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AN EVOLUTIONARY APPROACH IN MAGNETOTELLURIC INVERSION

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ABSTRACT

An evolutionary computation approach is described for the classical geophysics inverse problem of *magnetotelluric inversion*. This kind of solution to the problem is formulated as a stochastic, iterative optimisation problem, where the evolutionary algorithm operates as the optimiser. Some aspects of the approach are described, in particular three problem-specific operators that were defined: local search, homogenisation and spatial crossover. Comparison is then made between the solution obtained with the evolutionary computation method and another, that relies on a deterministic optimiser plus an entropy-based regularisation. Our results suggest that the evolutionary solution is more robust than the other, more classical approach.

Introduction

Evolutionary computation (EC) was introduced as a tool for the solution of complex problems. "In EC t*he algorithms are based on models of organic evolution. They model the collective learning process within a population of individuals each of which represents a search point in the space of potencial solutions to a given problem. The population evolves towards better regions of the search space by means of randomized processes of recombination, mutation and selection*" (Bäck, 1996). One of its advantages has been the ability to tackle problems that have not yet been completely solved; in these cases, it became unnecessary to know how to find *the* solution, but only to recognise how good a potential solution is, regardless the way it was generated.

Within the universe of evolutionary computation some practitioners argue for using the same simple and sufficiently tested evolutionary framework across the applications. The main justification for this view is the possibility of obtaining theoretical data (as, for example, the schema theorem or the building-block hypothesis (Goldberg 1989; Forrest and Mitchell 1992) that may support the use of evolutionary computation procedures. Such theoretical data have been obtained mainly for engines that could be called "canonical" and, in principle, would consist of: a) binary codification; b) fitness-proportional or rank-based selection; c) 1-, 2-, n-point or uniform crossover; and d) conventional mutation.

However, as well pointed out in Davis (1991), other researchers make a case for using *hybridisation,* where problem-dependent knowledge is introduced into the evolutionary computation engine, so as to attain an improved performance. According to Davis (1991) this can be done using three principles: 1) Using algorithms adapted to the problem in order to generate some of the individuals of the initial population; 2) Incorporating already known algorithm heuristics or procedures to the genetic operators; 3) Enriching the evolutionary algorithm with specific coding schemas.

Here we report on an evolutionary computation approach that was able to produce a solution to an inverse problem only through the design and use of *ad-hoc* evolutionary operators.

Previously (in a paper presented during the last edition of this conference) we tackled this problem and formulation through a gradient-based optimiser together with an entropy-based regularisation principle, and obtained good results. In this paper we review the problem cases therein, now with an evolutionary computation approach, and our results suggest that the latter solution is more robust than the previous, more classical approach.

The Problem

Obtaining the pattern of underground electric conductivities in some region of the Earth, based on measurements of the electromagnetic field at the surface, is a subject of great interest. This problem, called *magnetotelluric inversion,* is a classical problem that appears in many applications in geophysics (such as oil prospection, mining, underground water prospection, etc) and has high relevance for the exploration of regions that are difficult to study through conventional seismic methods (Ramos and Velho, 1996). Overall, what one wants to know is how to obtain underground (conductivity) data, once surface data (electromagnetic field) are known. The corresponding *forward problem* – obtaining electromagnetic fields at the surface, from underground patterns of electric conductivity – is solved through Maxwell's equations in a way that is much simpler than the inverse problem.

Several techniques may be used when looking for inverse problem solutions. The one discussed here is non-linear optimisation. The objective function is defined as the difference between field data (or, in the cases reported herein, synthetically generated data) and those produced by the forward model, which represents an error measure for the candidate solutions. This function is then iteratively minimised through an evolutionary computation procedure. This paper follows another, presented by Ramos and Velho (1996), which also uses an iterative method (although based on a standard gradient-based optimiser) in addition to explicit regularisation procedures. The problem here is the same, but the optimiser has been replaced by an evolutionary computation algorithm. In either case, the forward model involved is the same one presented in the original paper, where its mathematical description was presented. This forward problem will be completely ommitted here and will be used just as a 'black-box'. For simplicity of presentation (but with no loss of generality), only the magnetic field will be considered in this paper (in fact, in tune with what is done in Ramos and Velho (1996).

