Inverse Problems in Engineering: Theory and Practice:
3rd Int. Conference on Inverse Problems in Engineering
June 13-18, 1999, Port Ludlow, WA

ID03

IDENTIFICATION OF MATERIAL PROPERTIES BY THERMO-MECHANICAL TEST RESULTS

Marie Reynier
Laboratoire de Modélisation des Structures et Essais en Mécanique
1, chemin Desvallières 92410 VILLE-D'AVRAY-France
Laboratoire de Mécanique et Technologie.URA/CNRS850
61, avenue du Président Wilson 94235 CACHAN-France
Email: marie.reynier@cva.u-paris10.fr

Hacène Adrar
Laboratoire de Modélisation des Structures et Essais en Mécanique
1,chemin Desvallières 92410 VILLE-D'AVRAY-France
Email: hacene.adrar@cva.u-paris10.fr

ABSTRACT

This paper deals with the problem of identifying material parameters by measuring mechanical and/or thermal quantities. We propose different energy functions measuring the gap which exists between Analysis and Tests. They allow to compute indicators which will determine if a given set of experimentally obtained data is able to identify the desired thermal and/or mechanical material characteristics. These indicators are based on error measures on the constitutive relations. The so-called constitutive relations relate the load quantities (ie stress, heat flux vector) to the strain and/or temperature. These indicators measure the potentiel validity of the identification method in evaluating material parameters. The same functions in which the correction variables are introduced are also used to update the tensors associated with the material properties.

INTRODUCTION

The identification of structural parameters commonly uses a function describing the difference between the predicted results given by a parametric modelling and the experimentally obtained data. For complex structures the modelling is most often a finite element model described with initial values of structural parameters. These parameters should be corrected, minimizing the difference existing between Analysis and Tests. Several methods have been developed for updating problems and can be found in the literature, [see for example Friswell and Motterheads review paper in [1], Gladwell's in [2]]. Most methods are dedicated to mechanics and concern dynamic test data and identification techniques in thermal engineering are well developed [3] [4]. Here we look for a unified approach allowing to associate thermal and -mechanical data in order to improve the identification of material characteristics.

An error measure named error measure on the constitutive relation has been proposed by the LMT Cachan to update finite element models by means of dynamic data. The difference between modelling and experiment allows to build indicators having the great advantage of accurately locating the mismodeled areas. This measure does not directly use the gap between analytical and measured frequencies and mode shapes. It builds the shape of the displacement

field associated with the kinematic boundary conditions and the shape of the displacement field associated with the load. Experimental information are introduced taking the precision of measure into account. The principles of this method have been described in [5]].
The tuning strategy uses an iterative process with each iteration containing a location step followed by an evaluation step. The notion of error on the constitutive relation allows to define local indicators which have permitted to accurately locate modeling errors in updating procedures.
Here, we extend the notion of error on the constitutive relation to thermal and thermo-mechanical engineering.

ERROR MEASURES ON THE CONSTITUTIVE RELATION

The description of the gap between Analysis and Test uses an error measure on the constitutive relation. This measure has shown an efficient location power of the modeling error for updating problems.[7-9] where a given finite element model is improved using modal test results. An error measure on the constitutive relation has also been proposed for geometrically non-linear problems in [10-11].
Let us recall the principle as seen from the mechanical point of view and propose a measure in thermal and thermo-mechanical cases.
We consider the perfectly elastic behaviour of solids as undergoing infinitesimal strains. The boundary is assumed to be decomposed into two complementary subsurfaces ∂S_1 and ∂S_2. Let us consider the following elastic linear reference problem. In mechanics, it consists in finding the couple (σ, U) as a function of the space and of the time $\sigma(x,t)$ and $U(x,t)$ such that it is an admissible couple $\forall$ t $[0, \tau]$.
Implying that :
- it verifies the prescribed data

$$U = Ud \text{ on } \partial S_1 \text{ and } \sigma n = \sigma d \text{ on } \partial S_2 \quad (1)$$

