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INVERSE IDENTIFICATION AND UPDATING IN VIBRATIONS OF PIPING SYSTEMS

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ABSTRACT

This paper concerns the area of vibrations in piping systems of nuclear power plants. One needs to represent as best as possible the dynamic behaviour of those systems with respect to the measurements performed on a real structure. In a software developed by Electricite de France, this behaviour is idealised in mechanics with exact dynamic stiffness matrices, and in acoustics with plane waves. The proposed approach tries to combine in the same computation the identification of unknown inputs with the updating of unknown parameters. An objective function based on the difference between known forces and computed ones is minimised in order to find the solution. At the opposite of traditional methods, we are not forced to compute the objective function at sensor locations, but perform an exact condensation on what we call "test degrees of freedom" which are locations where external forces are well known.

NOMENCLATURE

$[Z]$	Dynamic stiffness matrix
$\{q\}$	Generalised displacements
$\{Q\}$	External forces
$\{C\}$	Measurement vector
ω	Pulsation
Ω	A set of pulsations: $\Omega = \{\omega_1, \cdots, \omega_n\}$
$\mathbf{p}$	Updating parameters vector

INTRODUCTION

The fluid filled piping networks are often huge and have a complicated behaviour. A typical example is the piping system of energy production plants which are often several hundred of meters length and include a lot of singularities. The vibrational analysis of these systems and the diagnosis of their functioning anomalies require the use of a mathematical model idealizing their behaviour. So, the knowledge of the solicitations and the boundary conditions acting on such a system is very important. In order to improve the agreement between a mathematical model and the physical behaviour of the real system, inverse and updating methods using measured response data are used. Those measurements can be performed on a system in real functioning conditions or in laboratory in order to identify a single component behaviour.

One of the difficulties encountered in a functioning analysis is the description of the interface conditions between the real structure and the ground. Moreover, piping systems analysis involves generally strongly coupled phenomena like fluid/structure interaction. They are often responsible of sources generation and also make the system behaviour depending on the functioning conditions which have to be taken into account during the analysis. This requires the dynamic knowledge of acoustic and vibration sources acting on the system. Actually, some of these sources cannot be idealized because of the lack of knowledge about them. So, boundary conditions at these locations have to be identified via an inverse method using measured data.

Once the identification of unknown boundary conditions is solved, some uncertainties remain on the model: physical parameters like Young modulus, sound velocity, etc. can still be badly estimated by the user. Updating methods are then used in order to identify those badly known parameters ((Natke, 1988), (Ewins, 1990)). Measurements performed on the real structure are used to build an objective function defining a distance between the model and reality. A generalized least squares method using exact dynamic stifness matrices derivation can then be performed to estimate unknown parameters.

We present in this paper an overview of our research in the field of experimental analysis which aims to characterize the dynamic behaviour of a part of a piping system and to identify and/or update its mechanical characteristics in real functioning conditions. The outline of this paper is as follows, first of all we present the boundary identification method, then we discuss about the updating process and show results for each part.

GENERAL APPROACH

We are interested here in a steady state and linear behaviour, so we use the frequency domain analysis. First, we developed a new approach to characterize the unknown boundary conditions which consists in identifying generalized forces and displacements at the interface between studied network and the ground. The main structure is assumed to be accurately modeled using **exact dynamic stiffness matrices** formulation. Spatial piping networks are then idealized with curvilinear elements. The pressure fluctuation inside the pipes is assumed to be propagated in plane waves and we take into account the fluid/structure interaction in elements like elbows or bifurcations.

In a second step, we use the same approach combined with a sensitivity method to update the structural model of the main network. Some known forces are identified and used to test the reliability of the inverse model. The most important feature of our method consists into its aptitude to combine both problems: identification of boundary conditions and structural updating of piping networks in real functioning conditions.

Figure 1 and Table 1 illustrate a schematic example of the decomposition of the different dofs we can find on an idealised structure. $\{Q\}$ are **external** forces applied on the structure, $\{q\}$ are the generalised displacements and an overlined vector $\{\overline{X}\}$ denotes a vector with numerically known values.

