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A SOURCE-DETECTOR METHODOLOGY FOR THE CONSTRUCTION AND SOLUTION OF THE ONE-DIMENSIONAL INVERSE TRANSPORT EQUATION

Adriana T. Kauati[1], Antônio J. Silva Neto[2,3] and Nilson C. Roberty[3]
[1] Biomedical Eng. Program, COPPE/UFRJ, CP 68510, CEP 21945-970, Rio de Janeiro, RJ, Brazil, kauati@peb.ufrj.br
[2] Instituto Politécnico, UERJ, CP 97282, CEP 28601-970, Nova Friburgo, RJ, Brazil, ajsneto@iprj.uerj.br
[3] Nuclear Eng. Program, COPPE/UFRJ, CP 68509, CEP 21945-970, Rio de Janeiro, RJ, Brazil, nilson@lmn.con.ufrj.br

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ABSTRACT

A source-detector methodology is presented for the construction of an inverse transport equation that once solved provides estimates for radiative properties and/or internally distributed sources in participating media. From the proper combination of source and detector pairs, a system of non-linear equations is assembled, taking also in consideration experimental data on the exit radiation from the medium. Test case results are also presented.

INTRODUCTION

The inverse analysis of radiative transfer in participating media has several relevant applications in engineering, medicine, geophysics, astrophysics and other research areas.

Ustinov (1978) estimated the extinction coefficient and aerosol particles concentration in atmospheres. Bakirov et al. (1986) proposed a methodology for the determination of mass concentration of soot particles in flames. McCormick (1979), McCormick and Sanches (1981), Ho and Özisik (1988), Sanchez et al. (1990), Subramaniam and Mengüç (1991), Nicolau et al. (1994), and Silva Neto and Özisik (1993, 1995), just to name a few, solved inverse problems for single scattering albedo, optical thickness and/or anisotropic scattering phase function estimation. Yi et al. (1992) estimated the location and strength of a bioluminescent radiation source. Fukshansky et al. (1991) estimated the absorption and scattering coefficients and the asymmetry factor of scattering in living plant leaves. Different types of radiation such as neutrons, gamma-rays and photons have been used for object identification in industry (non-destructive testing), and in medicine (diagnosis and therapy). In many of the techniques used, scattering is neglected, yielding relatively simple reconstruction problems. This is the case in Computerized Tomography and Single Photon Emission Computerized Tomography (SPECT).

When scattering has to be taken into account (McCormick, 1993, Mengüç and Dutta, 1994, Roberty and Oliveira, 1995), such as in Near Infrared Optical Tomography (NIROT), the reconstruction model becomes much more complex, non-linear, even requiring the computation of the radiation field. This particular tomographic problem is placed in the same context as radiative heat transfer in participating media and neutron transport in nuclear reactors, being the related physical phenomena (absorption, emission and scattering) modeled by the linearized Boltzmann equation.

Silva Neto and Roberty (1998, 1998a) have been working on a source-detector methodology for the estimation of radiative properties and internally distributed sources in participating media. In this work the methodology is presented, as well as test case results for extinction and scattering coefficients estimation in one-dimensional homogeneous media.

MATHEMATICAL FORMULATION OF THE DIRECT PROBLEM

A plane-parallel, gray, anisotropically scattering slab of thickness L, with transparent boundaries is subjected to an external collimated radiation source that may be positioned in different locations around the medium, as shown in Fig.1.

The mathematical formulation of this steady-state one-dimensional radiative transfer problem is given by:

$$\mu\frac{\partial\phi^{a,k_a}(x,\mu)}{\partial x}+\sigma_t(x)\phi^{a,k_a}(x,\mu)-\frac{1}{2}\int_{-1}^{1}\sigma_s(x,\mu',\mu)\phi^{a,k_a}(x,\mu')d\mu'=S(x,\mu)$$

$$\text{in } 0<x<L, -1\le\mu\le 1 \tag{1a}$$

$$\phi^{a,k_a}(b,\mu)=A\delta_{ab}f^{b,k_a}(\mu) \tag{1b}$$

where $\phi^{a,k_a}(x,\mu)$ is the radiation intensity, x is the spatial coordinate, μ is the direction cosine of the radiation beam with the positive x axis, $\sigma_t(x)$ is the total extinction coefficient (absorption + scattering), $\sigma_s(x,\mu',\mu)$ is the scattering coefficient, $S(x,\mu)$ is an internally distributed source, A is the amplitude of the strength of the external collimated radiation source and $f^{b,k_a}(\mu)$ represents its dependence with the polar angle. The indices a and b represent, respectively, the surface in which the source is located (a = 0 or a = L) and the surface for which the boundary condition is being written (b = 0 or b = L). The index k_a represents the location of the source, $k_a = 1, 2, \ldots, I_a$.