- it verifies the equilibrium equation

$$\operatorname{div} \sigma + f = \rho \ddot{U} \quad (2)$$

- and it verifies the initial time conditions.
Moreover it should verify the constitutive relation :

$$\sigma = \mathbf{H}\varepsilon(U) \quad (3)$$

The problem can be rewritten a formulation under the following:
To find the triplet $\mathbb{T}$ ($\underline{U}$, σ, $\underline{\Gamma}$) verifying :
- the prescribed data
- the constitutive relation $\sigma = \mathbf{H}\varepsilon(U)$ (4)
- the complementary relation $\Gamma = \rho \ddot{U}$ (5)

and verifying the initial time conditions.
implying that ($\underline{U}$, σ, $\underline{\Gamma}$) is admissible, and obeys the local equilibrium equation

$$\operatorname{div} \sigma + f = \Gamma \quad (6)$$

Let B be the space [{(U', σ', Γ') admissible, the initial time conditions (σ', Γ') verifiying the equilibrium}].

$$\mathbf{U} = \{ \underline{U}', \underline{U}' \ \partial S_1 = Ud, \underline{U}' \text{ regular}\} \quad (7)$$

The reference problem becomes :
To find ($\underline{U}$, σ, $\underline{\Gamma}$) $\in$ B such that it obeys the «constitutive» relations :

$$\sigma = \mathbf{H}\varepsilon(U) \text{ and } \Gamma = \rho \ddot{U} \quad (8)$$

This reference problem can finally be rewritten as:
To find ($\underline{U}$, σ, $\underline{\Gamma}$) $\in$ B such that it minimizies :

$$e^2 (U', \sigma', \Gamma') = \| \sigma' - \mathbf{H}\, \varepsilon(U')\|^2 + \|\Gamma' - \rho \ddot{U}'\|^2 \quad (9)$$

The norms used are energy norms:

$$\| \sigma \|^2 = \int_{t=0}^{\tau}\int_{\Omega} \sigma H^{-1} \sigma d\Omega dt \quad (10)$$

and

$$\| \Gamma \|^2 = \int_{t=0}^{\tau}\int_{\Omega} \Gamma \frac{1}{\rho} \Gamma d\Omega dt \quad (11)$$

The resulting $e^2(U, \sigma, \Gamma)$ is named error measure on the constitutive relation. If this measure is equal to zero, the solution is the reference solution.
To identify the characteristics by means of experimentally obtained data, we commonly introduce the measured quantities.
The mechanical problem denoted **(I)** becomes
To find ($\underline{U}$, σ, $\underline{\Gamma}$) $\in$ B such that it minimizies :

$$e^2_{exp} (U', \sigma', \Gamma') = \| \sigma' - \mathbf{H}\, \varepsilon(U')\|^2 + \|\Gamma' - \rho \ddot{U}'\|^2 + \| U' - U\exp\|^2 + \| \sigma' - \sigma\exp\|^2 + \| \Gamma' - \Gamma\exp\|^2 \quad (12)$$

$$\text{with } \| U \|^2 = \int_{t=0}^{\tau}\int_{\Omega} H\varepsilon(U)\varepsilon(U)\,d\Omega dt \qquad (13)$$

where the quantities Uexp, σexp , Γexp are the measured or experimentally obtained data. Most often, only displacement measures Uexp are available. Other measured quantities can a priori be introduced. If the model material properties are wrong. $e(U, \sigma, \Gamma) \neq 0$ and **H** and **ρ** should be corrected. The correction of these quantities $\Delta\mathbf{H}$ and $\Delta\rho$ are computed minimizing :

E^2_{exp}: $(U', \sigma, \Gamma, \Delta\mathbf{H}, \Delta\rho)$=======>

$$E^2_{exp}(U', \sigma, \Gamma, \Delta\mathbf{H}, \Delta\rho) = \| \sigma' - (\mathbf{H}+\Delta\mathbf{H})\varepsilon(U')\|^2 + \| \Gamma' - (\rho+\Delta\rho)\ddot{U}'\|^2 + \| U' - U\exp\|^2 + \| \sigma' - \sigma\exp\|^2 + \| \Gamma' - \Gamma\exp\|^2 \qquad (14)$$

In thermal procedure, the reference problem can be written in a symmetrical form assuming infinitesimal fluctuations in temperature :

To find the triplet T (q, T, η) verifying :

* the prescribed data

$T = Td$ on ∂S_{T1} and $qn = qd$ on ∂S_{T2} (15)

* the equilibrium equation $\operatorname{div} q + r = \eta$ (16)

