.

In such a problem, it is required to construct an admissible, physically realizable function (control) $\vec{U}_k$ (and corresponding state vector $\vec{X}_k$), that meets all constraints $\vec{a}_k$ and extremalizes the selected quality criterion. Necessary condition for the construction of this controlling function is connected with concept of *Controllability of Dynamic system*. Concept of *Controllability* consists in possibility of transferring dynamic system from one given state to another prearranged one in a definite time by means of piecewise continuous control. Concept of *Stochastic Controllability* (Kazakov, 1975, Kazakov and Artem'ev, 1987) is based on features of the covariance matrix of the estimate error. Let covariance a posteriori matrix of state error for continuous system appears as

$$P(t) = E\{[\hat{\vec{X}}(t) - \vec{X}_*(t)] * [\hat{\vec{X}}(t) - \vec{X}_*(t)]^T \Big| \tilde{\vec{Y}}(t),$$

$$t_0 \le t \le t\}$$

at mathematical expectation $E_{\tilde{\vec{Y}}(t)}[\hat{\vec{X}}(t) - \vec{X}_*(t)] = 0,$

where

$\vec{X}_*(t)$ is exact, theoretical value of state vector.

Then the system will be *stochastic controlled*, if, at given controlling function, trace of covariance matrix $trP(t)$ is bounded at any t®¥, and correspondingly, module of error is $|\hat{\vec{X}}(t) - \hat{\vec{X}}_*(t)|$ bounded at any t®¥. In other word, the result of random perturbation in the *stochastic controlled* system is bounded. However, if the condition of controllability is necessary one for the construction of this controlling function, identification of this function requires the extremalization of the selected quality criterion and the availability of the target of the control. The requirement of satisfiability of the system controllability can be weaken or sometimes, even, removed if it is given not the endpoint of final system state but some region of this final state. By the way, the latter is more fits naturally into the stochastic control as it takes into consideration the measurement errors and accuracy of determination of imposed constraints. Moreover, the solution of the ill-posed problems, which include optimum control ones, is supposed to be obtained as some region of feasible results (Matsevity and Moultanovsky, 1984).

It is necessary to define the concept of property of being ill-posed of optimum control or optimization problems. Due to inverse problems represent a particular case of optimization problems, all considerations about property of being ill-posed for inverse problems (Matsevity and Moultanovsky, 1982) are also true for optimum control problems. Generally speaking, ill-posedness of optimization problem is linked to an inadequate changing of optimal operator under arbitrarily small changes of probabilistic characteristic of initial data. At this case, the small, or, even, arbitrarily small changes of functionals (in terms of which the problem is stated) may leads to the rough estimate of the desired solution of controlling function, whereas above-mentioned functionals (or their extremums) appear very precise with this estimate.

The uniqueness of the obtained solution is not so important for the control problems because any of the obtained results appear as controlling function. However, the problem of optimization (search for optimum control) supposes the uniqueness of the constructed optimum control strategy.

STOCHASTIC PROBLEM OF OPTIMUM CONTROL AND STOCHASTIC PROBLEM OF PARAMETERS ESTIMATION

The behavior of the system described by stochastic equations (1) and (2) is fully controlled by means of time-discrete controlling signals $\vec{U}_k$. A search for this control action can be pursued either by solution of stochastic optimum control problem, or stochastic problem of parameter estimation. By virtue of duality theorem (Fedorenko, 1978, Fomin, et al, 1988), if it is utilizing the mean square quality criterion, it is always possible to switch from parameter estimate problem to equivalent optimum control problem. By the way, the optimality of the control is frequently resulted from the consistency of estimates. The definition of *consistency of estimates* here is the same as it has been used in reference (Matsevity and Moultanovsky, 1982): the estimate is consistent if its dispersion tends to zero when number of measurements approaches infinity.

In sufficiently general case of the control problem regulating the state $\vec{X}$ of the technical system, the quality criterion, minimized by the sequence of controls $\{\vec{U}\}$, may have the form (A & B denote as two real symmetric matrices):

$$I_k^{(1)} = E\left\{\sum_{i=1}^{k}\left[\vec{X}_i^T A_i \vec{X}_i + \vec{U}_{i-1}^T B_{i-1}\vec{U}_{i-1}\right]\right\} \qquad (3)$$

representing mathematical expectation of square form of the state and control vectors (i. e. accounting for the system and control behavior). Such a criterion makes it possible to take advantage of the theorem of division for synthesis of optimum control, if necessary. This theorem for the stochastic system supposes that for Gaussian random processes and generalized square quality criterion, the optimum law of control comprises an optimum filter for estimating state vector and an optimum linear regulator connected in series. Since the final target of control consists in the variation of the system state, it is preferable that the control by means of feedback takes into account all the history of the system (previous states) and the information on the current state of the system (including the results of current measurements). Here, one can see a direct connection with the stochastic problem for evaluation parameters (parametric identification of boundary conditions, thermophysical characteristics, etc.) at parallel determination of the state vector (temperature field). This connection in question is well seen from the example of writing functional minimized by the desired, in parametric identification, estimates obtained with the help of iterative filter (Matsevity and Moultanovsky, 1979, Matsevity and Moultanovsky, 1982, Matsevity and Moultanovsky, 1984):

$$\hat{I}_k^{(2)}\left[\vec{X}_k, \tilde{\vec{Y}}_k, j\right] = E\left\{\sum_{i=1}^{k}\left[\left\|H_i \hat{\vec{X}}_{i/i}^{(j)} - \tilde{\vec{Y}}_i\right\|^2_{R_i^{-1}} + \frac{1}{j(i)}\left\|\vec{X}_i - \hat{\vec{X}}_{i/i}^{(j)}\right\|^2_{[P_{i/i-1}^{(j)}]^{-1}}\right]\right\}, \qquad (4)$$

where

$\vec{X}_i$ is augmented state vector, that includes state vector proper and vector of unknown parameters, in particular, control vector;
H_i is the matrix of measurements, that consists of the zeros and units because augment state vector always includes the temperatures from the measurement vector;
$P_{i/i-1}$ is a covariance matrix of the prediction estimate's errors;
$j(i)$ is the number of iterations at i-th time step.

The functional (4) is comprised of two square forms has the form similar (in a sense) to the criterion (3). Therefore, it is possible to seek for the control strategy using the methods of identification theory, i. e. to carry out identification of the controlling action. Such an identification approach to the solution of optimum control problems is considered in all our research.

Instead of the fulfillment of the concept of controllability, this approach centers on the concept of parametric identifiability. Concept of *Parametric Identifiability* consists in possibility to determine unbiased and consistent estimate of parameters of mathematical model with the results of measurements of some coordinates or parameters during specific period of time. In other words, a concept of identifiability is closely associated with observability of the system. Concept observability was first coined by Kalman, (1960). In our research and papers we follow the most comprehensive definition of observability from reference (Meditch, 1969): System is referred to as *Fully Observable* if knowing its output vector $\left(\tilde{\vec{Y}}(t)\right)$, it is possible directly or indirectly to determine the system state $\vec{X}(t)$ at any interval of time $t_0 £ t £ t_1$. By this means, non-observable system cannot be identified, or put it in another way, it is impossible to identify parameters associated with non-observable states (Graupe, 1976). The concept of *Stochastic Observability* is similar to the notion of stochastic controllability and it assures the convergence of error of estimate of state vector to zero (or some finite value defined by the errors of measurements) with increasing number of discrete measurements (Feldbaum, 1966).

Turning back to theory of stochastic optimum control, point out that this theory supposes the construction of optimum control strategy by any method with attain of the control target and fulfilling all constraints $\vec{a}_k$ imposed on the controls or variable of state (Leondes, 1976). At this takes place, the best from all admissible control strategies will be controlling

sequence $\{\vec{U}\}$, that extremize pre-constructed quality criterion (functional), such as criterion (3). It is necessary to note here, that all sufficiently complex constraints in the problems of optimum determinate control must be represented in explicit way. In contrast to it, the problem of optimum stochastic control is formed only in such a way that all the constraints should implicitly enter the quality criterion. The algorithm of the controlling function search is constructed so that the prescribed quality function could be extremized. It should be added, that, since the function involving a set of random values will also be a random one, for solving the problem of optimization, one considers its mathematical expectation being a determinate value (e. g. functional (3) or (4)).