When performing a direct computation, only free dofs $(\{q_f\},\{\overline{Q_f}\})$ and clamped dofs $(\{\overline{q_c}\},\{Q_c\})$ are present. The model equilibrium at pulsation ω can then be written:

$$\begin{Bmatrix} \{\overline{Q_f}\} \\ \{Q_c\} \end{Bmatrix} = \begin{bmatrix} [Z_{ff}] & [Z_{fc}] \\ [Z_{cf}] & [Z_{cc}] \end{bmatrix} . \begin{Bmatrix} \{q_f\} \\ \{\overline{q_c}\} \end{Bmatrix} \quad (1)$$

Table 1. DECOMPOSITION OF DEGREES OF FREEDOM.

Boundary conditions		Generalised displacements	Associated forces
link		to identify: $\{q_l\}$	to identify: $\{Q_l\}$
known external force	test	to identify: $\{q_t\}$	known: $\{\overline{Q_t}\}$ to identify: $\{Q_t\}$
	free	unknown: $\{q_f\}$	imposed: $\{\overline{Q_f}\}$
clamped		imposed: $\{\overline{q_c}\}$	unknown: $\{Q_c\}$

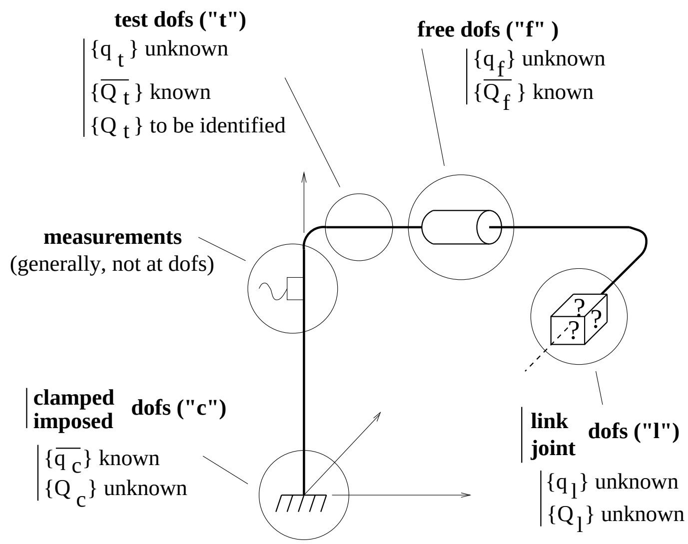

Figure 1. A TYPICAL PIPING SYSTEM.

and solved classically like in a finite element analysis:

$$\left| \begin{array}{l} \{q_f\} = [Z_{ff}]^{-1} . (\{\overline{Q_f}\} - [Z_{fc}] . \{\overline{q_c}\}) \\ \{Q_c\} = [Z_{cf}] . [Z_{ff}]^{-1} . (\{\overline{Q_f}\} - [Z_{fc}] . \{\overline{q_c}\}) + [Z_{cc}] . \{\overline{q_c}\} \end{array} \right.$$

When unknown boundary conditions are present, we introduce new kinds of dofs:

1- link dofs $(\{q_l\},\{Q_l\})$ which are places where neither generalised displacements, nor external forces are known. The model equilibrium can then be written:

$$\begin{Bmatrix} \{\overline{Q_f}\} \\ \{Q_c\} \\ \{Q_l\} \end{Bmatrix} = \begin{bmatrix} [Z_{ff}] & [Z_{fc}] & [Z_{fl}] \\ [Z_{cf}] & [Z_{cc}] & [Z_{cl}] \\ [Z_{lf}] & [Z_{lc}] & [Z_{ll}] \end{bmatrix} . \begin{Bmatrix} \{q_f\} \\ \{\overline{q_c}\} \\ \{q_l\} \end{Bmatrix} \quad (2)$$

but is now ill-posed because of the lack of known information. In order to solve this problem, measurements are introduced as we will see in the next section.