* the constitutive relation $q = -\mathbf{K}\nabla T$ (17)

and

* the complementary equation $\eta = c\dot{T}$ (18)

c and **K** should be corrected if $E^2_T(q, T, \eta) \neq 0$ minimizing

$E^2_{T\,exp}$: $(q', T', \eta', \Delta\mathbf{K}, \Delta\mathbf{c})$ ===>

$$E^2_{T\,exp}(q', T', \eta', \Delta\mathbf{K}, \Delta\mathbf{c}) = \|q' + (\mathbf{K}+\Delta\mathbf{K})\nabla T'\|^2 + \|\eta' - (\mathbf{c}+\Delta\mathbf{c})\dot{T}'\|^2 + \| q' - q\exp\|^2 + \| T' - T\exp\|^2 + \| \eta' - \eta\exp\|^2 \qquad (19)$$

The quantities containing the material characteristics **H** and **K** , and **ρ** and **c** have a symmetric behavior in the reference problem equations. The minimization of the error measures $E^2_{exp}(U, \sigma, \Gamma, \Delta\mathbf{H}, \Delta\rho)$ and $E^2_{T\,exp}$: $(q, T, \eta, \Delta\mathbf{K}, \Delta\mathbf{c})$ enables us to identify these material properties.

Remark 1:

In [13], the authors use a similar approach without introducing the complementary constitutive behavior that generalizes the thermal formulation. Another point is that our approach is based on reformulating the mechanical, thermal or thermo-mechanical problem as a problem in variation computation. For coupled equations,

Biot's variational principle can be used.

The norms used to express the rewritten thermal problem are those coming from this variational principle : $\delta\Psi + \delta D - \delta W = 0$ (20)

Biot's thermoelastic potential is given by :

$$\Psi = 1/2 \int_{\Omega} c(T - T_0)\frac{1}{T_0}(T - T_0)\,d\Omega \qquad (21)$$

The dissipation function is the

$$D = 1/2 \int_{\Omega} q_i \frac{1}{kT_0} q_i\,d\Omega \qquad (22)$$

The variation in the generalized virtual work is

$$\delta W = -\int_{\partial\Omega} (T - T_0)n_i\delta s_i\,dS \qquad (23)$$

Remark 2 :

The approach we have proposed here can be understood as a variant of the strategy proposed by Chavent et al. in [12] where the partial differential equation is considered a constraint in a least square approach. But here the most important terms are the first terms, and the experimental information marked by the underscript « exp » are introduced as additional terms to avoid the trivial solution 0 and to drive the solving.

THE COUPLED THERMO-MECHANICAL POINT OF VIEW

With the assumption of small perturbations, where the displacement gradient is small, the temperature field only differs from a reference temperature **T0** and the time rate of change of the temperature, the resulting thermoelastics problem is written using :

$-\sigma_M = \mathbf{H}\varepsilon(U)$ (24)

$-\sigma_T = (T-T0)\mathbf{A}$ and $\sigma_{TM} = \sigma_M + \sigma_T$ (25)

- the local equilibrium equations :

$$\operatorname{div}\sigma_{TM} + f = \rho\ddot{U} \qquad (26)$$

$$\mathrm{div}\, q + r = \mathbf{c} \ - T0\mathbf{A}\,.\varepsilon \qquad (27)$$

- the constitutive equations

$$\sigma_{TM} = \mathbf{H}\varepsilon(U) + (T-T0)\mathbf{A} \qquad (28)$$

$$q = -\mathbf{K} \quad T \qquad (29)$$

where **H** is the elasticity tensor, **A** the stress temperature tensor, **K** the conductivity tensor and ρ the density characterizing the material properties. The parameters of these tensors are the characteristics we look for in an identification problem.
The infinitesimal strain tensor is defined by the equation :

$$\varepsilon(U) = 1/2\,(\quad U + {}^{t} \quad U) \qquad (30)$$

In the isotropic case, **H** is characterized by Lame's coefficients and **A** and **K** are spherical tensors such that **A** = **a**Id and
K = **k**Id where k is the conductivity scalar
and **a** = - **(3λ + 2μ) α**
The basic equations cant hen be written as follows:
* Local equilibrium equations :

$$\mathrm{div}\,(\sigma_M + \sigma_T) + f = \boldsymbol{\rho}\ddot{\mathbf{U}} \qquad (31)$$

$$\mathrm{div}\, q + r = c\dot{T} - T_0 a\,\mathrm{tr}\dot{\varepsilon} \qquad (32)$$