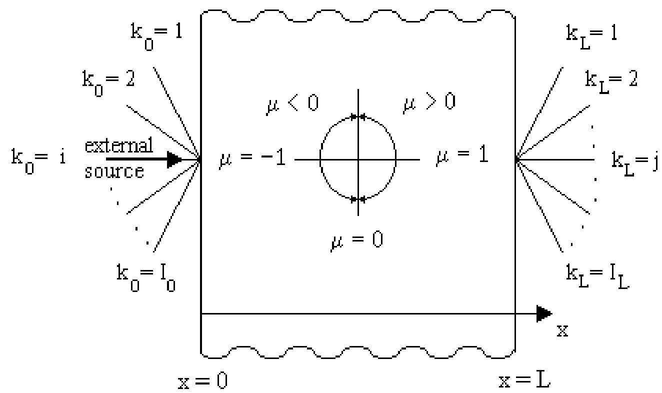

Figure 1 – Possible locations for the external collimated radiation source.

When the geometry, boundary conditions, material properties and the strength of the source are known, the radiation intensity distribution, $\phi^{a,k_a}(x,\mu)$ can be calculated. Problem (1) is then called the direct problem. On the other hand, when any of this information, or a combination of them, is unknown, but experimental measurements of the transmitted and/or reflected exit radiation are available, an estimation of the unknowns may be possible. This is known as the inverse problem.

MATHEMATICAL FORMULATION OF THE INVERSE PROBLEM

The inverse problem considered here involves the estimation of the source S(x, μ) and extinction and scattering coefficients, using the source detector methodology. From the direct problem, given by Eqs. (1), called here source problem, an adjoint problem is constructed, and is called detector problem. Convolving the source problem with the adjoint function, that consists of the solution of the detector problem, doing an integration by parts, and bringing the detector problem itself into the resulting equation, a system of non-linear equations is obtained. This system is called here the inverse transport equation (ITE). From the solution of the ITE, taking into account the experimental data on the exit radiation, the unknown quantities are estimated. These steps will now be described in more detail.

The Source Problem. This problem consists on the direct problem whose formulation is given by Eqs.(1), using estimates for the unknown quantities. These estimates are obtained along the iterative procedure adopted for the solution of the inverse problem.

The Detector Problem. For each location where a detector is positioned, an adjoint problem is formulated. This formulation is obtained from the source problem by reversing the direction of radiation transfer, i.e., by replacing μ by -μ,

$$-\mu\frac{\partial\phi^{*a',k_{a'}}(x,\mu)}{\partial x}+\sigma_t^R(x)\phi^{*a',k_{a'}}(x,\mu)-\frac{1}{2}\int_{-1}^{1}\sigma_s^R(x,\mu',\mu)\phi^{*a',k_{a'}}(x,\mu')d\mu'=q^*(x,\mu)$$
$$\text{in } 0<x<L, -1\le\mu\le 1 \qquad (2a)$$

$$\phi^{*a',k_{a'}}(b,\mu)=\delta_{a'b}g^{b,k_{a'}}(\mu) \qquad (2b)$$

where σ_t^R , σ_s^R and q* are reference functions for the unknowns σ_t , σ_s and S respectively.

In industry the reference values would be those of the material under investigation in perfect manufacturimg conditions. Any anomaly on the material properties or geometry could then be estimated using the inverse technique here described. In medicine the reference values would be those of healthy organs or tissue. We expect then that the inverse problem solution will show any deviation from these reference values.

The index a' represents the surface in which the sensor is located and $k_{a'} = 1, 2, \ldots, I_{a'}$, represents the position of the detector around that surface.