2- test dofs $\left(\{q_t\}, \{\overline{Q_t}\}\right)$ which are locations where external forces are known (like free dofs) but are going to be considered unknown and identified in a $\{Q_t\}$ vector. $\{Q_t\}$ is then compared to the known $\{\overline{Q_t}\}$ in order to build the objective function in the updating process. Locations where there is no external force are very good places for test dofs because the corresponding $\{\overline{Q_t}\}$ component is completely known to be null.

BOUNDARY CONDITIONS IDENTIFICATION

Development

A brief presentation of the boundary identification approach is given here. More details are provided in reference (Frikha, 1992). The unknown boundary conditions can describe the connections between the studied part of the network and the ground or the rest of the network. The inverse problem consists in computing the frequency spectra of the generalized displacements $\{q_l\}$ and the external generalized forces $\{Q_l\}$ at these connection dofs. Experimental data are the responses at a few points of the idealized structure. Either generalized displacements or internal forces (stress) can be used as measurements. Elements describe all the mechanical and geometrical characteristics of each part of the network by means of exact dynamic stiffness and transfer matrices. Those matrices are used to express the dynamic behaviour of a curvilinear element and to relate the measured components to the dofs:

$$\{C\} = [Z_C] . \{q\}$$

Assembling all elementary matrices leads to a global equation which relates the external forces $\{Q\}$ and the measurements $\{C\}$ to all displacements $\{q\}$, at the pulsation ω:

$$\left\{ \begin{array}{c} \{\overline{Q_f}\} \\ \{Q_c\} \\ \{Q_l\} \\ \{C\} \end{array} \right\} = \begin{bmatrix} [Z_{ff}] & [Z_{fc}] & [Z_{fl}] \\ [Z_{cf}] & [Z_{cc}] & [Z_{cl}] \\ [Z_{lf}] & [Z_{lc}] & [Z_{ll}] \\ [Z_{Cf}] & [Z_{Cc}] & [Z_{Cl}] \end{bmatrix} . \left\{ \begin{array}{c} \{q_f\} \\ \{\overline{q_c}\} \\ \{q_l\} \end{array} \right\} \tag{3}$$

The inverse problem consists in using network measured responses to identify the unknown quantities $\{q_l\}$ and $\{Q_l\}$. From equation (2), an exact condensation of clamped and free degrees of freedom is performed in order to dissociate the modeled data from the experimental one and to reduce the size of the inverse problem. That leads to a condensed dynamic stiffness relation of the modeled network part (eq. (4)) and to a transfer matrix relating in a global manner the measured responses to the link degrees of freedom (unknown of the inverse problem, eq. (5)):

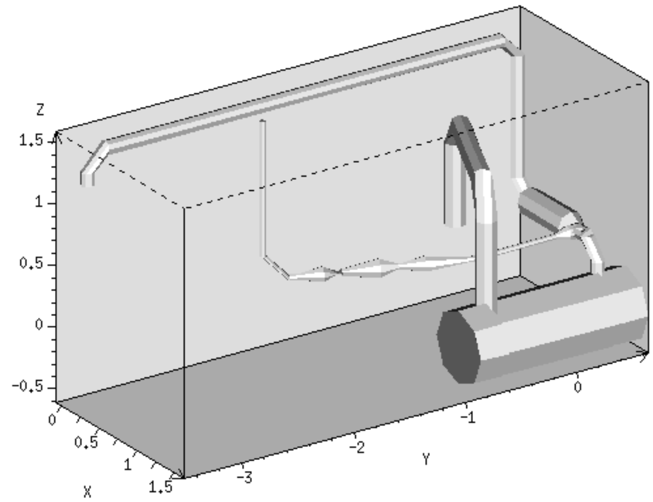

Figure 2. PIPING NETWORK FOR IDENTIFICATION.