* Constitutive equations :

$$\sigma_{TM} = \mathbf{H}\,\varepsilon(U) + (T-T0)\mathbf{a}\mathrm{Id} \qquad (33)$$

or

$$\sigma_{TM} = \mathbf{2\mu}\varepsilon(U) + \boldsymbol{\lambda}\,(\mathrm{tr}\varepsilon(U))\mathrm{Id} \ - (T-T0)\ \mathbf{(3\lambda + 2\mu)\ \alpha\ Id} \qquad (34)$$

and

$$q = -\mathbf{K} \quad T \qquad (35)$$

* equilibrium equations :

$$\left|\begin{array}{l} \mathrm{div}\,\sigma_{TM} + f = \Gamma \\ -\,\mathrm{div}\, q + r = \eta_{TM} \\ \sigma_{TM} = \sigma_M + \sigma_T \quad \text{and} \\ \eta_{TM} = \eta_M + \eta_T \end{array}\right. \qquad (36)$$

The constitutive relations become :

$$\sigma_T = (T-T0)\ \mathbf{(3\lambda + 2\mu)\ \alpha\ Id} \qquad (37)$$

$$\sigma_M = \mathbf{H}\varepsilon(U) \quad \text{and} \quad q = \mathbf{K} \quad T \qquad (38)$$

$$\eta_M = -\ T0\ \mathbf{a}\ \mathrm{tr}T' \quad \eta_T = \mathbf{c}, \qquad \Gamma = \boldsymbol{\rho}\,\ddot{\mathbf{U}}$$

Biot's variational principle is written as :

$$\delta\Psi + \delta D - \delta W = 0 \qquad (39)$$

and Biot's thermoelastic potential as:

$$\Psi = 1/2 \underset{\Omega}{\ } \lambda\varepsilon_{ii}\varepsilon_{ii} + 2\mu\varepsilon_{ij}\varepsilon_{ij} + c(T-T_0)\frac{1}{T_0}(T-T_0)d\Omega \qquad (40)$$

The dissipation function is

$$D = 1/2 \underset{\Omega}{\ } q_i \frac{1}{kT_0} q_i d\Omega \qquad (41)$$

The variation in the generalized virtual work is :

$$\delta W = \underset{\partial\Omega}{\ } \left(\sigma_{ij} n_j \delta u_i - (T-T_0) n_i \delta s_i\right) dS - \rho \underset{\Omega}{\ } \ddot{u}\,\delta u_i d\Omega \qquad (42)$$

In the most general case: a function evaluating the difference between analysis and test can be defined by :

$$F(\sigma_T, \sigma_M, U, T, \Gamma, q, \eta_M, \eta_T) = \| \sigma_T - (T-T0)\ \mathbf{(3\lambda + 2\mu)\ \alpha\ Id} \|^2 + \| \sigma_M - \mathbf{H}\varepsilon(U) \|^2 + \| q + \mathbf{K} \quad T \|^2 + \|\Gamma - \boldsymbol{\rho}\,\ddot{\mathbf{U}}\|^2 + \|\eta_M + T0\ \mathbf{a}\ \mathrm{tr}\varepsilon'\|^2 + \| \eta_T - \mathbf{c}\, T' \|^2 \qquad (43)$$

For stationary cases and taking the equation constraints into account, we obtain :

$$F(\sigma_T, \sigma_M, U, T) = \| \sigma_T - (T-T0)\ \mathbf{(3\lambda + 2\mu)}\ \alpha\ \mathbf{Id} \| + \| \sigma_M - \mathbf{H}\varepsilon(U) \| + \| q + \mathbf{K} \quad T \|^2 \qquad (44)$$

obeying :

$$\mathrm{div}\,(\sigma_M + \sigma_T) + f = \Gamma, \qquad (45)$$

$$\mathrm{div}\, q + r = 0 \qquad (\ 6)$$

The experimental information are introduced as in the previous section.