By imposing the coincidence of the location of the detector, $k_{a'} = 1, 2, \ldots, I_{a'}$, with those for the source, $k_a = 1, 2, \ldots, I_a$, function $g^{b,k_{a'}}(\mu)$ represents the measurement that would be obtained by the detector for the strength of the source located at that position,

$$g^{b,k_{a'}}(\mu)=\eta A f^{b,k_a}(-\mu), \qquad k_{a'}=k_a \qquad (3)$$

where η is the efficiency of the detector.

The Auxiliary Problem. Reversing again the direction of radiation transfer, i.e., replacing μ by -μ, and defining an auxiliary fuofnction

$$\phi(x,\mu)=\phi^*(x,-\mu) \qquad (4)$$

we obtain from the detector problem the following auxiliary problem

$$\mu\frac{\partial\phi^{a',k_{a'}}(x,\mu)}{\partial x}+\sigma_t^R(x)\phi^{a',k_{a'}}(x,\mu)-\frac{1}{2}\int_{-1}^{1}\sigma_s^R(x,\mu',\mu)\phi^{a',k_{a'}}(x,\mu')d\mu'=q(x,\mu)$$
$$\text{in } 0<x<L, -1\le\mu\le 1 \qquad (5a)$$

$$\phi^{a',k_{a'}}(b,\mu)=\delta_{a'b}\eta f^{b,k_a}(\mu) \qquad (5b)$$

The formulation of the auxiliary problem is the same as that for the source problem, with the exception that the former uses the reference values for the unknown quantities while the latter uses estimates obtained along the solution of the inverse problem.

The Inverse Transport Equation. In the first step on the Inverse Transport Equation (ITE) construction we multiply Eq. (1a) by the adjoint function $\phi^{*a',k'_a}(x,\mu)$, and integrate over the spatial and angular domain, x=[0,L], μ=[-1,1], respectively.

$$\int_{-1}^{1}\int_{0}^{L}\left[\mu\frac{\partial\phi^{a,k_a}(x,\mu)}{\partial x}\phi^{*a',k_{a'}}(x,\mu)+\sigma_t(x)\phi^{a,k_a}(x,\mu)\phi^{*a',k_{a'}}(x,\mu)\right]dxd\mu-$$
$$-\tfrac{1}{2}\int_{-1}^{1}\int_{0}^{L}\left[\phi^{*a',k_{a'}}(x,\mu)\int_{-1}^{1}\sigma_s(x,\mu',\mu)\phi^{a,k_a}(x,\mu')d\mu'\right]dxd\mu=$$
$$=\int_{-1}^{1}\int_{0}^{L}\left[S(x,\mu)\phi^{*a',k'_a}(x,\mu)\right]dxd\mu \qquad (6)$$

In fact Eq.(6) represents a system of $M=(I_o+I_L)x(I_{o'}+I_{L'})$ nonlinear equations, taking into account all possible combinations of source and detector locations.

Integrating Eq.(6) by parts, and plugging eq.(2a) into the resulting equation, we get

$$\int_{-1}^{1}\int_{0}^{L}\left[\sigma_t(x,\mu)+\sigma_t^R(x)\right]\phi^{a,k_a}(x,\mu)\phi^{*a',k_{a'}}(x,\mu)dxd\mu-$$
$$-\tfrac{1}{2}\int_{-1}^{1}\int_{0}^{L}\int_{-1}^{1}\left[\sigma_s(x,\mu',\mu)-\sigma_s^R(x,\mu',\mu)\right]\phi^{a,k_a}(x,\mu)\phi^{*a',k_{a'}}(x,\mu)d\mu'dxd\mu-$$
$$-\int_{-1}^{1}\int_{0}^{L}\left[S(x,\mu)\phi^{*a',k'_a}(x,\mu)-q^*(x,\mu)\phi^{a,k_a}(x,\mu)\right]dxd\mu=$$
$$=\int_{-1}^{1}\mu\left[\phi^{*a',k_{a'}}(x,\mu)\phi^{a,k_a}(x,\mu)\right]_0^L d\mu \qquad (7)$$

