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NOISE SOURCE IDENTIFICATION OF A RAILWAY CAR MODEL BY THE BOUNDARY ELEMENT METHOD USING SOUND PRESSURE MEASUREMENTS IN 2-D INFINITE HALF SPACE

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ABSTRACT

This paper is concerned with the noise source identification of a railway car model in a two-dimensional infinite half space by means of the boundary element method and a filter theory. It is assumed that the sound pressure is measured at a number of points in the field. The railway car is simplified into a two-dimensional model. The noise sources of the car model are located as a point source at the top of the car, as a distributed source on the surface of the car body and a point source at the bottom of the car. The noise source identification problem under consideration is stated such that a set of parameters representing the noise sources should be identified by minimizing the cost function of a square sum of sound-pressure magnitude difference between the measured and computed ones for all the measuring points. The values of parameters are iteratively modified by a step wise procedure based on the extended Kalman filter theory. Numerical simulation is carried out for a few models and the results obtained are discussed, whereby the advantages and disadvantages as well as the limitations of the proposed inverse analysis are revealed.

INTRODUCTION

Reduction of noise emanating from the vibrating structures is one of the most important subjects in engineering. For this purpose, modeling or identification of the noise sources should be done. This is a kind of inverse problems. Recently, computational approach has been frequently applied to such inverse problems, and successful results have been reported [1-3]. Most of these inverse analyses use numerical methods of analysis for direct problems together with the standard optimization method. However, it seems to be difficult to include measurement errors in the inverse analysis.

This paper presents a method of inverse analysis using the boundary element method and the filter theory to some inverse problems in acoustics. Then, it is applied to the inverse problem of noise source identification in a railway car model.

In the acoustic problems governed by the Helmholtz differential equation, we frequently have to consider infinite domains. The boundary element method is suitable for such infinite-domain problems and can provide an accurate solution considering exactly the radiation condition at infinity[4, 5].

In the inverse analysis using filter theory, it is assumed that a probabilistic system is subject to known excitation and the time-series data of measurement with a known distribution and amount of errors are available for analysis of the system. In the steady-state vibrating system, we may consider a time step as an iteration step of inverse analysis. In this study, we treat inverse analysis of such a steady-state vibrating problem in acoustics.

INVERSE ANALYSIS VIA BEM

In this study, we shall apply the boundary element method and filter theories to the noise source identification of a railway car model. The noise sources are expressed in terms of several parameters which are considered as a state vector in the filter theory. The unknown parameters are identified by minimizing a cost function which can be defined as a sum of magnitude of difference between the measured and computed sound pressures. The sound pressure is computed by the boundary element method which is reported separately [6, 7].

Boundary Element Analysis of Acoustic Field

It is assumed that the acoustic field is in a steady-state vibration with an infinitesimal amplitude. The governing differential equation is the Helmholtz equation which is expressed as

$$\nabla^2 p(\boldsymbol{x}) + k^2 p(\boldsymbol{x}) + f(\boldsymbol{x}) = 0 \tag{1}$$

where p is the sound pressure, f the forcing term and $k = \omega / C_0$ the wave number in which C_0 is the sound velocity and ω the angular frequency. If a point source is located at $\boldsymbol{x}_s$ in the acoustic field, $f(\boldsymbol{x})$ can be expressed by

$$f(\boldsymbol{x}) = A\delta(\boldsymbol{x} - \boldsymbol{x}_s) \tag{2}$$

where A is the intensity of the point source in [Pa], and $\delta(\,)$ is the Dirac delta function.

Including such a point source we may transform the differential equation into the following boundary integral equation[6,7]:

$$\begin{aligned}&\left[\int_\Gamma \left\{q^* - Q^*\right\} d\Gamma\right] p(\boldsymbol{y}) + \int_\Gamma q^* \left\{p(\boldsymbol{x}) - p(\boldsymbol{y})\right\} d\Gamma \\ &= -j\omega\rho \int_\Gamma p^* v(\boldsymbol{x}) d\Gamma + A p^*(\boldsymbol{x}_s, \boldsymbol{y})\end{aligned} \tag{3}$$

where $j = \sqrt{-1}$, ρ is mass density and v the particle velocity. The asterisked functions p^* and q^* are the fundamental solution of the Helmholtz equation and its flux, which are given for two-dimensional problems by

$$p^*(\boldsymbol{x}, \boldsymbol{y}) = -\frac{j}{4} H_0^{(2)}(kr), \quad q^*(\boldsymbol{x}, \boldsymbol{y}) = \frac{\partial p^*(\boldsymbol{x}, \boldsymbol{y})}{\partial n(\boldsymbol{x})} \tag{4}$$

The point $\boldsymbol{x}$ is the integration point and $\boldsymbol{y}$ the source point, whereas $r = |\boldsymbol{x} - \boldsymbol{y}|$ and $H_0^{(2)}$ is the *zero*th Hankel function of second kind.