VISIBILITY OF THE PERTUBATIONS USING MECHANICAL TEST RESULTS OR THERMAL TEST RESULTS

This section is developed from a mechanical point view for simplicity's sake.
To measure the efficiency of a set of experimentally obtained available data and identify the material characteristics, we will introduce simulated perturbations in the model, and then measure the capability of the indicators associated with the energy functions to detect these perturbations.
Let $\mathbf{p_{0i}}$ denote the value of the structural or material parameter found after an updating technique, and $\mathbf{p_i}$ the value obtained after introducing a possible bug in defect of the model. If we assume that the maximum defect is a given value $= p$, a range of variation is described with coefficients α_i such that:

$$p_i = p_{0i} + \alpha_i\,\Delta p \quad \alpha_i \in [-1, 1]\ \&\ i \in [1, m] \qquad (47)$$

that can be rewritten $\underline{p} = \underline{p}_0 + \Delta p\, \underline{\alpha}$, $\underline{p}$, $\underline{p}_0$ and $\underline{\alpha}$ being vectors with the dimension m.

Let **T(0)** denote the triplet (the mechanical triplet is ($\sigma(\underline{\alpha}=0)$, $U(\underline{\alpha}=0)$, $\Gamma(\underline{\alpha}=0)$), the thermal triplet is ($q(0)$, $T(0)$, $\eta(0)$)), computed by solving the problem presented in the previous section.

In a mechanical problem, the material characteristics become

$$\mathbf{H}(\underline{\alpha}) = \mathbf{H}(0) + \Delta\mathbf{H} \quad (48)$$

and

$$\rho(\underline{\alpha})=\rho(0)+ \Delta\rho \quad (49)$$

To evaluate how the chosen energy function **E** is affected by a perturbation $\underline{\alpha}$ of the model, the following problem **(II)** is solved:

To find the triplet ($\sigma(\underline{\alpha})$, $U(\underline{\alpha})$, $\Gamma(\underline{\alpha})$) minimizing :

E^2_0 : (σ', U', Γ') =>

$$E^2_0 (\sigma', U', \Gamma')=\| \sigma'-(\mathbf{H}+ \Delta\mathbf{H})\, \varepsilon(U')\|^2 + \|\Gamma'-(\rho+\Delta\rho)\, \ddot{U}'\|^2+ \| U' - U(0)\|^2 + \| \sigma'- \sigma(0)\|^2 + \|\Gamma'-\Gamma(0)\|^2 \quad (50)$$

The experimental information are here replaced by U(0), $\Gamma(0)$, $\sigma(0)$. If the measure E^2_0 (σ, U, Γ) is sensitive to the perturbation introduced by $\underline{\alpha}$ in

the model, the new triplet ($\sigma(\underline{\alpha})$, $U(\underline{\alpha})$, $\Gamma(\underline{\alpha})$) obtained by solving the problem **(II)** will contai a significant modification.

The model is now considered in the initial form $\underline{\alpha}=\underline{0}$ and the experimental data we introduce now are the modified triplet ($\sigma(\underline{\alpha})$, $U(\underline{\alpha})$, $\Gamma(\underline{\alpha})$). To measure the detectability of the $\underline{\alpha}$-defect the following problem **(III)** is solved:

To find the triplet (σ, U, Γ) minimizing :

$E^2_{\underline{\alpha}}$: (σ', U', Γ') =>

$$E^2_{\underline{\alpha}} (\sigma', U', \Gamma') = \| \sigma'-\mathbf{H}\, \varepsilon(U')\|^2+ \|\Gamma'-\rho\, \ddot{U}'\|^2+ \| U' - U(\underline{\alpha})\|^2 + \| \sigma'- \sigma(\underline{\alpha})\|^2+ \|\Gamma'-\Gamma(\underline{\alpha})\|^2 \quad (51)$$

If the indicators associated with the $E^2_{\underline{\alpha}}$ measure have a significant value, the defect is visible. The visibility of a defect associated with $E^2_{\underline{\alpha}}$ global indicator is given by :

$$V^2_\alpha = \frac{\|\sigma-H\varepsilon(U)\|^2+\|\Gamma-\rho\ddot{U}\|^2}{1/2(\|\sigma\|^2+\|H\varepsilon(U)\|^2)+1/2(\|\Gamma\|^2+\|\rho\ddot{U}\|^2)} \quad (52)$$

and to obtain the local indicators the energy norms are computed restricted to the sub-structures

DETAILED EXAMPLE OF THE COMPUTATION OF VISIBILITY INDICATORS IN THE MECHANICAL FRAMEWORK

As an example, using modal tests results, the constitutive relations are written as follows :

$$\sigma = \mathbf{H}e(U) \qquad \Gamma = - \rho\, \omega_{exp}^2\, U \quad (53)$$