$$\{Q_l\} = [Z_l] . \{q_l\} + \{Q'_l\} \tag{4}$$

$$\{C\} = [Z_C] . \{q_l\} + \{C'\} \tag{5}$$

$\{Q'_l\}$ and $\{C'\}$ includes all known forces and displacements at free and clamped dofs. Overabundant measurements are usually used to minimize the noise effects and equation (5) is solved for each pulsation using a least square method:

$$\{q_l\} = \left[[Z_C]^T [Z_C]\right]^{-1} . [Z_C]^T . \left(\{C\} - \{C'\}\right) \tag{6}$$

Replacing the identified vector $\{q_l\}$ in equation (4) gives the link external forces $\{Q_l\}$ and completely defines the boundary dynamic state:

$$\{Q_l\} = [Z_l] . \left[[Z_C]^T [Z_C]\right]^{-1} . [Z_C]^T . \left(\{C\} - \{C'\}\right) + \{Q'_l\} \tag{7}$$

Results

The system presented on figure 2 is a part of a nuclear piping network including pipes, elbows, supports and a pump (Moussou & al, 1999). The comparison between a first direct computation using CIRCUS code and on-site measurements gave bad adequacy: computed speed amplitudes were about a decade below the measured ones. Unsuccessful attempts were made to match the computations and the measurement values: calculation with smaller frequency step in order to catch the resonance maxima, introduction of non infinite stiffnesses for the supports, etc. We

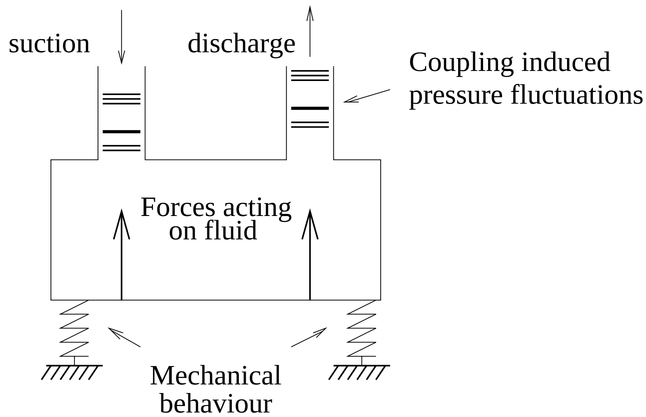

Figure 3. MOVING PUMP IDEALIZATION.

came to the conclusion that the pump did not behave as expected: it should be mobile, and the inner fluid should move with the pump.

Figure 3 shows the special model of the pump made in order to take into account the following hypotheses:

- the pump frame has a vertical stiffness,
- internal vibration forces are applied to the pump frame, which generates acoustic pressure fluctuations by fluid/structure interaction.

19 sensors (fig. 4) regularly located on the studied portion were used in order to identify the following unknown boundary conditions:

- the displacements of the pump mechanically considered as a rigid body,
- the acoustical flow sources on both sides of the pump,
- the acoustical pressure at each end of the studied part.

The identified source flow at the suction of the pump is shown on figure 5 in straight line. It's about a decade bigger than the code's one (dashed line). This can be explained by a bad idealization of the coupling phenomena induced inside the pump. The same result is obtained at the discharge of the pump.

Figure 6 shows the flow sources at the suction and the discharge of the pump. The order of magnitudes are the same, and the phases are in opposition below 100 Hz, as usual for an acoustical pump source. When looking at the mechanical vibrations of the pump, a rigid body movement was identified with a dominant vertical displacement. Figure 7 shows its spectrum which contains significant peaks at 25, 42, 60 and 110 Hz.

These identified sources were then injected in the whole piping network model, a new direct computation was performed and levels of speed magnitudes became similar to the measured ones.