It is interesting to note that equation (3) is the so-called regularized boundary integral equation in which no singular integral appears and the asterisked function Q^* is the flux of the well known fundamental solution p_L^* of the Laplace differential operator ∇^2, which is given by

$$Q^*(\boldsymbol{x}, \boldsymbol{y}) \equiv \frac{\partial p_L^*(\boldsymbol{x}, \boldsymbol{y})}{\partial n(\boldsymbol{x})} = -\frac{1}{2\pi r}\frac{\partial r}{\partial n} \tag{5}$$

The sound pressure at an internal point $\boldsymbol{x}$ can be calculated after boundary element analysis of equation (3), by using the following integral equation:

$$\begin{aligned}p(\boldsymbol{y}) = &-\int_\Gamma \left\{q^* - Q^*\right\} d\Gamma \cdot p(\boldsymbol{x}_0) - \int_\Gamma q^* \left\{p(\boldsymbol{x}) - p(\boldsymbol{x}_0)\right\} d\Gamma \\ &- j\omega\rho \int_\Gamma p^* v(\boldsymbol{x}) d\Gamma + p(\boldsymbol{x}_0) + A p^*(\boldsymbol{x}_s, \boldsymbol{y})\end{aligned} \tag{6}$$

where $p(\boldsymbol{x}_0)$ is the sound pressure at the point $\boldsymbol{x}_0$ on the boundary which is the nearest point from the source point $\boldsymbol{y}$.

Inverse Analysis Using Filter Theory

In the filter theory, it is assumed that measured data includes an error with a Gaussian distribution. There is the following relationship between the observation vector $\boldsymbol{y}$ of measured data for sound pressure and the state vector $\boldsymbol{z}$ of the parameters corresponding to the noise sources, that is,

$$\boldsymbol{y}_k = \boldsymbol{h}(\boldsymbol{z}_k) + \boldsymbol{v}_k \tag{7}$$

The nonlinear function is expanded into a Taylor series with respect to the state vector, and higher-order terms are neglected. Thus, we can obtain a linearized relation of equation (7) as follows:

$$\eta_k = \boldsymbol{y}_k - \boldsymbol{h}(\hat{\boldsymbol{z}}_{k-1}) + \boldsymbol{H}_k \hat{\boldsymbol{z}}_{k-1} \tag{8}$$

where k denotes iteration counter, and

$$\boldsymbol{H}_k = \left[\frac{\partial h_i(\boldsymbol{z}_k)}{\partial z_j} \right]_{\boldsymbol{z}_k = \hat{\boldsymbol{z}}_{k-1}} \tag{9}$$

This implies that the assumed values of the parameters can be revised using the following relation:

$$\hat{\boldsymbol{z}}_k = \hat{\boldsymbol{z}}_{k-1} + \boldsymbol{K}_k \left[\boldsymbol{y}_k - \boldsymbol{h}(\hat{\boldsymbol{z}}_k) \right] \tag{10}$$

where $\hat{\boldsymbol{z}}_k$ is an estimated set of the parameters $\boldsymbol{z}$ at the kth iteration, and $\boldsymbol{K}_k$ is the filter gain which is given for the extended Kalman filter [9] by

$$\boldsymbol{K}_k = \boldsymbol{P}_{k/k-1} \boldsymbol{H}_k^{\mathrm{T}} \left[\boldsymbol{H}_k \boldsymbol{P}_{k/k-1} \boldsymbol{H}_k^{\mathrm{T}} + \boldsymbol{R}_k \right]^{-1} \tag{11}$$

and for the projection filter [9] by

$$\boldsymbol{K}_k = \left[\boldsymbol{H}_k^{\mathrm{T}} \boldsymbol{R}_k^{-1} \boldsymbol{H}_k \right]^{-1} \boldsymbol{H}_k^{\mathrm{T}} \boldsymbol{R}_k^{-1} \tag{12}$$

In the above expressions, $\boldsymbol{P}_{k/k-1}$ is the covariance of the estimation errors of parameters at iteration $k-1$, and $\boldsymbol{R}_k$ the covariance of measurement errors at iteration k.