Defining the quantities

$$\Delta\sigma_t(x)=\sigma_t(x)-\sigma_t^R(x) \qquad (8a)$$
$$\Delta\sigma_s(x,\mu'\mu)=\sigma_s(x,\mu',\mu)-\sigma_s^R(x,\mu',\mu) \qquad (8b)$$

and using the definition of the auxiliary function given by Eq. (4), Eq. (7) is written as

$$\int_{-1}^{1}\int_{0}^{L}\Delta\sigma_t(x)\phi^{a,k_a}(x,\mu)\phi^{a',k_{a'}}(x,-\mu)dxd\mu-$$
$$-\tfrac{1}{2}\int_{-1}^{1}\int_{0}^{L}\int_{-1}^{1}\Delta\sigma_s(x,\mu',\mu)\phi^{a,k_a}(x,\mu)\phi^{a',k_{a'}}(x,-\mu)d\mu'dxd\mu-$$
$$-\int_{-1}^{1}\int_{0}^{L}\left[S(x,\mu)\phi^{a',k_{a'}}(x,-\mu)-q(x,-\mu)\phi^{a,k_a}(x,\mu)\right]dxd\mu=$$
$$=\int_{-1}^{1}\mu\left[\phi^{a,k_a}(x,\mu)\phi^{a',k_{a'}}(x,-\mu)\right]_0^L d\mu \qquad (9)$$

The system of non-linear equations represented by Eq. (9) is here called the Inverse Transport Equation (ITE). Solving the ITE, estimates for the unknown quantities are obtained.

A couple of comments are in order. As mentioned before, the source problem and the auxiliary problem are very similar. Therefore, the same method can be used for the solution of both problems. For the computational implementation, this means that the same computer program (actually subroutine) can be used for their solution. This is why we write the ITE in terms of the auxiliary function, instead of using the adjoint function.

On the right side of Eq. (9) the experimental data come into place. Table 1 summarizes how the boundary conditions and radiation exit measurements are taken into account.

Table 1- Radiation intensity $\phi(x,\mu)$ and auxiliary function $\phi^o(x,\mu)$ at the boundaries[1]

		source location			
		x=0		x=L	
boundary	x=0	$\mu > 0$	given = $Af^{o,ko}(\mu)$ [$\eta A f^{o,ko}(\mu)$]	$\mu > 0$	given = 0 [0]
		$\mu < 0$	measured [calculated]	$\mu < 0$	measured [calculated]
	x=L	$\mu > 0$	measured [calculated]	$\mu > 0$	measured [calculated]
		$\mu < 0$	given = 0 [0]	$\mu < 0$	given = $Af^{L,k_L}(\mu)$ [$\eta A f^{L,k_L}(\mu)$]

[1]$\phi^o(x,\mu)$ in brackets

THE SOLUTION OF THE INVERSE TRANSPORT EQUATION

To keep this presentation as simple as possible, we will consider the inverse problem of estimating only the extinction and the scattering coefficients. We first make an expansion of the unknown quantities given by Eqs. (8a-b),

$$\Delta\sigma_t(x)=\{\Delta\underline{\sigma}_{tn}\}^T\underline{\psi}(x)=\sum_{n=1}^{N}\Delta\sigma_{tn}\psi_n(x) \qquad (10a)$$

$$\Delta\sigma_s(x,\mu',\mu)=\sum_{l=0}^{J}\tfrac{2l+1}{2}\{\Delta\underline{\sigma}_{s\,ln}\}^T\underline{\psi}(x)P_l(\mu',\mu)=$$
$$\sum_{l=0}^{J}\tfrac{2l+1}{2}\sum_{n=1}^{N}\Delta\sigma_{s\,ln}\psi_n(x)P_l(\mu',\mu) \qquad (10b)$$

where $\underline{\psi}(x)$ is the base used for the spatial dependence representation, while for the angular dependence we used an expansion in Legendre polynomials. The indices n and l represent the spatial and angular discretizations, respectively.

The solution of the ITE, becomes now the problem of determining the coefficients $\{\Delta\underline{\sigma}_{tn}\}$ and $\{\Delta\underline{\sigma}_{s\,ln}\}$.