Above-mentioned control strategy as a function of measurements and previous controls $\vec{U}_k = \vec{n}\ (\tilde{\vec{Y}}_k, \vec{U}_{k-1})$, is suited to the control strategy for the closed-loop control system (Leondes, 1976), that is we have to deal with an optimum strategy of the closed-loop stochastic control minimizing the criterion of (3) type.

Stochastic control, as well as determinate one, is required to take into account the degree of indeterminacy of state vector's knowledge. Applied here principle of stochastic dual control supposes adaptation ("training") of system of stochastic regulator. This adaptation allows reducing the indeterminacy of state vector.

APPROACH TO THE CONTROL ON THE CONSTRAINTS PROBLEM BRINGING TO THE PROBLEM OF PARAMETER IDENTIFICATION

The statement of optimal stochastic control problem was proposed such that all constraints $\vec{a}_k$ imposed on the system are included into the vector of observation (measurements) $\tilde{\vec{Y}}_k$. Such an approach is appropriate for the types of systems that can operate on the constraints, i. e. the control target will be attained proceeding along the trajectory of limiting values of restricted parameters. These are the problems of maximum (or optimum) speed of respond, when the system transfers from its known initial state to its final one with maximum speed and without exceeding any of imposed constraints. Another type of these problems is the control problems with non-terminal (local) criterion. These problems suppose a given initial system state, the vector of final state is not restricted, and it is required to provide the extremum of given criterion at each current moment of time, minimizing therewith the deviations of the system from the given trajectory of measured parameters. In the latter problem the optimal control is provided at the cost of choosing of local-optimal control strategy.

The inserting of constraints into the vector of measurements allows simultaneously satisfying the above-mentioned necessary condition of optimal stochastic control: all constraints should be included into the quality criterion. In such a way the problem of stochastic optimal control reduces to the search for control strategy by means of methods of statistical parameter estimation. In such a statement, it is possible to construct common method and computational algorithm for parameter identification, optimization or diagnosis problems solving, and determination of the controlling sequence. By the way, usually in the problem of determination of the controlling sequence the constraints imposed on the phase coordinates are given from physical considerations and conditions of operation of the system. These constraints (or, in proposed statement, the vector of "measurements") are responding appropriately on the control signal. In that case, a priori checking of condition of observability and identifiability is not needed.

RECCURENT METHODS OF SOLVING OF OPTIMAL CONTROL PROBLEMS

It results from Bellman principle of optimality and theory of dynamic programming (Angel, et al., 1972), that, in the problems with discrete time, the process of determining the control strategy (controlling sequence) may be reduced to recurrent computation of separate members of this sequence $\{\vec{U}_K\} = \{\vec{U}_1, \vec{U}_2, ..., \vec{U}_K\}$. As this take place, principle of optimality itself is a means for solving k-step problem of minimization of functional (3)

$$I_{k\ \min}^{(1)} = \underbrace{\min}_{k} E\Big(....\Big(\underbrace{\min}_{k=2} E\Big(\underbrace{\min}_{k=1} E\Big\{ \sum_{i=1}^{k} \Big[\ \vec{X}_i^T A_i \vec{X}_i + + \vec{U}_{i-1}^T B_{i-1} \vec{U}_{i-1} \Big]\Big\}\Big)\Big)....\Big)$$

by recasting this k-step problem to *one*-step problem.

One possibility of the solution of this problem is the recurrent method of iterative filter (Matsevity and Moultanovsky, 1979, Matsevity and Moultanovsky, 1982), which has been created for the solution of parametric and non-parametric identification problems. The sequence of estimates $\left\{\hat{\vec{X}}_{k/k}^{(j)}\right\}$ are obtained with the help of iterative filter, minimizes the quality functional (4). This filter rests on the linearized mathematical model (2) written in the finite difference matrix form

$$\vec{X}_{k+1} = \Phi_{k+1,k}\ \vec{X}_k + F_{k+1,k}\ \vec{U}_k + G_{k+1,k}\ \vec{W}_{k+1}, \qquad (5)$$

where $F_{k+1,k}$, $F_{k+1,k}$ $G_{k+1,k}$ are the transition matrices.

However, there is little point in the use of the iterative filter for the construction of the comprehensive method for the solution of both identification and control problems. The matter is that the good points of iterative filter, such as high accuracy of the solutions, the availability to use number of iterations as a regularizing parameter, or the possibility to adopt the expression $\dot{\vec{Z}} = 0$ as an equation for unknown parameters (Matsevity and Moultanovsky, 1979, Matsevity and Moultanovsky, 1982), are accompanied by significant

disadvantages. Firstly, the linearization of the initial mathematical model and corresponding substitute of this model by the equation (5) is always required. Secondly, the transition matrices of this system should be known with certainty, whereas, during the iterative filter calculations only their estimates, even if refined by iterative process, obtained. Thirdly, due to enormous dimension of algorithm's vectors and matrices, a huge computer's memory volume and speed is required. The iterative-filter-based method of Adaptive Iterative Filter (AIF), first discussed in the references (Matsevity and Moultanovsky, 1988, Matsevity and Moultanovsky, 1991, Moultanovsky, 1996) allows to avoid these disadvantages and to solve the stated problem. Since this method does not require the linearization of initial equation (2), and sometimes even the equation (1), it makes possible to eliminate extra errors of solution that associated with system linearization.

The quality functional that taking into consideration both the model fit and control vector behavior and allowing at each time step to search for conditionally-optimum control, can be written in the form close to expression (4)

$$\hat{I}_k^{(3)}\left[\vec{Z}_k, \tilde{\vec{Y}}_k, j\right] = E\left\{ \sum_{i=1}^{k} \left[\left\| h\left[\hat{\vec{Z}}_{i/i}^{(j)}\right] - \tilde{\vec{Y}}_i \right\|_{R_i^{-1}}^2 + \frac{1}{j(i)} \left\| \vec{Z}_i - \hat{\vec{Z}}_{i/i}^{(j)} \right\|_{\left[P_{i-1/i-1}^{(j)}\right]^{-1}}^2 \right] \right\}. \quad (6)$$

The required optimum control strategy $Z_k = \{\vec{Z}_i\}$ searches as the best estimate in the sense of the estimate mean square error minimum. Minimization over all controls limited by the domain a ($\left|\vec{Z}_k\right| \le a_k$) being carried automatically since all the constraints a_i imposed on the system are included into the vector of "measurements" $\tilde{\vec{Y}}_k$. Thus, at each k-th time step one identifies the estimates of conditionally-optimum controls $\vec{Z}_k$, the complex of which will make up the general strategy of control Z_k transferring the system under study into its final state.

One more significant source of extra errors of identification is a preliminary approximation of desired unknown function. Either polynomial, or spline, or other very accurate approximation is still approximation, which is why it contributes the extra estimate errors. That is why our approach of pointwise identification of unknown parameters (Matsevity and Moultanovsky, 1982, Matsevity and Moultanovsky, 1986) along with stochastic way of problem solving and with method of iterative calculations at each time step, makes it possible to avoid such kind of extra errors of estimates.