In the inverse analysis using the filter theory mentioned above, the sensitivity matrix $\boldsymbol{H}_k$ is computed by means of the finite difference scheme, and the boundary element method is twice applied to evaluate the sound pressure at each iteration.

In our previous paper [8] treating boundary shape identification of blast furnace hearth, the extended Kalman filter was rather tough than the projection filter. Therefore, we shall also use the extended Kalman filter for the present inverse problem.

The main flow of the proposed inverse analysis is illustrated in Fig.1.

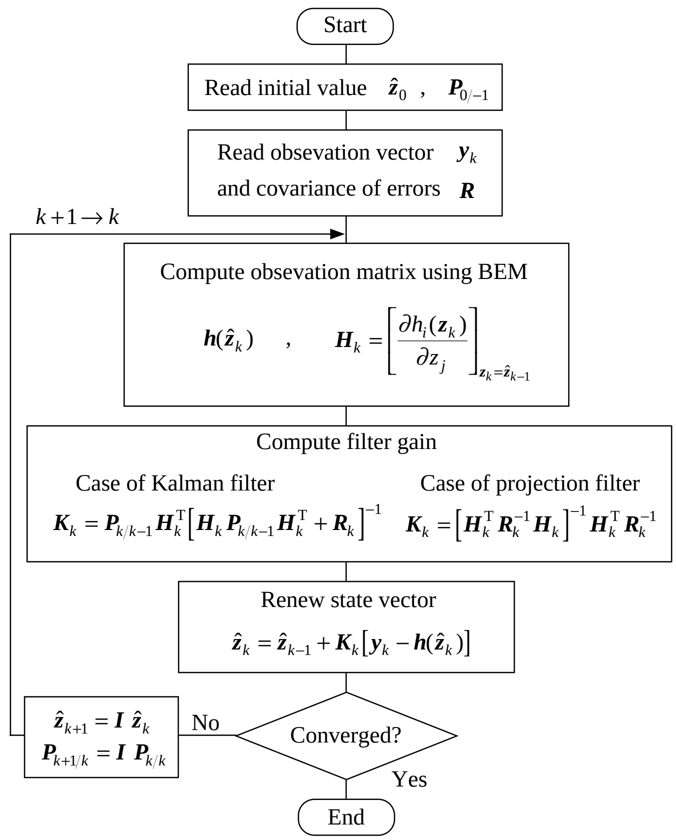

Fig. 1 Main flow of inverse analysis

NUMERICAL SIMULATION

The railway car model considered in this study is shown in Fig.2. The inside domain of the car model is not taken into account as the acoustic field under consideration, that is, we consider the exterior acoustic field surrounding the car model surface. In the analysis, we also take into account the symmetric conditions with respect to the center plane of the railway and also the rigid condition of the ground surface. Therefore, only the half the boundary of car model surface should be discretized into boundary elements.

Furthermore, it is assumed that point sources are located at points F and G and the boundary portion BCD is a distributed source, while the other boundary of car model is rigid. The noise sources are assumed to vibrate with different frequencies.

Table 1 summarizes the input data to be used for computation of the "measured data" of sound pressure by the boundary element method for the present numerical simulation of inverse analysis for noise source identification. The boundary is discretized into a series of boundary elements with quadratic interpolation. The boundary portion BCD is discretized into a boundary element in which B, C and D denote nodal points of the boundary element. Real and imaginary parts of sound pressure in [Pa] are shown in Table 1. The measured data of numerical simulation for the present inverse analysis are produced by boundary element analysis using the target values shown in Table 1.