Plugging Eqs. (10a-b) into Eq. (9), we obtain for the problem with no internal sources,

$$\sum_{n=1}^{N}\Delta\sigma_{tn}\int_{0}^{L}\psi_n(x)\left[\int_{-1}^{1}\phi^{a,k_a}(x,\mu)\phi^{oa',k_{a'}}(x,-\mu)d\mu\right]dx -$$
$$-\tfrac{1}{4}\sum_{l=0}^{J}(2l+1)\sum_{n=1}^{N}\Delta\sigma_{s\,ln}\int_{0}^{L}\psi_n(x)\left\{\int_{-1}^{1}\phi^{a',k_{a'}}(x,-\mu)\times\right.$$
$$\left.\times\left[\int_{-1}^{1}P_l(\mu',\mu)\phi^{a,k_a}(x,\mu')d\mu'\right]d\mu\right\}dx$$
$$=\int_{-1}^{1}\mu\left[\phi^{a,k_a}(x,\mu)\phi^{a',k_{a'}}(x,-\mu)\right]_0^L d\mu \tag{11}$$

System (11) has E=(J+2)xN unknowns and $M=(I_0+I_L)x(I_{0'}+I_{L'})$ equations. For $I_0 = I_{0'}=I_L=I_{L'}=K$, we get $M=4K^2$ equations. If we want the system to be overdetermined, i.e. M>E, we can expect to recover $\{\Delta\underline{\sigma}_{tn}\}$ and $\{\Delta\underline{\sigma}_{sln}\}$ at not more than M/E = 4 K^2/(J+2)xN elements each, i.e. $N<4K^2/J+2$. For isotropic scattering, J=0, we get $N < 2K^2$.

Considering $\psi_n(x)=1$, $x_n<x< x_{n+1}$, n=1, 2, 3,…, N-1, with $x_o=0$ and $x_N=L$, and isotropic scattering, Eq. (11) is further simplified, yielding

$$\sum_{n=1}^{N}\Delta\sigma_{tn}(x_{n+1}-x_n)\int_{-1}^{1}\phi^{a,k_a}(x_{n+1/2},\mu)\phi^{a',k_{a'}}(x_{n+1/2},-\mu)d\mu -$$
$$-\tfrac{1}{4}\sum_{n=1}^{N}\Delta\sigma_{son}(x_{n+1}-x_n)\int_{-1}^{1}\phi^{a',k_{a'}}(x_{n+1/2},-\mu)\int_{-1}^{1}\phi^{a,k_a}(x_{n+1/2},\mu')d\mu'd\mu =$$
$$=\int_{-1}^{1}\mu\left[\phi^{a,k_a}(x,\mu)\phi^{a',k_{a'}}(x,-\mu)\right]_0^L d\mu \tag{12}$$

System (12), as well as system(11), can be written in a more compact form as

$$\sum_{j=1}^{E}W_{m,j}Z_j = h_m,\ \ m=1,2,...,M \tag{13}$$

where

$$\underline{Z}=\left\{\Delta\underline{\sigma}_{tn},\Delta\underline{\sigma}_{s\,ln}\right\}^T \tag{14}$$

W_{mj} are coefficients obtained integrating each term on the left side of Eq. (12) and h_m are obtained by using the proper combinations of the results of the source and auxiliary problems and the experimental data according to Table1 on the right hand side of the same equation.

As system (13) may be ill-conditioned, with possible problems of existence and uniqueness, an action by line, or action by block, algorithm may be suitable for its solution. The relatively smaller convergence rate of such methods when compared with other methods, is compensated by its robusteness on the treatment of ill-conditioned problems.

Using the Bregman distance with the entropy function (Bregman, 1967)

$$D(\underline{Z},\overline{\underline{Z}})=\sum_{j=1}^{E}Z_j\ln\frac{Z_j}{\overline{Z}_j}+\overline{Z}_j-Z_j \tag{15}$$

where $\overline{Z}$ is a vector with reference values for the unknowns, we write the Lagrangian considering one line Bregman projection

$$L_m(\underline{Z},\overline{\underline{Z}})=D(\underline{Z},\overline{\underline{Z}})-\lambda_m\left(\sum_{j=1}^{E}W_{m,j}Z_j-h_m\right) \tag{16}$$

Regularization occurs through the projection on convex sets (each line on system (13)).

In the presence of noise a squared residues norm is added to Eq.(16) (Elfving, 1989) yielding a Tikhonov like nonlinear regularization functional (Engl et al., 1996). With Tikhonov's functional an additional regularization is included by a small spectral translation.