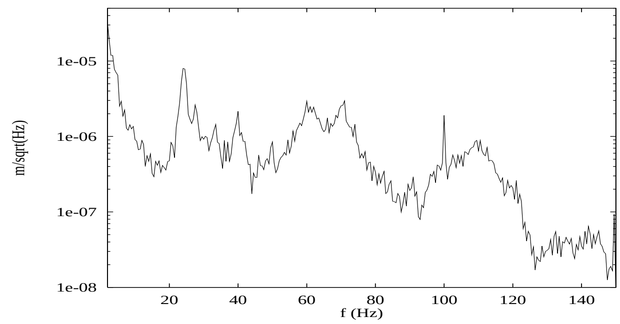

Figure 4. A MEASURED SENSOR EXAMPLE.

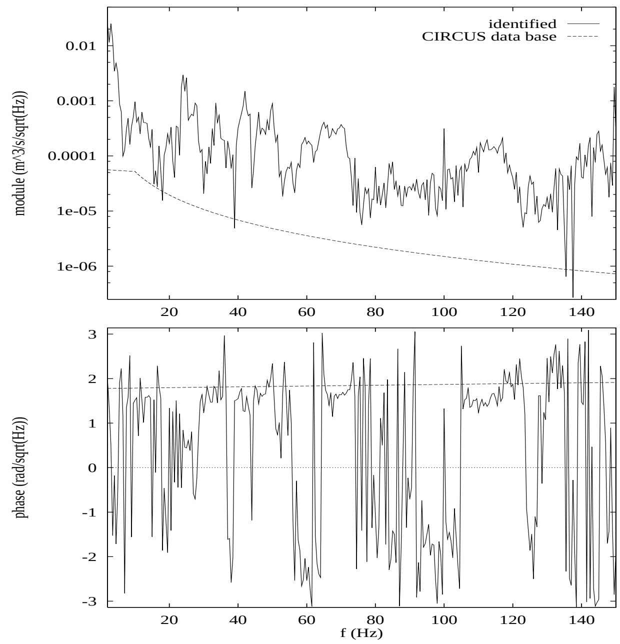

Figure 5. COMPARISON IDENTIFICATION VS DATA BASE.

DYNAMIC UPDATING

Development

We present in this section a new updating method (Gaudin, 1996) for piping network having unknown boundary conditions. In the last section, the modeled part of the network and the sensors measuring its response provide an estimation of boundaries dynamic state. Obviously, like any measuring device, the accuracy of the identified boundary conditions depends on the knowledge accuracy of its physical behaviour. The calibration of such device often improve that knowledge by measuring a set of well-known quantities. This principle is used here in order to improve the knowledge of the physical characteristics of the tested network. So, besides really unknown boundary conditions, a set of

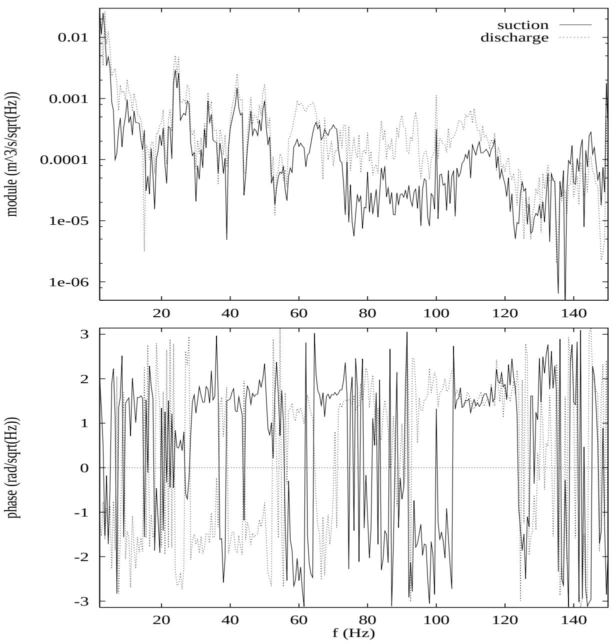

Figure 6. IDENTIFIED VOLUMIC FLOW SOURCES.