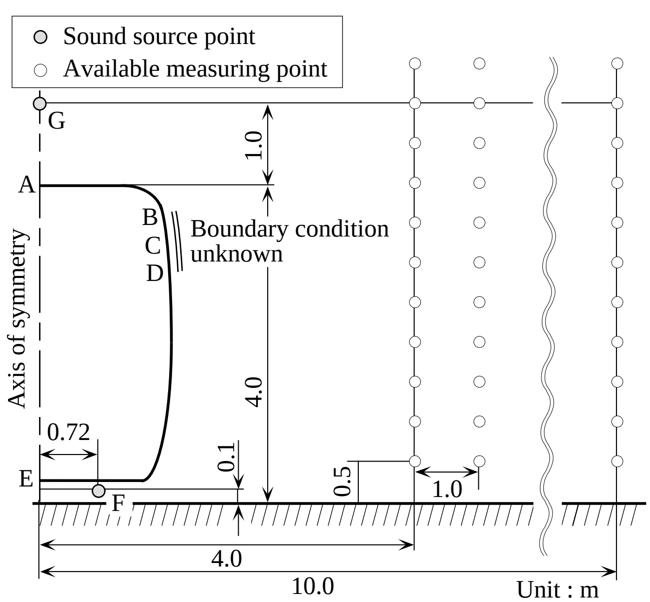

Fig.2 Two-dimensional car model

In this numerical simulation, it is assumed that the covariances of measurement and estimation errors are, respectively, as follows:

$$\boldsymbol{R}_k = 1.0 \times 10^{-6} \boldsymbol{I}\,, \qquad \boldsymbol{P}_{0/-1} = 1.0\ \boldsymbol{I} \tag{13}$$

where $\boldsymbol{I}$ is the unit matrix. It is noted that the assumed value of covariance $\boldsymbol{R}_k$ above corresponds to about 3% measurement error in sound pressure.

Table 1 Target values of parameters

Sound source	(Re, Im)	Frequency [Hz]
B	(0.5, 0.0)	500
C	(1.0, 0.0)	
D	(0.5, 0.0)	
F	(2.0, 0.0)	1000
G	(2.0, 0.0)	2000

It is noted that convergence of the present inverse analysis may depend on the initial values of the parameters to start iterative computation. In the numerical simulation, the initial values are assigned in an appropriate manner so that they change from -20 to 20 [Pa]. It can be seen that any case of the initial values provides a converged solution of inverse analysis, although a different iteration number is required.

It is assumed that the measuring points of sound pressure are located in the two-dimensional acoustic field with 7 columns with equal space between 4.0[m] and 10.0[m] from the center line of the model, and vertically 11 points with equal space of 0.5[m]; Totally, 77 points are assumed. The real and imaginary parts of sound pressure at these measuring points are given by boundary element analysis under the target values of parameters shown in Table 1.

We now select 5 measuring p0ints among the 77 measuring points mentioned above in the following three ways.

Type A : 5 points uniformly located
Type B : 5 points with lowest sensitivities
Type C : 5 points with highest sensitivities

The seisitivity above denotes an absolute value of the component of the sensitivity matrixis defined by equation (9), which is computed as sound pressuure's first derivative with respect to the values of parameters at each iteration step by the finite difference method. Fortunately, the same points are chosen at each iteration step for Types A to C. The locations of the selected measuring points for the above three types are shown in Fig.3.

To evaluate convergence of the present iterative inverse analysis, we use the cost function defined by

$$W_k = \frac{1}{M}\sum_{i=1}^{M}\left|\frac{h_i(\hat{\boldsymbol{z}}_k) - h_i(\boldsymbol{z}_k)}{h_i(\boldsymbol{z}_k)}\right|^2 \tag{14}$$

where M is the number of measuring points.

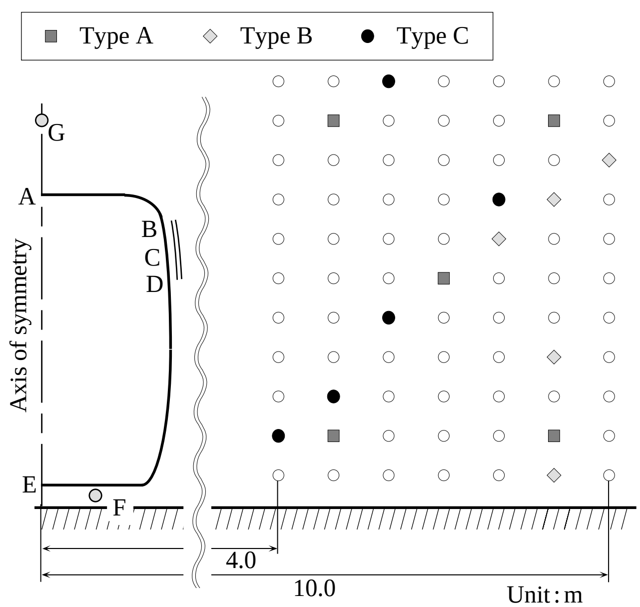

Fig.3 Measuring points

The numerical results obtained are shown in Fig.4. The initial values of parameters to be identified are assumed as shown in Table 2.