The Evolutionary Method

Fitness Function

Every individual in the population is represented by a matrix of 7 x 10 real numbers, each one referring to a rectangular slice of underground material (in fact, a prism cut by a half-plane) of unknown conductivity. These conductivities multiply the value $4\pi \times \omega \times 10^{-10}$ and are given in mhos/m; the dimensions of the underground rectangles are taken as $\Delta y = 10$ km and Δz varying from 1 to 10 km. The objective function to be maximised is

$$f = \frac{1}{(1+K)^{\varepsilon}}$$

where f is the fitness of an individual, K is a parameter (kept constant, equal to 0.01, in the present case) that allows for the selective pressure to be tuned, and ε is the *magnetic error* given by

$$\varepsilon = \sum_{i=1}^{440} \left| H_p - H_c \right|$$

In this expression H_p is the component of the magnetic field generated from field measurements (or, out of the simulated measurement, in the case of synthetic data) and H_c is the calculated component by the evolutionary engine. The number 440 results from the fact that measurements are being made at 11 points on the surface of the earth, in 20 frequencies (ω, varying from 0.0001 and 0.01 Hz), yielding both real and imaginary components of the magnetic field at those points. In a situation with real data ε would be the only error capable of being obtained. However, since the present case uses synthetic data, the original individual the evolutionary process will try to reconstruct is already known. So, although the evolutionary search is guided by the error ε, it is possible to define a second error measure, named *conductive error* given by

$$E = \sum_{j=1}^{70} \left| C_p - C_c \right|$$

where E (conductive error) is the absolute values adding up of 70 differences between standard and calculated conductivities. The reason for presenting these two kinds of error resides in the fact that, due to noise (a typical ocurrence in the inverse problem context), minimisation of the magnetic error not necessarily entails minimisation of the conductivity error. In fact, pushing minimisation of the former error too far, may lead to an increase of the latter. Naturally, the evolutionary search process has to avoid being misled by this feature.

Problem-Specific Evolutionary Operators

The numerical results reported below were obtained using three problem-specific evolutionary operators – local search, homogenisation, and a specific crossover operator named SPAUC (SPAtial Uniform Crossover) – all of them described below.

SPAUC

The SPAUC operator considers each individual as a two-dimensional entity. Accordingly, when genetic material is interchanged, full rectangular patches of the candidate solutions are involved. The operator implements the neighbourhood concept both vertically and horizontally. This action prevents two common problems that appear when a two-dimensional individual is represented in one dimension, namely, the possibilities that: a) two neighbouring vertical positions in the chromosome no longer be neighbours; and b) originally distant elements get closer to each other than they should (for example, as happens when the last value on a row and the first value in the subsequent row are involved).

In order to implement the SPAUC operator, we used the following algorithm: a) n horizontal cuts are generated on random sites across the individual; b) k vertical cuts are generated on random sites across the individual (amounting to r patches, $r = (n+1)\times(k+1)$) ; c) a V mask of size r is generated having random binary values; d) given the two ancestors that will be operated by SPAUC, the offspring to be generated will obey the following rule: patches with a mask value equal to 0 are filled up with values from the first parent, while patches with a mask value equal to 1 take their values from the corresponding patch in the second parent. Notice that the longer the mask V, the closer SPAUC will resemble the standard uniform crossover. Detailed description of SPAUC, as well as various other aspects related to it and the other genetic operators we used can be found in (Navarro *et al*, 1999).

Local Search

Local search is implemented as simple a hill-climbing algorithm. At every 5 generations the attempt is made to improve the fitness of the fittest individual by aproximately 13 cycles, of 140 trials each (the number of cycles is an average figure, since it starts smaller and increases throughout the generations). The value 140 derives from the fact that the chromosome is 70 points long, and each one of them is subjected to two small random increments (one positive and the other negative), which represent probing two neighbouring positions in the fitness landscape. These probed positions with higher fitness are stored until the end of the cycle, when all the changes are applied at once, thus generating a new, higher fitted individual that replaces the original. This is not a novel approach, though; in fact, Bäumer (1996), studying a similar problem stated that: "*the most efficient way is to start with a genetic algorithm for calculating a population of nearly optimal models which then are used as initial models for local search methods*".

Homogenisation

This operator is based upon the notion that neighbouring regions will have greater probability of having similar conductivities. At every 5 generations an attempt is made to improve the fitness of the fittest individual through an average of 13 cycles of 3 trials each. Every trial starts with the generation of a rectangular random patch and continues, firstly, by replacing all values in the patch, for one of the existing values – thus generating an homogeneous patch – and checking the resulting fitness; secondly, by repeating the same procedure, individually for all the other values in the patch. The patch that results the best (improved) performance, if any, replaces the original.

The parameter values associated to the local search and homogenisation operators (5 generations, 13 cycles, 3 trials) were not obtained after an exhaustive search. Rather, they resulted from a trial-and-error process that was performed until satisfactory results were found. At the moment, we are deepening the analyses of the effect of parameter values in the behaviour of the model.

We used the genetic algorithm package GALLOPS (Goodman 1996), modified by the 3 operators above that were added to it. The following parameters were used: population size of 100, tournament selection of size 15, 3% mutation per conductivity value, and crossover at a rate of 85%.