Writing the Euler-Lagrange equation, and solving for Z_j

$$\frac{\partial L_m(\underline{Z},\overline{\underline{Z}})}{\partial\underline{Z}}=0\quad\rightarrow\quad Z_j=\overline{Z}_j\exp(\lambda_m W_{m,j}) \tag{17}$$

and plugging Z_j into Eq (13), one obtains the Lagrange multiplier λ_m from the solution of the resulting nonlinear equation with Newton's method,

$$\lambda_m^{k+1}=\lambda_m^k-\frac{\sum_{i=1}^{E}W_{m,i}\overline{Z}_i\exp\left[\lambda_m^k W_{m,i}\right]-h_m}{\sum_{j=1}^{E}W_{m,j}\ \overline{Z}_j\exp\left[\lambda_m^k W_{m,j}\right]} \tag{18}$$

where k is the iteration counter.

From Eq. (17),

$$Z_j^{k+1}=\overline{Z}_j\exp\left[\lambda_m^{k+1}W_{m,j}\right] \tag{19}$$

$$Z_j^{k}=\overline{Z}_j\exp\left[\lambda_m^{k}W_{m,j}\right] \tag{20}$$

Plugging Eq.(18) into Eq.(19) and then using Eq.(20), we write the action by line algorithm known as MART (Multiplicative Algebraic Reconstruction Technique) (Censor and Lent, 1981, Reis and Roberty, 1992):

for k=0 until maximum iteration number
for m=1, 2, …,M (one line at a time)
for j = 1, 2, …, E

$$Z_j^{k+1}=Z_j^k\exp\left[-\frac{\sum_{i=1}^{E}W_{m,i}Z_i^k-h_m}{\sum_{i=1}^{E}W_{m,i}^2\ Z_i^k}W_{m,j}\right] \tag{21}$$

where k is the iteration counter.

With the determination of the vector of unknown coefficients, $\underline{Z}$, we obtain an estimate for the unknowns given by Eqs. (8) and (10). As the coefficients $W_{m,j}$ in Eq.(13) depend on the solution of the source problem, that depends on the estimated values of the extinction and scattering coefficients that we want to determine, we have an iterative procedure.

The Solution Algorithm. The iterative procedure is summarized as follows:

1. choose an initial guess $\underline{Z}^o$;
2. solve the auxiliary problem, Eqs. (5a-b), to obtain $\phi^{a,k_a}(x,\mu)$;
3. solve the source problem, Eqs. (1a-b), to obtain $\phi^{a,k_a}(x,\mu)$;
4. assemble the ITE using Eqs. (12) and (13) as well as Table1;
5. solve system (13) using, for example, the action by line algorithm given by Eq. (21);
6. calculate $\sigma_t(x)$ and $\sigma_s(x,\mu',\mu)$ using Eqs. (8);
7. terminate if a stopping criteria established a priori is satisfied, e.g. $\|\underline{Z}^{k+1}-\underline{Z}^k\| < \varepsilon$. Otherwise, go back to step 3.

RESULTS

To demonstrate the feasibility of the solution of the inverse problem with the source-detection approach described in the previous sections, we will present some preliminary results obtained for a homogeneous, gray (with no spectral dependence), isotropic scattering medium. Furthermore, the intensity of the internally distributed source is considered negligible in comparison to the intensity of the radiation coming from the external source.

In Fig.2 are presented the results from several simulations considering hypothetical experimental data without measurements errors. The circles represent the exact values of the unknown properties, $(\sigma_s; \sigma_t)$ = (0.1; 0.15); (0.6; 0.8); (0.7; 0.8); (0.2; 0.9); (0.8; 1.0) and (1.0; 1.0) and the crosses represent the initial guesses employed. These were made equal to the reference values, $(\sigma_s^{k=0}; \sigma_t^{k=0}) = (\sigma_s^R; \sigma_t^R)$ = (0.1; 0.15); (0.4; 0.6); (0.6; 0.8); (0.7; 0.8); (0.8; 1.0) and (1.0; 1.0). The units are cm^{-1} for both σ_s and σ_t. Convergence to the exact values in all simulations shown in Fig.2 were obtained within no more than fifteen iterations. All computational implementation has been done with MATLAB, and each simulation has taken approximately forty minutes of CPU time on a IBM compatible personal computer with 200 MHz Pentium processor.