Table 2 Target values and initial values of parameters

Sound source	Target values	Initial values
G	(2.0 , 0.0)	(20 , 16)
F	(2.0 , 0.0)	(12 , 8)
B	(0.5 , 0.0)	(4 , -4)
C	(1.0 , 0.0)	(-8 , -12)
D	(0.5 , 0.0)	(-16 , -20)

It can be seen that Type C provides the best convergence and Type A shows a moderate rate of convergence, whereas Type B shows the worst rate of convergence. If we set the tolerance limit of convergence as $\log_{10} W_k < -4$, we can conclude that in Type C a converged solution is obtained after 5 iterations and in Type A after 27 iterations, whereas in Type B no convergence is realized even after 30 iterations.

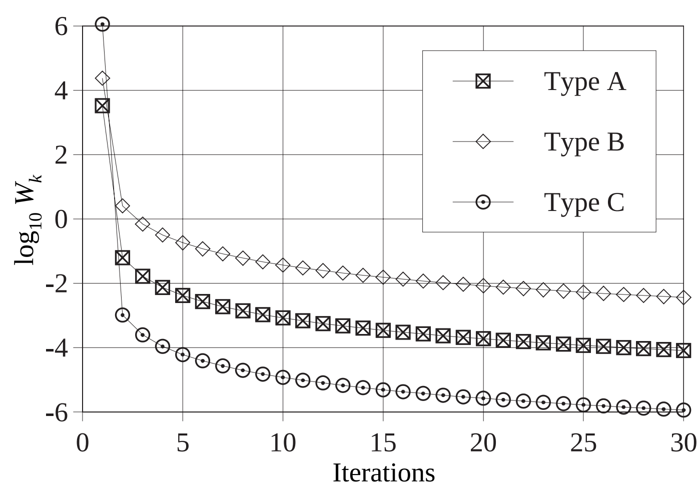

Fig.4 Convergence properties

Table 3 Parameter values after 5 iterations

	Type A	Type B	Type C
G	(2.1439 , 0.1768)	(2.2855 , 0.3754)	(1.9943 , -0.0063)
F	(2.0474 , 0.0555)	(1.9293 , 0.2637)	(2.0346 , 0.0181)
B	(0.5206 , 0.0675)	(0.7829 , 0.1398)	(0.5156 , 0.0178)
C	(0.8510 , -0.1124)	(0.9602 , 0.2035)	(0.9699 , -0.0209)
D	(0.4633 , -0.0079)	(0.0883 , -0.2138)	(0.5046 , -0.0044)

Table 4 Parameter values after 30 iterations

	Type A	Type B	Type C
G	(2.0241 , 0.0297)	(2.0487 , 0.0643)	(1.9990 , -0.0010)
F	(2.0079 , 0.0093)	(1.9878 , 0.0449)	(2.0058 , 0.0030)
B	(0.5034 , 0.0113)	(0.5485 , 0.0239)	(0.5026 , 0.0029)
C	(0.9749 , -0.0189)	(0.9932 , 0.0349)	(0.9949 , -0.0035)
D	(0.4938 , -0.0013)	(0.4294 , -0.0364)	(0.5007 , -0.0007)

CONCLUSION

The present paper has presented a new inverse analysis method for some inverse problems in acoustics, which uses the boundary element method and the filter theory. The method of inverse analysis was successfully applied to noise source identification of the two-dimensional railway car model. It is demonstrated through numerical simulation for inverse analysis of the model that convergence of iterative computations can be accelerated if the measuring points of sound pressure are selected in consideration of higher sensitivity. Inverse analysis could be successful even if a very limited number of measuring points are employed. Furthermore, it is interesting to note that the measuring points uniformly located in the filed can provide a good convergence of inverse analysis.

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