ADAPTIVE ITERATIVE FILTER AND ITS FEATURES

The form of the functional (6), with consideration for identification of vector $\vec{Z}_k$ explained above, supposes that the search of the estimates of the desired parameters is carried out with the aid of the Adaptive Iterative Filter. The fundamental algorithm of AIF can be written as following

$$\hat{\vec{Z}}_{k+1/k+1}^{(j)} = \hat{\vec{Z}}_{k+1/k+1}^{(j-1)} + K_{k+1}^{(j)}\left[\tilde{\vec{Y}}_{k+1} - \hat{H}_{k+1}^{(j)}\hat{\vec{Z}}_{k+1/k+1}^{(j-1)}\right]; \quad (7)$$

$$K_{k+1}^{(j)} = P_{k/k}\left[\hat{H}_{k+1}^{(j)}\right]^T \left\{ \hat{H}_{k+1}^{(j)} P_{k/k} \left[\hat{H}_{k+1}^{(j)}\right]^T + R_{k+1} \right\}^{-1}; \quad (8)$$

$$P_{k/k} = \left[I - K_k^{(i)}\hat{H}_k^{(i)}\right] P_{k-1/k-1} \times \left[I - K_k^{(i)}\hat{H}_k^{(i)}\right]^T + K_k^{(i)} R_k \left[K_k^{(i)}\right]^T; \quad (9)$$

$$\hat{H}_{k+1}^{(j)} = \left\{\frac{\P\vec{Y}}{\P\vec{Z}}\right\}_{k+1}^{(j)}. \quad (10)$$

Here, the vector $\hat{\vec{Z}}_{k+1/k+1}^{(j)}$ is the unbiased, with minimum dispersion, estimate obtained for the vector of the parameters being defined at the j-th iteration of the $k+1$-st time step on the basis of the vector measurements $\tilde{\vec{Y}}_{k+1}$; $P_{k/k}$ is the covariance matrix of the estimate errors; R_{k+1} is the covariance matrix of the measurement errors and admissible deviations from the constraints imposed on the system in the control problem; K_{k+1} is the weight matrix; $\hat{H}_{k+1}^{(j)}$ is the non-stationary artificial matrix of measurements, which is calculated, and respectively changed, "corrected" (is refined) at each current iteration j. This matrix incorporates all internal coupling of the thermal system and takes into account the estimates obtained. The $\hat{H}_{k+1}^{(j)}$ matrix terms represent the partial derivatives of the measured parameters with respect to the estimated (identified) ones. The calculation of the artificial measurement matrix by the numerical method requires the solution of the equations of the process under study several times at each iteration, that is solving the number of the direct heat transfer problems at each iteration.

The mathematical model of the thermal system formalized in the form of a matrix-vector equation relates the estimation $\hat{\vec{X}}_{k+1}^{(j)}$ of the system state vector $\vec{X}_{k+1}^{(j)}$ at j-th iteration to the estimates of vectors of state and control, and vector of perturbation all at the previous $j-1$-st iteration, describes the

Markov process, and represents a non-linear stochastic equation similar to the equation (2):

$$\hat{\vec{X}}_{k+1}^{(j)} = \vec{f}\left(\hat{\vec{X}}_{k+1}^{(j-1)}\right) + j\left(\hat{\vec{X}}_{k}^{(j-1)}\right)\hat{\vec{U}}_{k+1}^{(j-1)} + \\ + y\left(\hat{\vec{X}}_{k+1}^{(j-1)}\right)\vec{W}_{k+1}. \quad (11)$$

For the solution of the equation (11), estimates $\hat{\vec{Z}}_{k+1/k+1}^{(j-1)}$ substitute control vector $\hat{\vec{U}}_{k+1}^{(j-1)}$ in the right-hand side of this equation. The results of solving of equation (11) represent the estimate of the state vector, starting from $\hat{\vec{X}}_1$.

The limitation of the iterations' number "i" (equation (9)) or the stop-criterion is used as the regularizing (tolerant) factor of the iteration process at each time step (Matsevity and Moultanovsky, 1992, Alifanov, et al, 1995, Moultanovsky and Khawaja, 1997, Moultanovsky, 1997, Moultanovsky, 1998a). This number "i" is selected from the condition of the agreement of mean square errors (s) of the measurements with the value of general discrepancy, both over k moments of time.

Returning to the unbiasedness of the estimate $\hat{\vec{Z}}_{k+1/k+1}^{(j)}$, it notes that its unbiasedness depends upon the accuracy of statistical linearization of initial mathematical model (1) and (2) or (11). However, the huge advantage of AIF is that the linearization of mathematical model is required only for the calculation of the measurement matrix (10), and, in general, it is not needed for the computation of estimates (7) or (11). That is why, first, the obtained estimates $\hat{\vec{Z}}_{k+1/k+1}^{(j)}$ can be counted as unbiased, and, second, extra inadequacy between process under study and its mathematical model, introducing by the model's linearization, is excluded. One more great advantage of AIF is that the method is highly tolerant to the possible measurement's anomalies.

Because of ill-posedness, incorrectness of problems under study, the solution obtained by means of algorithm (7) -- (10) with regularizing stop-criterion represents the optimization with restriction. The optimization is herein taken to mean that the quality functional, such as functional $\hat{I}_k^{(3)}$ from expression (6), reaches its extremum by means of estimate (7). However, it is appropriate at this point to recall that the case at hand is an identification of the estimates from the admissible, regular, stable domain, rather than the limitations in the control problem. Indeed, second member of functional (6), which depends upon the number of iterations, is responsible for the regularization of the solution of ill-posed problem by means of stop-criterion of iterative process. Our investigation of convergence of filter's iterative process indicates that the following inequality is fulfilled in any "k"

$$\hat{I}_k^{(3)}\left[\hat{\vec{Z}}_{k/k}^{(j+1)}, \tilde{\vec{Y}}_k\right] < \hat{I}_k^{(3)}\left[\hat{\vec{Z}}_{k/k}^{(j)}, \tilde{\vec{Y}}_k\right]$$

and the sequence $\hat{I}_k^{(3)}[j]$ converges to the true minimum of the goal functional at this time step. That is why Adaptive Iterative Filter falls in the category of admissible.

Going back to the principle of dual control by Feldbaum, involving simultaneous carrying out of two functions (identification of the object's characteristics and control of the object with the purpose of bringing it to the required state), it should be noted that our approach to the solution of the control problem makes it possible to take advantage of the principle of dual control in pure form, without resorting to the assumptions of the theorem of division. The disadvantage of the methods using the theorem of division was discussed earlier. In fact, at constructing the optimum regulator, the method of AIF allows to give up a separate algorithm (the optimum filter) for the identification of the state vector. Estimates of the state vector found with the help of equation (11) are used for computation of the matrix of measurements, i. e. first function of the principle of dual control is implemented automatically.

Summing up everything said above, one could make a conclusion that an adaptive iterative algorithm of control, which allows for the principle of dual control by Feldbaum, has been constructed.

With the help of proposed method of AIF and suggested calculation procedure, the number of problems has been solved. These problems include: the construction of a optimum law for starting a turbomachine, the optimization of procedure of vehicle evaporator or heater selection, the identification of optimum thermal mode of post-implantation activation annealing of semiconductor materials, the optimization of slag-granulation process.

TURBOMACHINE CONTROLLING PARAMETERS IDENTIFICATION

One of the major problem of controlling heat power engineering system is the problem of identification of an optimum law of starting turbine followed by driving machine to its operating conditions. Maximum values of the temperature stresses and temperature differences in the turbine casing as well as steam maximum temperature in its flow passage act in this problem as imposed constraints that allow the constructing an optimum graph in question. A graph making it possible to put the set in operation on the border of admissible values of the parameter will be an optimum solution or, at least, quasi-optimum solution if one considers a piecewise constant graph (step function) as an optimum function (Feldbaum, 1966).

Adaptive Iterative Filter approach makes it possible to construct an optimum law for starting turbomachine. This problem was illustrated by investigation of steam turbine high-pressure cylinder internal casing. A part of this casing cross-section being shown in Fig. 1.

The initial mathematical model of element being investigated (the equations of thermal conductivity and boundary conditions) has the form

$$\frac{1}{r}\frac{\P}{\P r}\left[r\,l\ (T)\frac{\P T}{\P r}\right]+\frac{1}{r^2}\frac{\P}{\P j}\left[l\ (T)\frac{\P T}{\P j}\right]= \tag{12}$$

$$= c_V\,.\,(T)\frac{\P T}{\P t};$$

$$l\ (T)\frac{\P T}{\P j}\ \Big|_{\substack{j=0;\\ j=p/2}}=0; \tag{13}$$

$$-\,l_s(T)\frac{\P T}{\P r}\ \Big|_{r=R_{\text{int}}}=h_1(T-T_{m1}); \tag{14}$$

$$-\,l_s(T)\frac{\P T}{\P r}\ \Big|_{r=R_{\text{ext}}(j)}=h_2(T-T_{m2}); \tag{15}$$

Turbomachine load schedule was considered to be known making it possible to calculate the heat transfer coefficients h_1 and h_2 on both internal, radius R_{int}, and external, radius R_{ext}, casing surfaces (Fig. 2). The optimum laws of steam temperature T_{m1} and T_{m2} changes were defined. As a constraints vector $(\vec{a}\ )$ appeared temperature stresses, actually, constraints on temperature gradient $\frac{\P T}{\P r}\ \Big|_{T=T_{21}}\le C_1$ and temperature differences $DT=T_{15}-T_{18}\le C_2$ in the casing as well as constraints on maximum value of steam temperature(s) $T_{m1}\le C_3$, $T_{m2}\le C_4$. A case was investigated when dependence between steam temperature at the external and internal circuit of heat transfer is known $T_{m2}=0.88T_{m1}$. In that case the "measurement" vector and matrix of "measurements" can be written as following

$$\tilde{\vec{Y}}=\{\ \tilde{C}_2,\tilde{C}_1,\tilde{C}_3\ \}_k^T;$$

$$\hat{H}_k=\{\ \frac{\partial(\Delta T)}{\partial T_{m1}};\quad \frac{\partial}{\partial T_{m1}}\Big[\ \frac{\partial T}{\partial r}\ \Big|_{T=T_{21}}\ \Big];\quad 1\ \}_k^T.$$

The problem under study in such an event is the problem of maximum speed of response. The solution of this problem with desired control parameter T_{m1} is shown in Fig. 3. The lower plot corresponds to the desired control law (temperature T_{m1}, $C_3=500$) and the upper one coincide to "measured" parameters ($C_1=40$, $C_2=60$).