Let $B_{\omega\, exp}$ be the space **{(U', σ', Γ), U' U, (σ', Γ) verifies the equilibrium}.**

The reference problem becomes :

To find (U, σ, Γ) $\in B_{\omega\, exp}$ obeying the «constitutive» relations.

$$E^2_{\underline{\alpha}} = \| \sigma-\mathbf{H}\, \varepsilon(U)\|^2 + \|\Gamma-\rho\, \omega_{exp}^2\, U\|^2 + \| U' - U(\underline{\alpha})\|^2 \quad (54)$$

and

$$\|\sigma\text{-}H\varepsilon(\underline{U})\|^2 = \int_\Omega tr\left[(\sigma\text{-}H\varepsilon(\underline{U}))H^{-1}(\sigma-H\varepsilon(\underline{U}))\right]d\Omega \quad (55)$$

$$\|\rho\omega^2_{exp}\underline{U}+\Gamma\|^2 = \int (\rho\omega^2_{exp}\underline{U}+\Gamma)\frac{1}{\rho\omega^2_{exp}}(\rho\omega^2_{exp}\underline{U}+\Gamma)d\Omega \quad (56)$$

The global visibility of the defect is then computed by :

$$V^2 = \frac{\|\sigma - H\varepsilon(\underline{U})\|^2 + \|\Gamma + \rho\omega^2_{exp}\underline{U}\|^2}{D/2} \quad (56)$$

The visibility per sub-structure is then given by the computation of the energy norms only on the considered sub-structure:

$$V^2 = \frac{\|\sigma - H\varepsilon(\underline{U})\|^2_{Sb} + \|\Gamma + \rho\omega^2_{exp}\underline{U}\|^2_{Sb}}{D/2} \quad (57)$$

and

$$D = \|\sigma\|^2 + \|K\varepsilon(\underline{U})\|^2 + \|\underline{\Gamma}\|^2 + \|\rho\omega^2\underline{U}\|^2 \quad (58)$$

If the defect is visible, the sub-structure containing the default will be well localized by the visibility indicators. The same approach allows to write visibility indicators associated with thermical quantities and/or thermo-mechanical quantities.

In the following section we illustrate the behaviour of the visibility indicators.

VALIDATION

Example1 :

The first example is a damped free beam free disctized into 15 finite elements The mechanical characteristics are :

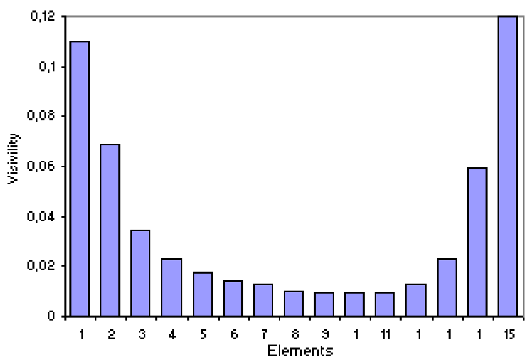
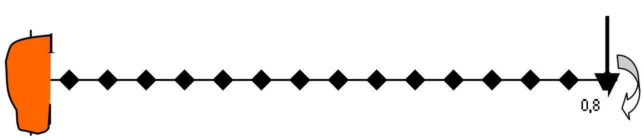


fig2 Visibility of a defect of 20% on the element 1&15 computed with 30 experimental modes and 30 DOF

Young's modulus : E=0.75 10^{11} pa
Area : S=10^{-4} m²
Total length :L=0.8 m
Density : D=0.78 kg/m3



The defects close to embedding are generally more visible when the information are complete (fig2). The defects in zones having a strong deformation energy or a strong kinetic energy are visible.

Figure 3 shows that the defects are visible when all experimental information are available : they have positive or negative values

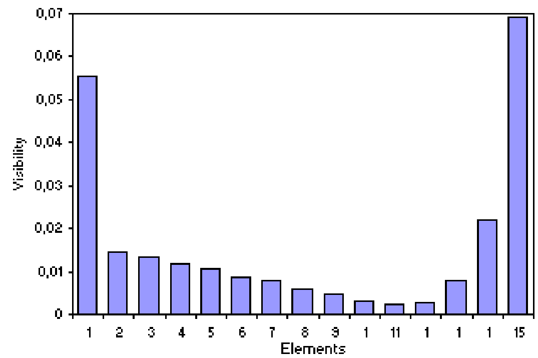


fig4 Visibility of a defect of 20% on the elements 1 & 15 computed with exciting modes (1&30) and 30 DOF

In figure 4, the experimental information are reduced to 2 modes, but the sensor location is optimal. The visibility is preserved[12].