As $\sigma_t \geq \sigma_s$, all meaningful test cases have to be taken at or above the dashed line in Fig.2.

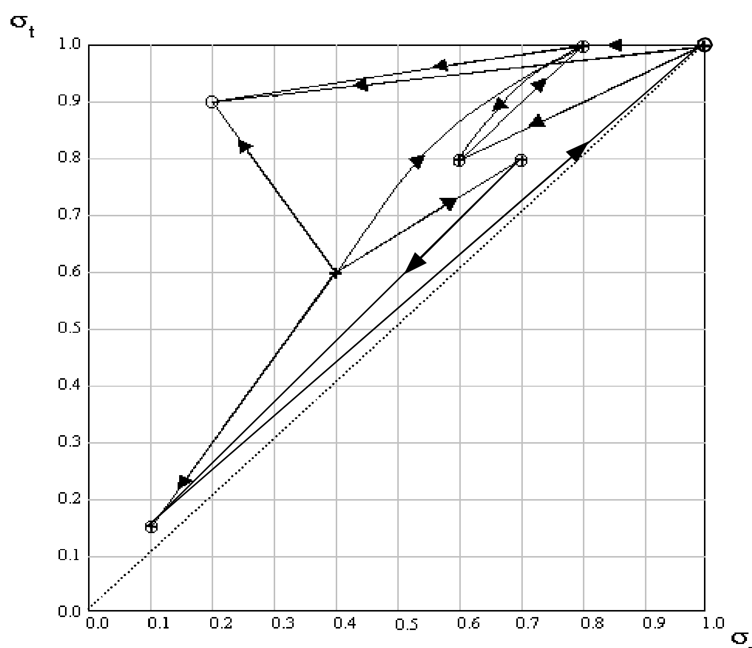

Figure 2 – Simulation results for test cases without measurement error.
o exact values + initial values = reference values

Numerical experiments have shown that convergence to the exact values is achieved for any value of the initial guesses, except when they are relatively far and greater than the exact values. As in real applications one does not know whether the reference values are far from the exact values or not, a refinement was introduced in the algorithm presented in the previous section. As soon as convergence is achieved, say at iteration k^*, the reference properties are replaced by the estimated properties $(\sigma_s^R; \sigma_t^R) = (\sigma_s^{k=k^*}; \sigma_t^{k=k^*})$, and the iterative procedure is restarted from the beginning. If the estimated values at the previous iteration cycle ($\sigma_s^{k=k*}$; $\sigma_t^{k=k*}$) are indeed a solution of the inverse problem, the new cycle of iterations stops at the very first iteration, otherwise it will go on until convergence is again achieved. The replacement of the reference values is done once more and a new cycle of iteration is initiated. This procedure is repeated as many times as necessary until no more variation is observed on the estimated properties.

Just to give an example of this situation, in Table 2 are presented the estimates for the scattering and extinction coefficients at each iteration for a case with (σ_s; σ_t) = (0.1; 0.15) and (σ_s^R; σ_t^R) = (0.6; 0.8). After repeating the iterative procedure just one time convergence to the exact values has been achieved.

As often is the case, less experimental data is available than the number of unknowns to be estimated, therefore uniqueness of the solution can not be assured. In such situation one must try different initial guesses if more than one solution is to be found. We have to keep in mind, though, that our main objective is to find deviations from the reference values (flaws in materials or anomalies in biological tissue), and in that sense any solution that deviates from the reference may be good enough.

In Fig. 3 are presented the results for a test case with the exact values (σ_s; σ_t) = (0.5; 0.9) with the initial guesses and reference values ($\sigma_s^{k=0}$; $\sigma_t^{k=0}$) = (σ_s^R; σ_t^R) = (0.2; 0.9) considering simulated experimental data with 1%, 5% and 10% error with respect to the maximum measured radiation intensity. For each run shown in Fig.3, a different set of simulated experimental data was used. Each set of experimental data was obtained by adding random errors to the exact calculated values of the exit radiation.