However, our experience shows that the most reliable results are obtained on prescribing some random domain around the exact values of the constraints (C_1, C_2, C_3), that is, the sizes of these domains change in random way by the normal law with the value of mean square deviation $s_1=0.03C_i$ (Fig. 4). In other words, these constraints were prescribed with an accepted interval of deviation (accuracy of determination), i. e. white Gaussian noise with s equal to 3% of appropriate value was applied. By the way, such a pattern of C_i prescribing, as it was noted, corresponds better to the real processes.

OPTIMUM THERMAL MODES IDENTIFICATION OF POST-IMPLANTATION ACTIVATION ANNEALING OF SEMICONDUCTOR MATERIAL

Optimum thermal modes of post-implantation activation annealing of semiconductor materials were identified. In other words, it was constructed an optimum strategy for controlling the modes of activation annealing, which is related to the satisfaction of the following conditions: 1) by the end of annealing plate surface layer must be heated to the required temperature T_{sur} ; 2) final profile of the concentration $C(x)$ should be close to the prescribed one; 3) total time of the process should be minimum accounting for the satisfaction of the constraints imposed on temperature gradients and temperature differences across the plate thickness. Clearly, the problem in question belongs to the class of control problems with maximum speed of response.

The density of heat flux, governing the conditions of heating, was considered as a parameter under identification. In the control problem this parameter is defined as conditionally optimum control. In that event the "measurement" vector appears as

$$\tilde{\vec{Y}}_k=\left\{T_{sur};\Delta\ T_{\max};\left(\frac{\P T}{\P x}\right)_{\max}\right\}^T,$$

where

T_{sur} is the required temperature of the surface layer (1065 K);

DT_{max} is the maximum temperature difference across thickness (limit is 320 K);

$(\P T/\P x)_{max}$ is the maximum gradient (limit is $3*10^6$, K/m).

These constraints were prescribed with an accepted interval of deviation (accuracy of determination), i. e. white Gaussian noise with s equal to 3% of appropriate value was applied.

In the process of identification the following occurs: if any one of three calculated components of "measurement" vector (the last term of the right-hand member of the main AIF equation (7)) $\{\hat{\tilde{Y}}_{k+1}\}[i]=\{\hat{H}_{k+1}^{(j)}\hat{\tilde{q}}_{k+1/k+1}^{(j-1)}\}[i]$ reaches or exceeds its maximum value (allowing for the accepted interval of deviations), the iterative process stops and the estimate $\hat{\tilde{q}}_{k+1/k+1}^{(j)}$ at the next step is artificially set as equal to zero and remains so until the above component enters within the limit of the constraints. In other words, the expression (7) is substituted by the following equation:

$$\hat{\vec{q}}^{(j)}_{k+1/k+1} = \begin{cases} \hat{\vec{q}}^{(j-1)}_{k+1/k+1} + K^{(j)}_{k+1}\left[\tilde{\vec{Y}}_{k+1} - \hat{H}^{(j)}_{k+1}\hat{\vec{q}}^{(j-1)}_{k+1/k+1}\right] \\ by \quad \{\hat{\vec{Y}}_{k+1}\}[i] < \{\tilde{\vec{Y}}_{k+1}\}[i],\ i=1,2,3 \\ \hline 0, \\ by \quad \{\hat{\vec{Y}}_{k+1}\}[i] \geq \{\tilde{\vec{Y}}_{k+1}\}[i],\ i=1,2,3 \end{cases}, (16)$$

The least deviation from the required final profile of admixture concentration *C(x)* is determined by the following conditions. First criterion ("quality" criterion of the obtained admixture ions profile) assumes that the admixture mass content beyond the layer d must not exceed 10% from the admixture mass content in the activated layer. Second criterion (calculated in terms of the difference between the maximum and minimum values of concentration in the layer) determines the uniformity of admixture distribution across the activated layer. Numerically, first criterion can be written by the expression

$\frac{\int_d^{3d} C(x)dx}{\int_0^d C(x)dx} < 0.1\int_0^d C(x)dx$ and second one by the term

$$C(x)_{\max} - C(x)_{\min} \leq 0.34 \quad C(x)_{\max}, \qquad 0 \leq x \leq d,$$

where the values of the multipliers 0.1 and 0.34 are taken from the results of numerical experiment on simulation of plate activation annealing.

These criteria in question are taken into account in the computational algorithm of Adaptive Iterative Filter in the form of the conditions for shutting-down the technological process. This process stops on the first satisfaction of both inequalities.

Figure 5 shows the results of numerical experiment on construction of an optimum strategy to control the activation annealing of an arsenide-gallium plate, implanted by boron ions, with oxide silicon coating. From the bottom to top are located: optimum law of change of surface heat flux density and correspondingly (top graph) laws of change of surface temperature T_{sur}, maximum temperature difference across the plate thickness DT_{max}, and maximum gradient $(\P T/\P x)_{max}$. It is seen that the local peaks of the identified function $q(t)$correspond to one of the value

$\tilde{\vec{Y}}_k = \left\{ T_{sur}; \Delta\ T_{\max}\ ; \left(\frac{\P T}{\P x}\right)_{\max} \right\}^T$ reaching its limit. The process shut-down occurred at t *=2.92 sec*. In other words, by this moment the required concentration profile was reached.

OPTIMIZATION OF SLAG-GRANULATING PROCESS

The problem is being considered at this example was appeared from the technological metallurgical processes wherein the materials change their phases. Quality of the resulting product as well as the capacity of the technological process is predominantly determined by the time taken to product gets its ultimate state. That is why the problem was set up as control problem of optimum speed of response. As it was already said, the solution of the similar problems implemented in terms of the optimum control theory involves preliminary construction of a quality criterion (3) that should reach its extremum with appropriately identified controlling signals.

Technological process of cooper fusion's slag-granulating was being investigated. The special drum-crystallizers were used and they served as well for the slag's heat recovery. The slag-granulating unit is shown in Fig. 6. This unit is built-up from two heat removing drums 1. These drums are positioned under the slag's outlet 2. Molten slag 3 crystallizes on the surfaces of the rotational drums and later on the solid layer of slag is cropped by knives 4. The fundamental parameters that affect technological process are: the size of drum's surface in contact with slag, the intensity of cooling, and the speed of drums' rotation. Because the size of unit was specified and intensity of cooling was determined by the type of coolant, the decisive parameter might be in such a case only the speed of drums' rotation. By this means, this speed or, that is the same, time of contact of working surface with slag is the sought-for parameter in the problem being considered. Because the width of solid layer is uniquely determined by the value of speed of drum's rotation, one can talk about the control of the slag's congelation process. This process is depicted in Fig. 7. Here the boundary B_p (dash line) substitutes for the liquid/solid transition region because its width is two orders of magnitude less than its length. The possibility of the setting up 2-dimension problem is determined by the uniform of the level and temperature of the molten slag and intensity of cooling along the whole length of the drums, that is any perpendicular to the drum's axis section has the same temperature distribution. The substitution of the liquid/solid transition region by the boundary B_p and possibility to solve 2-dimension problem brought the process under study to the Stefan's problem. However, because of using the stochastic approach to the identification of the controlling action, the phase transformation temperature $\tilde{T}_p$ accounts liquidus T_l and solidus T_s temperatures at each time step

$$\tilde{T}_p = f\left(\bar{T}_p \pm \Delta T_p\right), \qquad (17)$$

where $\bar{T}_p$ the average phase transformation temperature, $DT_p = \left(T_L - T_S\right)/2$.