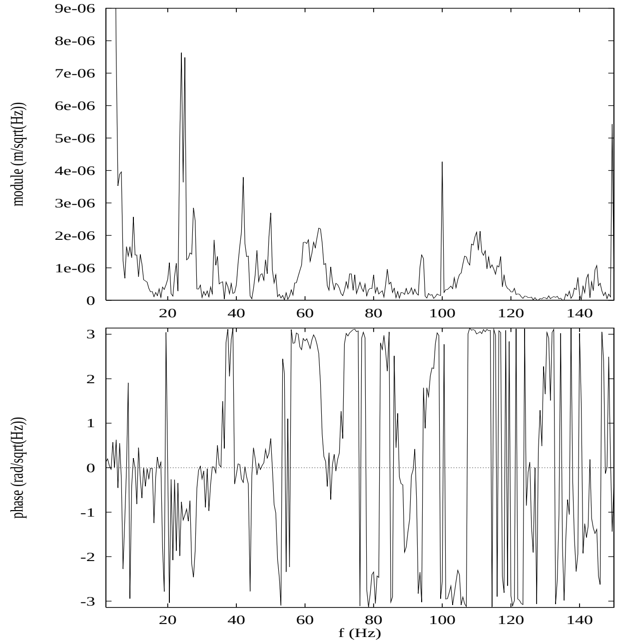

Figure 7. IDENTIFIED VERTICAL DISPLACEMENT.

well known boundaries are identified $(\{Q_t\})$, and compared to the known ones $\left(\{\overline{Q_t}\}\right)$. This difference is used to test and improve the accuracy of the network model. Actually, the mechanical behaviour of the instrumented piping network is idealized using a mathematical model parameterized by its physical characteristics. Its calibration consists in correcting that parameters and agreeing them with the real ones. That is well known as an "updating procedure". The inverse problem expressed in equations 4 and 5 becomes after including all dofs to be identified ("link" ones: $\{q_l\}$ and "test" ones: $\{q_t\}$, see Table 1):

$$\left\{ \begin{array}{c} \{Q_l\} \\ \{Q_t\} \end{array} \right\} = \begin{bmatrix} [Z_{tt}] & [Z_{lt}] \\ [Z_{tl}] & [Z_{tt}] \end{bmatrix} . \left\{ \begin{array}{c} \{q_l\} \\ \{q_t\} \end{array} \right\} + \left\{ \begin{array}{c} \{Q'_l\} \\ \{Q'_t\} \end{array} \right\} \tag{8}$$

$$\{C\} = [Z_C] . \left\{ \begin{array}{c} \{q_l\} \\ \{q_t\} \end{array} \right\} + \{C'\} \tag{9}$$

Matrices $[Z_{..}]$, vectors $\{Q'_l\}$, $\{Q'_t\}$ and $\{C'\}$ depend on the structural parameters that have to be corrected. The number of sensors has to be greater or equal to the number of identified dofs $(\{q_l\}, \{q_t\})$. Solving (9) and replacing identified $\{q_l\}$ and $\{q_t\}$ in (8) gives an estimation of $\{Q_l\}$ and $\{Q_t\}$:

$$\left\{ \begin{array}{c} \{Q_l\} \\ \{Q_t\} \end{array} \right\} = \begin{bmatrix} [Z_{tt}] & [Z_{lt}] \\ [Z_{tl}] & [Z_{tt}] \end{bmatrix} . [Z_C]^+ . \left(\{C\} - \{C'\}\right) + \left\{ \begin{array}{c} \{Q'_l\} \\ \{Q'_t\} \end{array} \right\} \tag{10}$$

where $[Z_C]^+$ is the generalized inverse of $[Z_C]$. Model updating consists in finding the best $\mathbf{p}$ vector which minimizes the distance between identified external forces $\{Q_t\}$ and known ones $\{\overline{Q_t}\}$:

$$\{\varepsilon(\omega,\mathbf{p})\} = \left\{\{Q_t(\omega,\mathbf{p})\} - \{\overline{Q_t}(\omega)\}\right\} \tag{11}$$