In figure 5, by choosing the number and the position of degrees of freedom in an optimal way, the visibility is preserved.

Example2 Truss structure

The experimental information are complete on the degrees of freedom which are all measured. The indicators show that 30 modes are enough to detect the defects (fig 6 et 7). The visibility is very good, measuring all degrees of freedom.

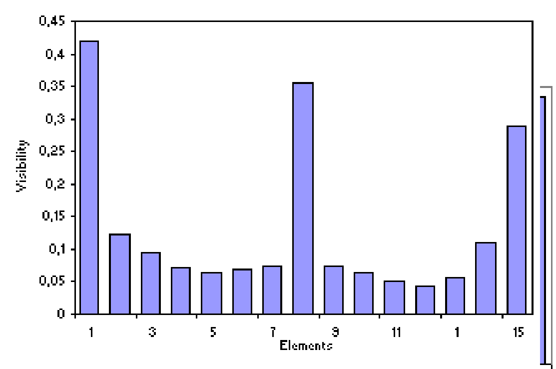


fig5 Visibility of a defect of -80% on the elements 1, 8 & 15 computed with exciting modes (1-2-30) and 30 DOF

Elements

fig3 Visibility of a defect of -80% on the elements 1, 8 & 15 computed with 30 experimental modes and 30 DOF.

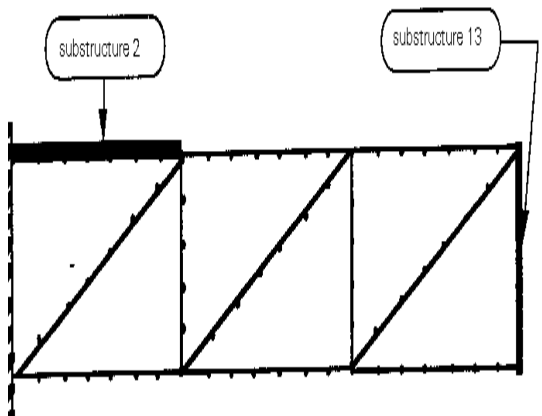


fig6 Defect of 30% on the substructures 2 and 13

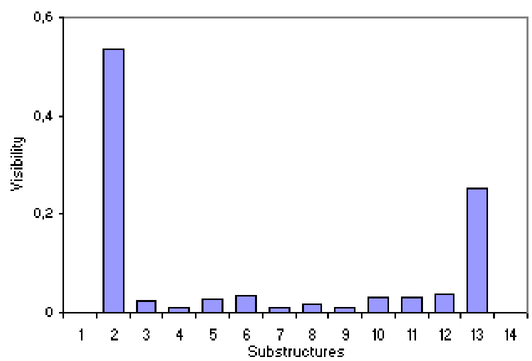


fig7 Visibility of a defect of 30% on substructures 2 & 13 computed with 30 modes and 216 DOF

In figure 8 below, the experimental information

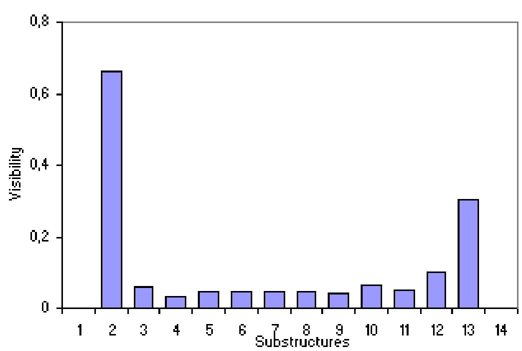


fig8 Visibility of a defect of 30% on the substructure 2 & 13 computed with 30 modes and 30 DOF

are very limited. Only 30 dof sensors are measured. On the other hand the indicators of visibility show that the preliminary optimization of sensor sites ensures the visibility.