Second stage takes place without change of slag's phase and mathematical model of the process at this stage is expressible in terms of mathematical model for the heat transfer phenomenon for the solid body. That is why heat transfer processes at the first and second stages can be considered separately.

Because the surface's curvature affects the results of the heat transfer inside the drum just for the drum's diameter less

than 0.2m and the process under study has such a minimal diameter more than 0.5m, the mathematical model of the granulating process can be represented in Cartesian coordinate system (Fig. 8) (for both stages of the process):

$$\frac{\partial}{\partial x}\left[\lambda_{sl.S}(T)\frac{\partial T}{\partial x}\right]+\frac{\partial}{\partial y}\left[\lambda_{sl.S}(T)\frac{\partial T}{\partial y}\right]= = C_{V\,sl.S}(T)\frac{\partial T}{\partial t}; \quad (18)$$

$$\frac{\partial}{\partial x}\left[\lambda_{d}(T)\frac{\partial T}{\partial x}\right]+\frac{\partial}{\partial y}\left[\lambda_{d}(T)\frac{\partial T}{\partial y}\right]=C_{Vd}(T)\frac{\partial T}{\partial t}; \quad (19)$$

$$-\lambda_{d}(T)\frac{\partial T}{\partial y}=h_c(T-T_c); \quad (20)$$

$$\lambda_{sl.S}(T)\frac{\partial T_{sl}}{\partial y}=\lambda_{d}(T)\frac{\partial T_d}{\partial y}; \qquad T_{sl}=T_d; \quad (21)$$

$$\lambda_{sl.L}(T)\frac{\partial T}{\partial n}\Big|_{B_p}=\lambda_{sl.S}(T)\frac{\partial T}{\partial n}\Big|_{B_p}+ +\frac{L\,r_{sl.S}(T)}{N}V(t); \quad (22)$$

$$-\lambda_{sl.S}(T)\frac{\partial T}{\partial y}\Big|_{l_1<x\le l;}=h_a(T-T_a)+ +e\,C_0\left[\left(\frac{T}{100}\right)^4-\left(\frac{T_a}{100}\right)^4\right]; \quad (23)$$

$$T_{sur}\Big|_{0\le x\le l_1;}=T_L; \qquad T_{sur}\Big|_{x=0;\ \Delta\le y\le\Delta+d;}=T_L; \quad (24)$$

$$-\lambda_{sl.S}(T)\frac{\partial T}{\partial x}\Big|_{x=l;\ \Delta\le y\le\Delta+d;}=h_a(T-T_a)+ +e\,C_0\left[\left(\frac{T}{100}\right)^4-\left(\frac{T_a}{100}\right)^4\right]; \quad (25)$$

$$\lambda_{d}(T)\frac{\partial T}{\partial x}\Big|_{x=0;\ x=2pR;\ \ 0\le y\le\Delta;}=0. \quad (26)$$

Here, $\lambda_{sl.S}$ and $C_{Vsl.S}$ are the solid slag thermal conductivity and specific heat when $T<\tilde{T}_p$; $\lambda_{sl.L}$ and $C_{V\,sl.L}$ are the liquid slag thermal conductivity and specific heat, respectively, for $T\ge\tilde{T}_p$; λ_d and C_{Vd} are the thermophysical characteristics of the drum's material; h_c is the heat transfer coefficient between the drum's surface and the coolant; $V(t)$ is the growth velocity (rate) of the solid layer of slag; N is the intensification coefficient of the crystallization (Guyko, 1986); L is the latent crystallization heat; e is the range of the blackness; h_a is the heat transfer coefficient from the slag to air; T_c, T_a, and T_L are temperatures of coolant, ambient air, and liquid (molten) slag, accordingly; C_0 is the Stefan-Boltzmann constant; d is the width of the solid slag layer; D is the width of the drum's wall. The intensification coefficient N takes into account the speed of drum's rotation and it ranges from *1* to *2* in order of increasing drum's speed from zero to the value that are matched by the maximum productivity of the unit.

In order of obtain maximum productivity of the unit, it is necessary to optimize the relationship between the width of the slag's layer and speed of the drum's rotation. From first sight, the productivity can be raised to its maximum by increasing the width of the slag layer. However, this increasing, in its turn, requires the decreasing of speed resulting in a diminution of productivity. The greatest productivity can therefore be achieved by optimizing the width of the layer and the speed. This speed will determine time of all process from the liquid phase of slag to its solid phase with specific temperature that allows to crop solid slag by knives. To put this another way, two optimization problems occurred: optimization of the slag crystallization's thermal process (first stage) and optimization of solid slag's extra cooling process (second stage).

Investigation of heat transfer within the slag's layer has shown that the heat flux to the coolant along the Y-axis is greater by 2 - 3 orders of magnitude than the flux along the X-axis. Taking into account this statement, as well as relationship between system's linear dimensions (the characteristic dimensions of the drums and the liquid slag's pool are greater by 2 – 3 order of magnitude than the width of the slag's layer), it makes sense to consider 1-dimension model of thermal system (Fig. 8, section AB) moving along the X-axis. In this model, temperature changes along the Y-axis, whereas the boundary conditions at the top and bottom of the section (Fig. 8, section A'B') change as system moves along the X-axis. Time that required for the section to travel pathway l_1 is equal to time of crystallization boundary travel from the drum surface (layer width $d = 0$) to the needed width d. By this means, the desired at our problem time is uniquely defined by the velocity of the motion of solid-liquid boundary. Because there is the necessity of the simultaneous solution of the slag-drums systems, we are forced to set up 1-dimension problem for the whole drum's thermal system. However, there is sufficiently large heat removal along the X-axis that will be taken into account by the way of substitution of the real drum's material thermal conductivity λ_d by the efficient thermal conductivity λ_e.

The most complicated problem is the problem of choosing of supporting, "measured" parameters (constraints). It is only the temperature of phase separation (the solid-liquid boundary) can be used in system under study as a supporting (constraining) factor. In such an event, according to the terminology of the control theory, we can speak of a transition

from the problem of optimum/maximum speed of response to the problem of follow-up (non-terminal) control, when at any moment of time it is necessary to optimize parameters of system that moving along a known trajectory $\tilde{T}_p = f[x(t)]$ of transition between the initial $\tilde{T}[x(t_0)]$ and final $\tilde{T}[x(t_k)]$ states. Moreover, in such a case, we actually have arrived to the parametric identification problem when one has to identify the velocity (*V(t)*) of solid-liquid boundary motion (in fact, this is the velocity of crystallization) based on the "measurements" of $\tilde{T}_p$. According to the definition of reference Matsevity and Moultanovsky, (1982), the stated problem is the geometric inverse heat transfer problem, where the measurement equation can be presented as a following expression:

$$\left(\tilde{T}_p\right)_k = Ah[V_k] + \left(\P T_p\right)_k. \qquad (27)$$

Here, $\tilde{T}_p$ is the crystallization temperature, that is moving with desired (subject to identification) velocity V_k at each *k*-th moment of time; *Ah[*]* is the operator of relation between estimated and "measured" parameters; $\left(\P T_p\right)_k$ is the stochastic (white Gaussian sequence) error, that takes into account $?T_p$ and the discreteness of the model under study, that is the jump of temperature between nodes in series.

The search of the parameters (controls) is carried out with the aid of AIF method. In connection with identification of only parameter V_k and taking into consideration the measurement equation (27), the main adaptive iterative filter equation (7) and expression (10) for the "measurement" matrix can be significantly simplified and take the following form:

$$\hat{V}^{(j)}_{k+1/k+1} = \hat{V}^{(j-1)}_{k+1/k+1} + K^{(j)}_{k+1}\left[\left(\tilde{T}_p\right)_{k+1} - \hat{H}^{(j)}_{k+1}\cdot\hat{V}^{(j)}_{k+1/k+1}\right]; \qquad (28)$$

$$\hat{H}^{(j)}_{k+1} = \left\{\frac{\P T_p}{\P V}\right\}^{(j)}_{k+1} = \left\{\frac{\P\left[Ah\left(\hat{V}\right)\right]}{\P V}\right\}^{(j)}_{k+1}. \qquad (29)$$

In that event all parameters in formulas (7) through (10) are the scalars $\tilde{T}_p, \hat{H}, P, R, I$ as well as the desired velocity V of motion of solid-liquid boundary. $\tilde{T}_p$ is defined as normal Gaussian sequence with expectation $E\left[\tilde{T}_p\right] = \bar{T}_p$.