All derivatives of $\{\varepsilon(\omega,\mathbf{p})\}$ may be computed analytically. The assembling technique of stiffness matrices and their derivatives is similar and uses the same assembling guides. Then, the minimization of the objective function is performed using a first order Taylor expansion of the error vector $\{\varepsilon(\omega,\mathbf{p}+\Delta\mathbf{p})\}$:

$$\{\varepsilon(\omega,\mathbf{p}+\Delta\mathbf{p})\} = [S(\omega,\mathbf{p})] . \Delta\mathbf{p} + \{\varepsilon(\omega,\mathbf{p})\} \tag{12}$$

$$\text{where :} \ [S(\omega,\mathbf{p})] = \left[\frac{\partial\{\varepsilon(\omega,\mathbf{p})\}}{\partial p_1} \ \ldots \ \frac{\partial\{\varepsilon(\omega,\mathbf{p})\}}{\partial p_n} \right]$$

The number of columns of $[S(\omega,\mathbf{p})]$ equals the number of updating parameters and its number of rows equals the number of test degrees of freedom. We introduce damping by means of a complex Young modulus $Ee^{2j\zeta}$ where ζ is the damping coefficient. So when damping is present, both sensitivity matrix and error vector are complex. Updating parameters being real numbers, the error vector and the sensitivity matrix are splitted into their real and imaginary parts. Hence, the solution consists in minimizing the non linear objective function $\{\varepsilon(\Omega,\mathbf{p})\}^T . \{\varepsilon(\Omega,\mathbf{p})\}$

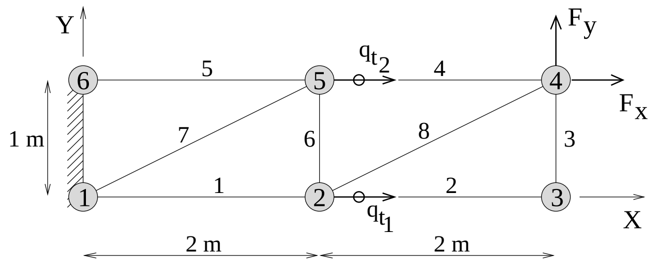

Figure 8. FRAMED NUMERICAL TEST.

where Ω is a set of frequencies and:

$$\{\varepsilon(\Omega,\mathbf{p})\} = \begin{Bmatrix} \Re(\{\varepsilon(\omega_1,\mathbf{p})\}) \\ \Im(\{\varepsilon(\omega_1,\mathbf{p})\}) \\ \vdots \\ \Re(\{\varepsilon(\omega_n,\mathbf{p})\}) \\ \Im(\{\varepsilon(\omega_n,\mathbf{p})\}) \end{Bmatrix} \tag{13}$$

The identification of actual unknown boundary conditions is performed in the same time using equation (9). At the beginning of the minimization procedure, the identified unknowns are probably erroneous. These identified values are being corrected progressively and simultaneously with model parameters updating. Their final value should be the correct one if the mathematical model used to idealize the tested network is able to reproduce its physical behaviour and if the chosen sensors provide a information which make observable the updated parameters and the unknown boundary conditions.

Results

In order to make a simple validation of the proposed approach, we tested the framed structure shown on figure 8. This frame contains 8 circular bars, 6 nodes and is clamped at the left side.

A first evaluation of the dynamical behaviour of the frame was performed with $F_x = F_y = 100\,N$. Displacements were extracted at 8 locations (X and Y directions at nodes 2, 3, 4 and 5, see fig. 9), in the frequency range $[1;200\,Hz]$. Dimensions and material characteristics were:

- $\phi_{bars} = 10\,mm$
- $E = 2.1\,10^{11}\,N/m^2$
- $\rho = 7800\,kg/m^3$
- $\nu = 0.3$
- no damping

In this example, the unique updating parameter is chosen to be the Young modulus of bars 7 and 8. This value is initially