For the same defect size of the defect, the

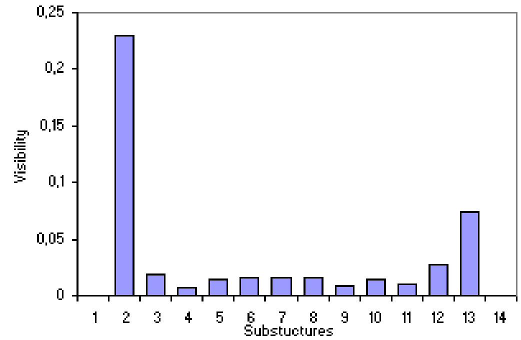


fig9 Visibility of a defect of 30% on the substructures 2 & 13 computed with 5 experimental modes and 30 DOF

visibility on substructure 13 is less significant with only 5 measured modes. (fig9).

Figure 10 shows that the visibility is not

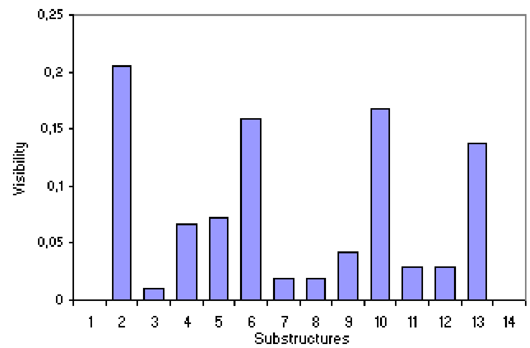


fig10 Visibility of a defect of 30% on the substructures 2 & 13 computed with 5 experimental modes and 30 DOF

preserved if the sensors location are not optimized

CONCLUDING REMARKS

In the previous tested cases we have shown the efficiency of the proposed indicators to measure the possibility of identifying a given default or a given material characteristics variation These indicators are particularly interesting when it is impossible to measure the entire displacement field, which is often the case.

Future works will consist in applying these indicators on thermo-mechanical loads in order to detect thermo-mechanical variation properties in a more accurate way.

REFERENCES

[1] Gladwell, G. M. L. (1996). "Inverse Problems in Vibration - ii", *Applied Mechanics Review*, 49, N∞ 10, Part2, S.25.

[2] Friswell M.J., and Motterhead J.E., Special Issue (1998). Model Updating - a Survey, Mechanical System and Signal Processing, *MSSP Journal,* vol. 12 Nb 1, January.

[3] Inverse problems in Engineering Mechanics", Eds Bui , Tanaka et al., A.A.Balkema, Rotterdam, Brookfield, 1994.

[4] 2nd International Conference on Inverse Problems in Engineering, 9-14 June1996, Port aux Rocs,- Le croisic.

[5] LadevËze, P., Nedjar, D. and Reynier, M. (1994).Updating of finite element models using vibration tests, *A.I.A.A. Journal*, vol. 32, N∞ 7, pp. 1485-1491

[6] N.M. Maia, M. Reynier, "Error Localization for Updating Finite Element Model using Frequency-Response-Functions", Proceedings of the 12th International Modal Analysis Conference - IMAC XII - Hawaii, U.S.A., Févr. 1994, pp. 1299-1308.

[7] Reynier, M., Ladevèze P., Feuardent V., "Selective Errors Location Indicators for Mass and Stiffnes Updating". Proc. of the International Symposium on "Inverse problems in Engineering Mechanics", Nagano, Japon, pp.24-27 March, 1998.

[8] M. Reynier, B. Nedjar "Damage Detection on Elastic Beams :the Non-linear Formulation" proceedings of the 2nd International Conference on Inverse Problems in Engineering, 9-14 June1996, Port aux Rocs,- Le Croisic.

[9] Chavent G., Kunisch K., Roberts J. E., Primal Dual Formaulations for parameters Estimation Problems, N°2891, Mai 1996, Rapport INRIA, Institut National de Recherche en Informatique et en Automatiuqe, ISSN 0249-6399.

[10]Bonnet M., Bui H.D., Maigre H., Planchard J., Identification of heat conduction coefficient : application to non destructive testing, IUTAM Symposium, Tokyo, Springer Verlag, 1992, pp.475-487.

[11] Ladevèze P., 1993, "Erreur en relation de comportement en dynamique : théorie et application au recalage de modèle de structure", Rapport Interne n° 150, L.M.T. Cachan - France.

[12] B. Belkacem, M. Reynier, H. Abou-Kandil "Tools for optimal or near optimal sensor location", dans Integrated Design and Manufacturing in Mechanical Engineering, Kluwer Academic Publisherx, Boston/London, pp.143-152.