Table 2 – Estimated values for σ_s and σ_t at each iteration.

	iteration #	σ_s (cm $^{-1}$)	σ_t (cm $^{-1}$)
$\sigma_s^R = 0.6$; $\sigma_t^R = 0.8$	1	0.0843	0.1878
	2	0.4370	0.5580
	3	0.2810	0.3974
	4	0.3497	0.4681
	5	0.3198	0.4373
	6	0.3329	0.4508
	7	0.3272	0.4449
	8	0.3297	0.4475
	9	0.3291	0.4468
$\sigma_s^R = 0.3291$; $\sigma_t^R = 0.4468$	1	0.0176	0.0626
	2	0.1282	0.1786
	3	0.0911	0.1412
	4	0.1028	0.1528
	5	0.0991	0.1491
	6	0.1003	0.1503
	7	0.0999	0.1499
	8	0.1000	0.1500
$\sigma_s^R = 0.1000$; $\sigma_t^R = 0.1500$	1	0.1000	0.1500

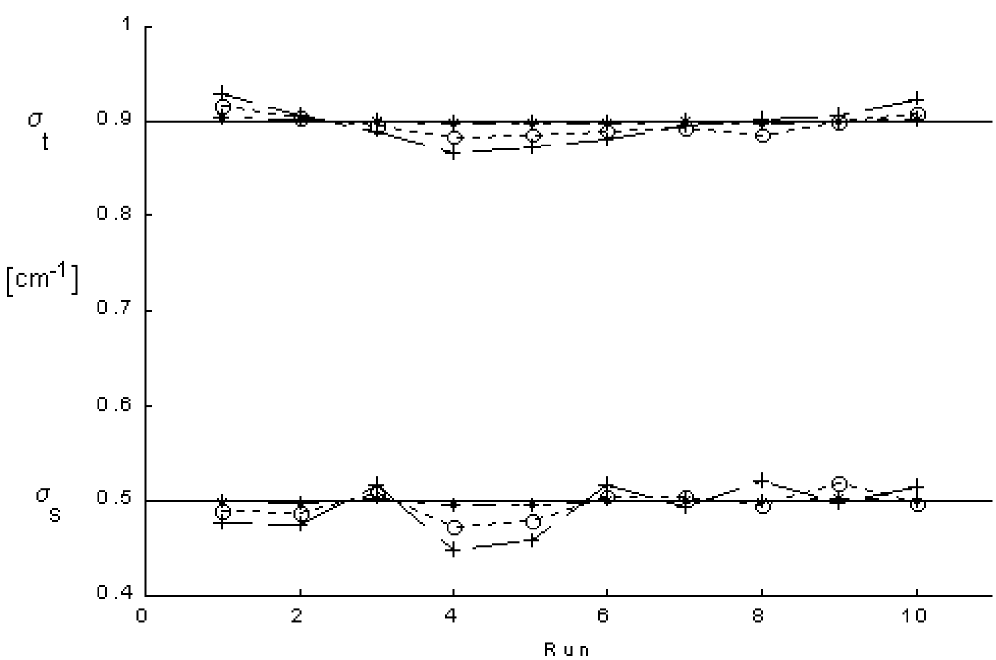

Figure 3 – Simulation results for test cases with measurement errors.
___ exact values * – – – – * 1% error
o----o 5% error +___+ 10% error

In all simulations presented here we have considered 10 possible source and detector locations, being 5 on each side of the slab. For the solution of the direct problem we have used the discrete ordinates method.

CONCLUSIONS AND FUTURE WORK

The results obtained so far are very encouraging. The method seems to perform well regarding both accuracy and computational performance (reasonable memory and CPU time requirements).

At the moment we are working towards the simulations dealing with internally distributed sources, heterogeneous and isotropically as well as anisotropically scattering media. In such cases the computation of the coefficients of the linear system given by Eq.(13) becomes more involved, and the number of unknowns to be estimated scales up quickly.

We are also starting the modeling of the problem for two-dimensional regions. Although the inverse problem in higher dimensions (2-D and 3-D) becomes more involved computationally, we expect the method presented here to perform even better, because more experimental data may be taken into account.

Mathematical and numerical analysis of several aspects such as existence, uniqueness and stability must be performed in order to get the full potential of the method.

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