The slag-granulation process under study is shown in Fig. 6. The radius of the drums and its length were 0.3m and 1.6m, respectively. The walls of drums were made from bimetal copper-steel, where width of copper was 0.03m and width of steel was 0.01m. Copper surface has contacted with the slag. The following characteristics have been used for the calculation: latent heat of crystallization was $L = 273\cdot10^3$, J/Kg; crystallization temperature was $\bar{T}_p = 1100^0$C, $\P T_p \cong 0.05\cdot\bar{T}_p$; specific heat was C_p = *{(155+0.05(T-200)/T}10*^{3}, J/(Kg*K); liquid and solid slag density were r_L = *3000*, Kg/m^3, r_S = *3600*, Kg/m^3, respectively; initial liquid slag temperature was $T_L = 1250^0C$; the temperature of cutting of slag (after extra cooling) was 700^{0}C. The slag's thermal conductivity is $l(T = 200^0C) = 1.2$, W/(m*K); $l(T = 400^0C) = 1.7$, W/(m*K); $l(T = 800^0C) = 2.7$, W/(m*K); $l(T = 1200^0C) = 4.4$, W/(m*K).

For the implementation of the identification algorithm the finite difference approximation of the initial mathematical model of the process under study has been made (approximation error was $0(h^2+Dt)$). The estimates of *V(t)* defines at the first stage of solution. This velocity provides a decisive for the determination of the drum's rotation speed *W* since the crystallization process should be completed by the moment of time of the outcome of the slag layer from the liquid pool (Stage I, Fig. 7).

The identification results are shown at the Table 1, where *d* is the width of the slag layer at its outcome from the liquid pool and it equals the width of this layer at the Stage II (Fig. 7, 8), d_1 is the variable width of this solid layer inside the pool during the crystallization process at the Stage I.

Research of the obtained results lends support to the validity of the common tendency that the velocity V_k decreases as the width d_1 increases and it does not depend on the value *d*. Once V_k has been identified, it is easily to determine time t_1 of the completion of the crystallization process for every possible width *d* and thereafter to define the drum's rotation speed *W*. This speed can result from the expressions: $W = l_1/t_1$, m/sec; $n = W´60/(2pR)$, rpm.

Because the value l_1 is variable from *1/4* to *1/8* of the circle length for the different types of slag-granulating drums, the dependence *n = f(d)* are represented as region with boundaries corresponding to the speeds *W* for the extreme values of l_1, that are *1/4* and *1/8* of the circle length (Fig. 9).

Table 1.Velocity identification.

Velocity of solid-liquid boundary motion **V·10^4, m/sec.**				
d,**m**	0.001	0.0015	0.002	0.003
$\delta_1=\delta/10$	4.3	3.4	2.5	0.8
$\delta_1=2\delta/10$	3.2	2.4	1.6	0.3
$\delta_1=3\delta/10$	3.0	1.9	1.2	0.24
$\delta_1=4\delta/10$	2.75	1.7	1.1	0.25
$\delta_1=5\delta/10$	2.6	1.65	1.1	0.25
$\delta_1=6\delta/10$	2.2	1.3	1.1	0.24
$\delta_1=7\delta/10$	2.0	1.25	1.1	0.23
$\delta_1=8\delta/10$	1.9	1.2	1.1	0.22
$\delta_1=9\delta/10$	1.8	1.15	1.0	0.2

According to our estimates, the length *l* (completion of both stages I and II) is always less than 3/4 of the circle length resulting in determination of the disposition of knives at the unit as represented in Fig. 7. Circle length from the start of the stage I to the end of stage II, where the knives are disposed, is equal to 3πR/2. Because the dependence *n = f(d)* are represented as

region, the productivity function $P = f_1(d)$ has the similar view (Fig. 10).

Analysis of the last statement discloses that an increase of the slag layer's width in excess of 3 – 4 mm will cause the productivity sharply to decline. This one testifies about impracticability to get layer more than 3 – 4 mm, while the reduction of d will lead to significant rise of P that is the result of similar increase of n (Fig. 9). However, the lowering $d \leq 0.0005m = 0.5\ mm$ in an attempt to increase the number of revolution makes no sense because it can lead to the appearance of the liquid slag on the solid slag's surface outside the stage I (Fig. 7).

This investigation has been done for the most complicated two-drum slag-granulater. Because the thermal processes are invariant and $d << l$ for all types of units, the obtained results are also true for any kind of granulaters. That is why the distinctions between dependencies $P = f_1(d)$ derived for the different types of units bear just the quantitative nature. Using the obtained results (Fig. 10), it is easy to calculate the geometric parameters of the designing unit of the dry granulation process with a given productivity.

OPTIMIZATION OF SIZE OF HEAT EXCHANGERS OF VEHICLE HVAC SYSTEM (CONCEPT)

The idea of using inverse methods (and, specifically, AIF method) for identification of heat transfer between the air and vehicle heat exchanger has been discussed in the references Moultanovsky and Khawaja, (1997), Moultanovsky, (1997), Moultanovsky, (1998a), and Moultanovsky, (1998b). By means of the same stochastic approach to the identification problem solution, the precise results of heat fluxes (heat transfer coefficients) were obtained. These heat fluxes or heat transfer coefficients were utilized for the design of heat exchanger core material on the basis of nomogram created as an interconnections between core thermal conductivity (material), air-device heat transfer, and airflow rate (evaporator) or heater inlet temperature (heater).

With the same approach, the problem of design optimization of HVAC heat exchanger can be solved. The following is the example of mathematical simulation of the stated problem. Mathematical model of the evaporation process for the vehicle bunk unit evaporator (Moultanovsky, 1998a) are the concern of the present paper. Omitting all reasoning and discussion presented at the reference (Moultanovsky, 1998a), we will immediately skip to the heat flux identification results. The heat flux between the air and outer evaporator surface depending on evaporator surface temperature was identified in the reference (Moultanovsky, 1998a) by means of AIF method (Fig. 11). The identified heat flux is a function of specific heat exchanger core. The total heat transfer of heat exchanger depends upon the air mass flow coming through the device and device's frontal area. So, at the same air flow rate the heat transfer is defined by surface area of the evaporator.

Actually, the frontal area should be separated from the whole apparatus because, on the one hand, the surface area is a most significant factor in influencing the evaporator heat transfer and capacity. That is, the greater this area, the better the evaporator performance. On the other hand, the frontal area of the evaporator is usually limited by the space available for the device. However, it is very often that during the design procedure the envelope given for the evaporator to be filled up is too large for the required heat transfer. Hand in hand with this the money issue is one of the most decisive factors of the HVAC system design. It is reasonable that the smaller evaporator of the same type is the cheaper one. To put it differently, the heat transfer requirements can be satisfied by the apparatus with the smaller frontal area. This is an excellent example of the design optimization by means of inverse methods.

Turning back to the heat flux of the bunk evaporator (Fig. 11) and taking into consideration that its frontal area is equal to S_e=0.2×0.188=0.0376 m^2, one can obtain the similar curve for the total evaporator heat transfer G. With the help of enthalpy analysis the analogue graph can be created for the evaporator capacity. Utilizing the same approach and statistically treating the obtained data, we can bring into being the nomogram, which is serving as an interconnection between the size (frontal area) of the evaporator, air-device total heat transfer, and evaporator outer surface temperature (Fig. 12). This nomogram enables us to choose the preliminary inverse selection (design) of evaporator size, which satisfy the heat transfer or evaporator capacity requirements. For example, for a given total heat transfer value, we can draw a horizontal line so that to meet heat transfer requirements for all range of surface temperatures. Evidently, at the required evaporator total heat transfer of 1,800 Watts (Fig. 12), the frontal area of the device should be chosen no more than 60% of the surface (0.0376 m^2) of tested evaporator.