In addition to the use of homogenisation alone, it was also used in association with the SPAUC operator, when, after recombination, each one of the regions created undergoes the action of homogenisation. But while homogenisation alone is applied with 100% probability to the best individual only, in association with SPAUC it is applied to all individuals created out of the recombination (i.e., potentially to the entire population), and with low probability (5% in this case, to each generated region).

Results

The SPAUC-based evolutionary process is compared to a classical iterative approach for the problem, that uses a sophisticated gradient-based optimiser – the routine E04UCF from the NAG library (NAG, 1988) – plus an entropy-based regularisation process. This experiment is fully performed in the presence of 1% gaussian noise, with the same amount for both approaches. The comparisons were made in four well-defined test cases, meant to explore significant problem features. In each test case the algorithms try to reconstruct the conductivity pattern represented by it.

Test Case 1

The first test case is represented in Table 1, where an underground region is represented by a matrix of conductivity values; it is assumed that a rectangular block with conductivity 100 is to be found immersed within a rocky background with conductivity 10. Ramos and Velho (1996) investigated the same problem running their model 4 times and obtaining an average conductive error (Eavg) of 453.8. Their best solution is shown in Table 2. The same problem was run 6 times with the evolutionary, SPAUC-based approach, yielding an average conductivity error of 210.5, and the fittest individual is represented in Table 3.

Table 1: Test case 1									
10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	10.0	10.0	10.0
10.0	10.0	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0

Table 2. Best solution for teste case 1 found in Ramos and Velho (1996)									
10.02	10.25	9.64	10.09	10.03	9.58	10.15	10.07	9.97	9.94
10.02	9.64	**97.10**	**100.00**	**88.98**	**97.62**	**100.00**	10.07	9.95	9.94
9.79	9.64	**97.10**	**99.97**	**88.98**	**97.62**	**100.00**	10.07	9.95	9.93
9.78	9.63	10.08	10.02	12.38	16.85	9.56	10.06	9.95	10.54
9.78	9.62	10.08	10.03	12.38	16.85	9.57	9.97	9.95	10.54
10.25	9.63	10.08	10.02	9.58	10.16	9.57	9.97	9.94	10.55
10.25	9.63	10.09	10.02	9.58	10.15	10.09	9.97	9.94	10.55

Table 3. Fittest individual for test case 1 found by the evolutionary approach									
9.89	10.17	9.72	10.34	9.44	10.51	9.73	10.24	10.25	10.19
9.62	9.85	**96.78**	**98.84**	**90.48**	**98.27**	**94.57**	10.29	9.16	10.89
10.61	10.98	**99.84**	**99.96**	**74.33**	**99.47**	**92.17**	10.43	8.78	10.58
10.26	8.83	8.63	16.27	7.06	11.48	12.91	11.19	11.29	10.20
8.82	11.75	10.60	6.95	11.81	11.81	8.85	10.90	9.73	9.45
12.86	6.99	10.98	12.41	8.28	4.22	6.95	8.22	11.74	11.15
4.20	34.73	5.26	10.29	3.23	9.74	19.85	11.25	10.44	6.43

Other Test Cases

Tables 4, 5 and 6 represent the other test cases. Table 4 refers to a problem with two underground patches to be found, one defined by a block of material with conductivity 10 times as much that of the background, and another with 1/10 the background conductivity. Test case 3, represented by Table 5, depicts a big and irregular conductive region that stretches throughout the background. Finally, test case 4 shows a very small patch of low conductivity material, embedded in rock, as represented in Table 6.

The results achieved by the gradient-based otimisation technique plus regularisation, and by the evolutionary procedure are summarised in Table 7, for test cases 2, 3 and 4. In the table, the minimum and average values of the conductivity error (E) and magnetic errors (ε) found in each case are presented for both approaches.

Table 4: Test case 2									
10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	**1.0**	**1.0**	10.0	**100.0**	**100.0**	10.0	10.0	10.0
10.0	10.0	**1.0**	**1.0**	10.0	**100.0**	**100.0**	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0

Table 5: Test case 3									
10.0	**100.0**	**100.0**	10.0	10.0	10.0	10.0	10.0	10.0	10.0
10.0	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	10.0	10.0	10.0
10.0	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	**100.0**	**100.0**	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	**100.0**	**100.0**	**100.0**	10.0	10.0
10.0	10.0	10.0	10.0	10.0	**100.0**	**100.0**	**100.0**	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0

Table 6: Test case 4									
10.0	10.0	10.0	10.0	**1.0**	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0