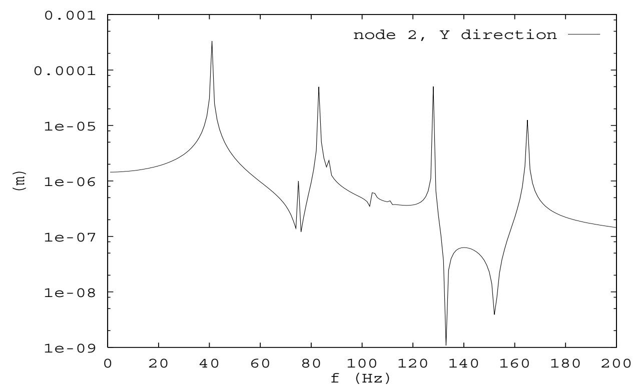

Figure 9. MEASUREMENT SAMPLE.

perturbed and equals $1.8\,10^{11}\,N/m^2$. We also assume that external forces exciting the frame (F_x and F_y) are unknown. In order to perform the identifying/updating process, we have to fix two other variables: the frequency choice and the location of test degrees of freedom. Considering the unique updating parameters, we need a few frequencies. The frequency choice is set to $\Omega = \{40, 90, 140\,Hz\}$. If measurements were perturbed with noise, more frequencies should be taken in order to smooth the effect of noise on parameters estimation. Concerning the location of test degrees of freedom, no extensive exploration of the best location to place those dofs was done. For this kind of problem, we don't need a lot of them. In the proposed example, we choose the X direction at nodes 2 (q_{t1}) and 5 (q_{t2}).

The identifying/updating process applied on this simple example gives the results shown on figures 10 and 11. We can see that it takes 12 iterations to reach the good agreement ($F_x = F_y = 100\,N$ and $E = 2.1\,10^{11}\,N/m^2$).

CONCLUSION

We present in this paper an efficient approach for experimental analysis of piping system response subjected to vibratory and acoustical solicitations. The main feature of the developed method is that it is suitable to be used in real functioning conditions, taking into account unknown boundary conditions. The solicitations applied to the analyzed structure do not have to be known or controlled, but measurements performed on the real system permits the identification of those unknown boundary conditions and the updating of unknown parameters. The original feature of this work is that we can perform both computations in the same procedure.

The complicated and strong coupling existing in piping networks make them a good application of our developments. However, these ideas are probably useful for all in-situ analysis problems.

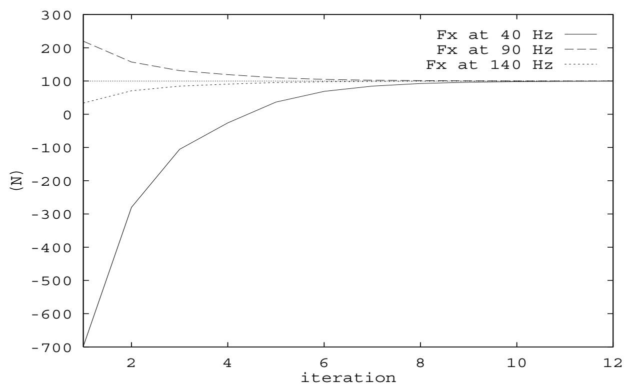

Figure 10. ONE OF THE IDENTIFIED FORCES (F_x).

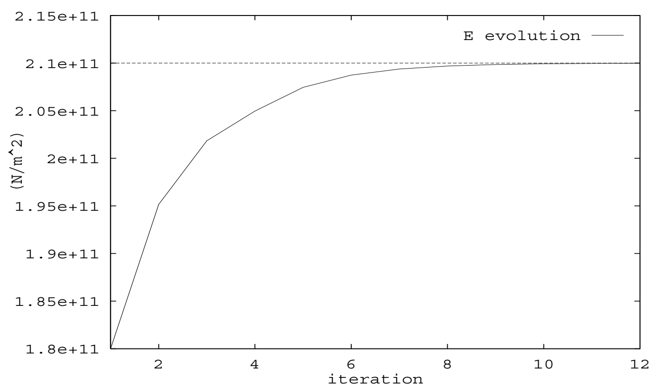

Figure 11. YOUNG MODULUS EVOLUTION.

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