CONCLUSIONS

1. The proposed approach and methodology of Adaptive Iterative Filter can be used for the solution of various thermal control and optimization problems.
2. Approach of Adaptive Iterative Filter method leads to very effective and valuable solution of the control problems by means of parameter identification methods.
3. Number of various control problems solved shows the comprehensive character of proposed methodology.

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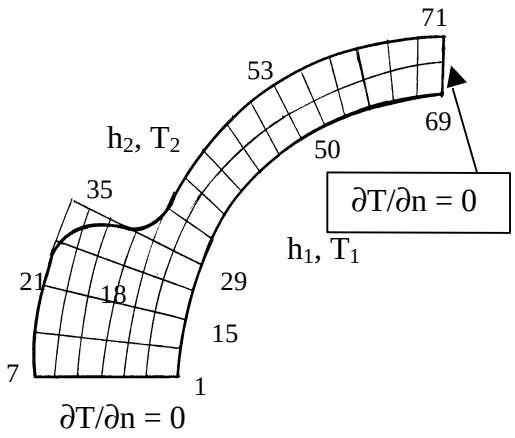
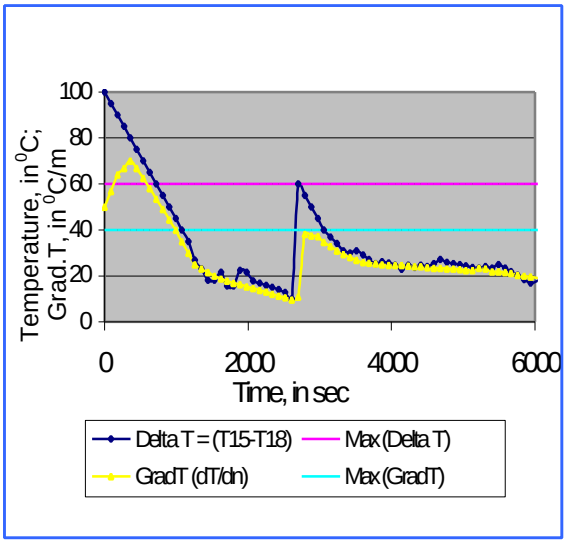


Fig. 1. Cross-section of Turbine High Pressure Cylinder Internal Casing



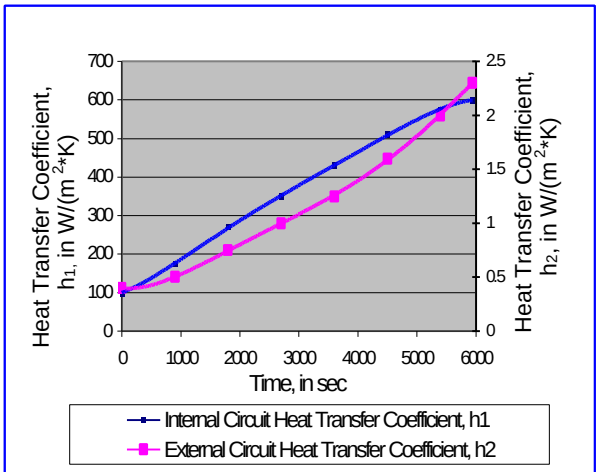


Fig. 2. Turbomachine Load Schedule

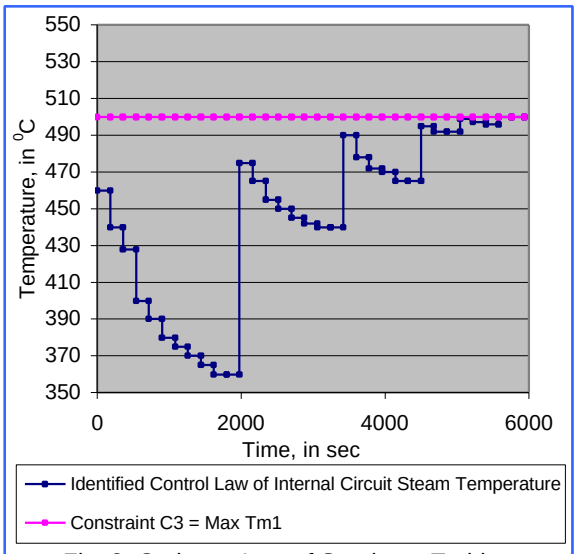
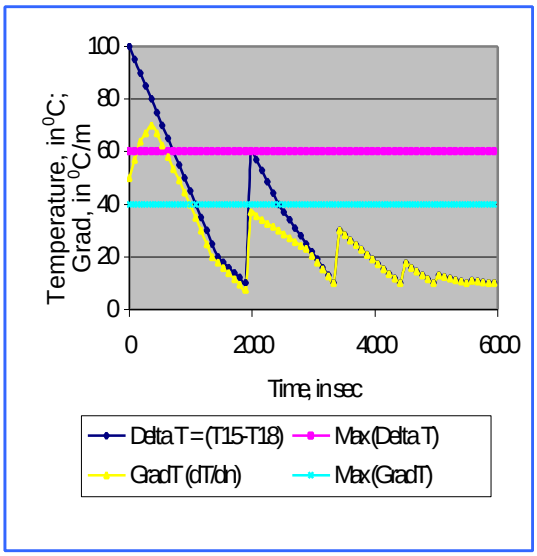
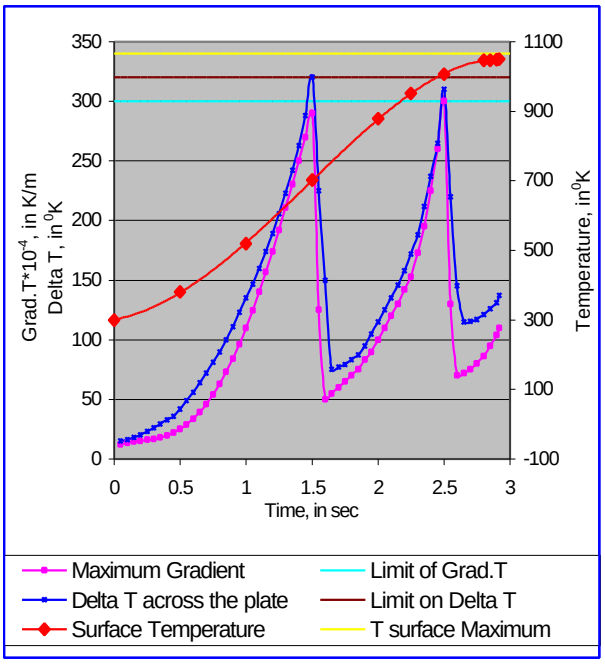


Fig. 3. Optimum Law of Starting a Turbine





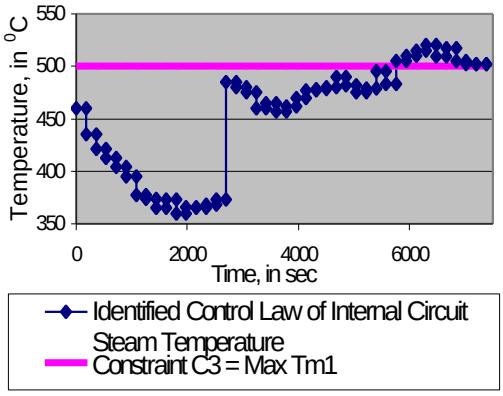


Fig. 4. Optimum Law of Starting a Turbime (random character of the constraints)

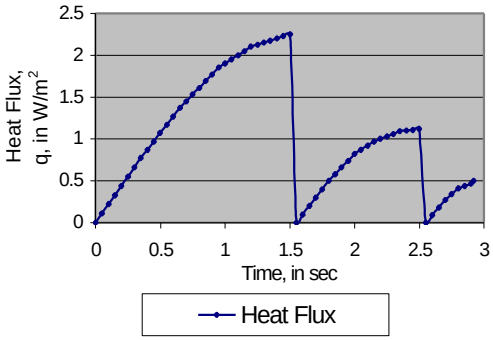


Fig. 5. Optimum Law (Heat Flux) to Control the Activation Annealing

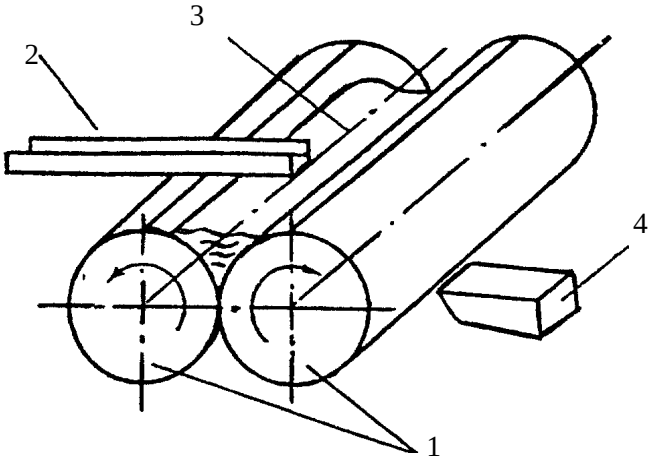


Fig. 6. Slag Granulation Unit.