Table 7: Summary of results between two techniques for the test cases 2, 3 and 4

Test Case	Gradient-Based + Entropic Regularisation					Evolutionary Approach				
	# runs	E min.	E avg	ε min	ε avg	# runs	E min.	E avg	ε min	ε avg
2	5	7.3	258.5	0.1356	0.4980	3	150.7	229.5	0.2891	0.4480
3	1	714.5	--	0.6116	--	3	185.5	447.3	0.1721	0.2272
4	3	32.3	109.4	0.3770	3.9821	2	61.12	64.59	0.1555	0.1646

In test case 2, one of the five runs with the gradient-based approach yielded an excellent result which, represents a clear shift away from the average (notice the columns with the minimum figures); however, when all runs are considered, the superiority of the evolutionary approach appears, since both errors are smaller for this approach. In test case 3 the averages are not included because only the result of a single run was available; for this problem instance, the results obtained from the evolutionary computation are clearly better. And finally, in test case 4 the minimum conductive error is better in the gradient-based process, but the minimum magnetic error and both averages values are better in the evolutionary approach. Overall, the conclusion, therefore, is that the evolutionary technique is showing more robustness than the other, more classical approach.

Bäumer (1996), studying a similar problem, makes the comparision between two methods: genetic algorithm and simulated annealing. This study pointed at the superiority of the non-linear methods, such as the evolutionary (in contrast to the classical), since those are less dependent on the initial solution used to trigger off the search process. As suggested from the minimum errors found by the evolutionary approach in our experiments, the same point can be made here.

Concluding Remarks

The first point to notice is that our experiments have once more suggested that the use of canonical, ready-to-use genetic algorithms seems to be only appropriate for simple problems. In more complex, real-world situations – such as magnetotelluric inversion, a high dimensional inverse problem – in order to attain satisfactory results it seems necessary to add problem-specific knowledge into the evolutionary engine.

In the case we reported we explicitly relied on the following facts: individuals are two-dimensional and should be handled as such; conductivity distributions in the underground of the earth should follow homogenous patches; and simple hill-climbing algorithm should be used to help improve the results.

Another point to be remarked is that the evolutionary engine presented better results than a conventional optimiser, although at higher cost. So, while the best result from the Ramos and Velho (1996) test case 1, establishes an approximate amount of 14×10^3 objective function calls, a similar result found here requested approximately 100×10^3 calls. Fine-tuning the evolutionary method so as to decrease such a cost is a real possibility that we are currently investigating.

However, on real-world data, where processing time gives place to robustness, an evolutionary technique such as the one we presented may be a more attractive alternative. In fact, in real-world situations, where noise and lack of information about the prospected underground areas are the general rule, robustness issues, such as independence on the initial solutions of the search process, the use of a stochastic optimisation method (such as the one presented herein) is a very appealing alternative. Another possibility, as concluded by Bäumer (1996), is using evolutionary computation in order to find the best region of the search space, and then performing an efficient local search so as to find the best point of that region.

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REFERENCES

Bäumer, O. “Inverting Magnetotelluric Data Using Genetic Algorithms and Simulated Annealing”. *Inverse Methods: interdisciplinary elements of methodology, computation and applications*. Springer-Verlag, 1996.

Bäck, T. *Evolutionary Algorithms in Theory and Practice.* Oxford University Press. 1996.

Davis, L. *The Handbook of Genetic Algorithms*. Van Nostrand Reinhold, 1991.

Forrest, S. and Mitchell, M. “Relative Buildind-Block Fitness and the Building-Block Hypothesis”. *Foundations of Genetic Algorithms 2*, Whitley, D. (ed.). Morgan Kaufmann, 1992.

Goldberg, D.E. *Genetic Algorithms in Search, Optimisation and Machine Learning*. Addison Wesley Publishing Company, 1989.

Goodman, E.D. *The Genetic Algorithm Optimised for Portability and Parallelism System*. Michigan State University, 1996.

NAG. “E04UCF Routine”, *NAG Fortran Library Mark 13*, Oxford, UK, 1988.

Navarro, P.L.K.G.; de Oliveira, P.P.B.; Ramos, F.M. and Velho, H.F.C., “Magnetotelluric inversion using problem-specific genetic operators”. In: W. Banzhaf, J. Daida, A.E. Eiben, M.H. Garzon, V. Honavar, M. Jakiela, and R.E. Smith (eds.). *GECCO-99: Proceedings of the Genetic and Evolutionary Computation Conference*, San Francisco, CA, Morgan Kaufmann, 1999. To appear.

Ramos, F.M. and Campos Velho, H.F. (1996): "Reconstruction of Geoelectric Conductivity Distributions Using a Minimum First-Order Entropy Technique", In *2nd International Conference on Inverse Problems on Engineering*, Le Croisic, France, Vol. 2 , pp. 199-206.