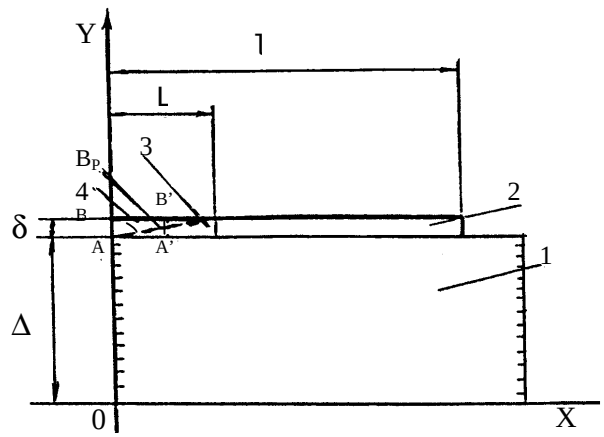


Fig. 8. Model Granulation Process in Cartesian Coordinates
1. Cooling drum, 2. Solid phase in the extra cooling region (stage II),
3 & 4. Solid and Liquid phases in the crystallization zone (stage I),
B_P. Crystallization front in the Stefan problem

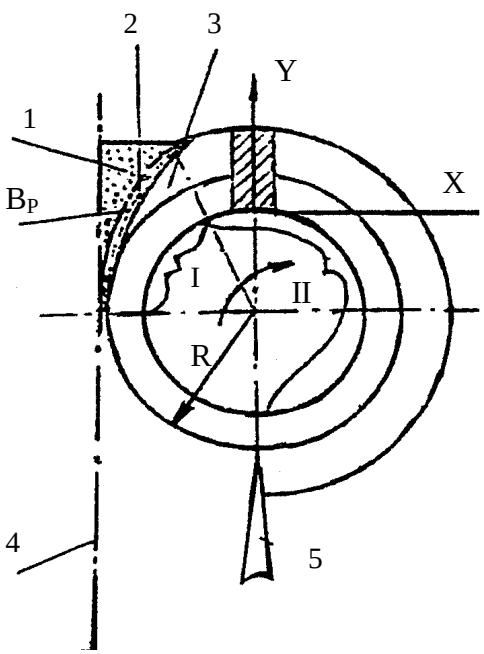


Fig. 7. Process of Slag-Granulating
1. Liquid slag, 2. Liquid-Solid transition region, 3. Solid slag, 4. Symmetric axis of the unit, B_p. Crystallization front in the Stefan problem, I. First stage of the crystallization process, II Second stage

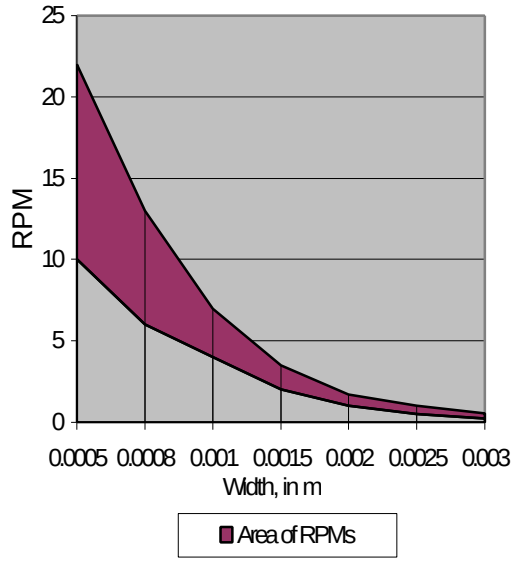


Fig. 9. Number of Drum Revolution as a Function of Layer Width

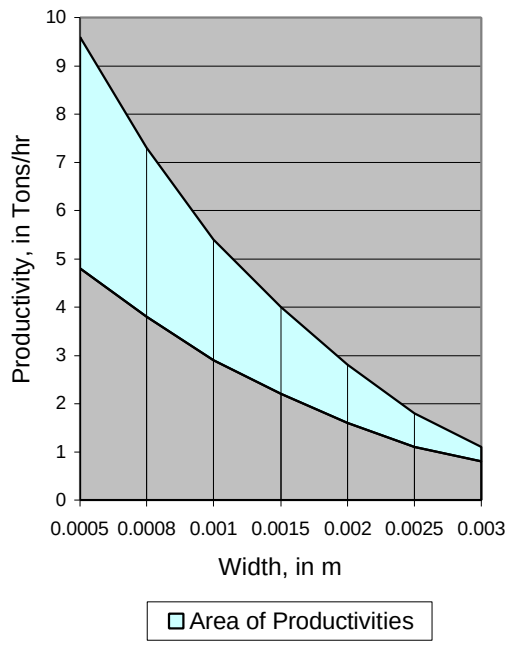


Fig. 10. Granulation Productivity as a Function of Layer Width

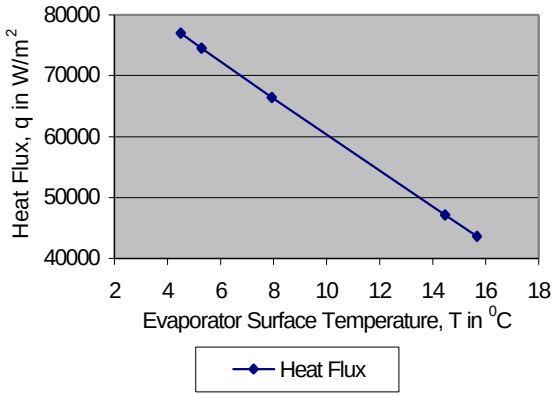


Fig. 11. Boundary Conditions between Vehicle Rear Evaporator & Ambient Air

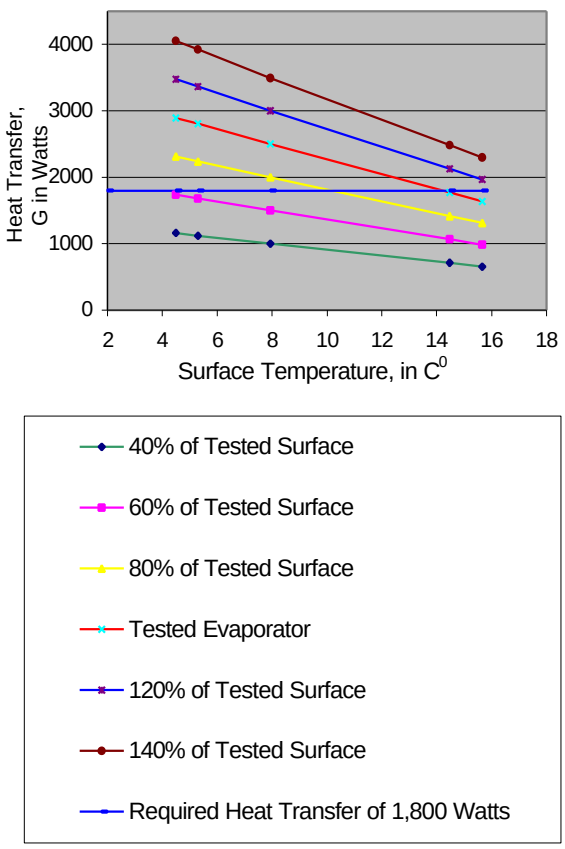


Fig. 12. Heat Transfer between Evaporators with Various Sizes (Frontal Area) & Ambient Air