Inverse Problems in Engineering: Theory and Practice
3rd Int. Conference on Inverse Problems in Engineering
June 13-18, 1999, Port Ludlow, WA, USA

MA03

EFFECTIVE APPROACHES USING COMBINATORICS TO SOLVE INVERSE PROBLEMS IN 2-D SYSTEMS

Edward Rydygier

The Andrzej Soltan Institute for Nuclear Studies,
PL-05-400 Otwock-Swierk, Poland
Phone: (+4822) 7180555, Fax: (+4822) 7793481
E-mail: rydygier@nov.iem.pw.edu.pl

Zdzislaw Trzaska

Warsaw University of Technology
Institute of the Theory of Electrical Engineering and Electrical Measurements
ul. Koszykowa 75, PL-00-662 Warsaw, Poland
Phone: (+4822) 6607543, Fax: (+4822) 6284568),
E-mail: trz@nov.iem.pw.edu.pl

KEYWORDS:

2-D distributed systems, inverse problems, identification of field sources, Fibonacci sequence

ABSTRACT

In this paper, new, effective approaches to solve third and second order difference equations are introduced. These approaches consist in the use of new computational tools constructed by the authors of this paper during previous investigations in a field of combinatorics and the Fibonacci hyperbolic trigonometry. Particularly, to work out suitable combinatorial algorithms the monic power 1-D and 2-D polynomials generated by modified numerical triangles and the hyperbolic Fibonacci functions defined by successive elements of the Fibonacci sequence are used. The construction of numerical algorithms is described and commented. The new computational approaches can be effectively applied for studies of various discrete-continuous systems that are occurred in practice. Also the idea of combinatorial approach is used to solve different inverse problems particularly identification problems. Especially, the identification of field sources in 2-D systems is studied very exactly. The elaborated algorithms were tested with special benchmark functions and also the experimental verification using measurement data was done. On the basis of presented studies and results of computer simulations, it can be found that the combinatorial method of solving inverse problems is effective and easy to use. The advantages of this method are recurrence equations defined monic polynomials, a high accuracy of calculations, and ease to an implementation in the MATLAB package

NOMENCLATURE

C	: constant
CFh	: cosine hyperbolic Fibonacci function
$cFh_Q(k)$	: generalized cFh function
$f_n, f(p)$	: element of the Fibonacci sequence
$f_{m,n}, f(m,n)$	: discrete value of sources' function f
f	: field sources' function
$\mathbf{f}$	: matrix of discrete values of function f
f_p	: sources' function for inverse problem
$\mathbf{f}_p$	: matrix of discrete values of function f_p
$f_1(.,.)$	: benchmark sources' function
$\mathbf{F}$	: matrix of Fourier series coefficients for f
$\phi = 1 + \rho$	: constant
MNT1,MNT2	: modified numerical triangles
$\psi(.,.)$	: auxiliary function in body torsion problems
$P_n(x)$	: polynomial generated by MNT2 triangle
ρ	: golden ratio ($\frac{1+\sqrt{5}}{2}$ = 1.618033 …)
q_k	: scalar parameter
q	: constant sources' function

Q	: parameter as complex matrix
SFh	: sine hyperbolic Fibonacci function
$sFh_Q(k)$	: generalized sFh function
$\mathbf{S}$	: matrix of values of sine functions
U	: potential function
$T_n(x)$	: polynomial generated by MNT1 triangle
$T(.,.)$	: temperature distribution function
$\mathbf{T}$	: matrix of discrete values of function u
$u_{m,n}$, $u(m,n)$	: discrete values of function u
$\mathbf{U}$	: matrix of Fourier series coefficients for u

INTRODUCTION

Studies on problems described by the 2-D and 3-D models have been intensively developed at many world scientific centers in order to explain different phenomena encountered not only in mathematics but also in electrical engineering, mechanics, economics, biology, medicine, and even in social sciences (Kaczorek, 1985, Groetsch, 1993, Anger, 1990, Kurpisz, 1995, Tikhonov, 1995, Engl, 1996, Neittaanmaki, 1996).

Recently, it can be observed that the inverse problems are of increasing interest both in scientific centers and industry. The studies on inverse problems are carried out in two directions. One is a development of the theory and numerical methods and the second is a improvement of measurement technology. Inverse problems exist in many branches of the natural sciences and engineering such as mathematics (theory and methods), statistics, geophysics, seismology, astrometry, astrophysics, optics, and image restoration, plasma diagnostics, electrodynamics, scattering in elementary particles physics, medicine (medical imaging, impedance tomography, electrocardiogram interpretation). The problem of modelling the physical reality with suitable differential equations systems is relatively uncomplicated in the finite dimensional setting but becomes very difficult for various partial differential equations such as wave, heat, electromagnetic ones. When it is impossible, or difficult, to obtain an exact solution of the partial differential equations governing a continuous system, the system is reduced to discrete form (John, 1978, Anger, 1990, Tikhonov, 1995).

In this paper an effective method for computational solutions of direct and inverse problems described by the 2-D models is presented in relation to distributed parameter systems using discrete spatial coordinates. This new approach named combinatorial method consists in the use of new computational tools developed by authors of this paper during previous investigations in a field of combinatorics and the Fibonacci trigonometry (Rydygier, 1997, Trzaska, 1993a, 1993b, Trzaska, 1996, Trzaska, 1997).

COMPUTATIONAL TOOLS

Presently, a growing interest is observed in development of methods using a combinatorial analysis based on conceptions and objects from modern combinatorics. The combinatorial analysis is applied in the theory of crystalograghy, cryptology or selected optimization problems of decision making, scheduling and graph theory (Akgul, 1992). In this paper, it will be shown that in a field of engineering problems, various structures of the so-called numerical triangles and hyperbolic Fibonacci functions can be used for modelling and numerical analysis of distributed parameter systems (Bergum, 1994, Ross, 1996).

MODIFIED NUMERICAL TRIANGLES

Monic non-zero polynomials which generate the first modified numerical triangle, *MNT1*, are defined by the following recurrence (Trzaska, 1996)

$$T_{n+2}(x) = (2 + x)T_{n+1}(x) - T_n(x), \quad n = 0, 1, 2, \ldots \qquad (1)$$

with $T_0(x) = 1$ and $T_1(x) = 1 + x$ as initial elements. From the above recurrence, the following polynomials can be calculated

$T_0(x) = 1$
$T_1(x) = 1 + x$
$T_2(x) = 1 + 3x + x^2$
$T_3(x) = 1 + 6x + 5x^2 + x^3$
$T_4(x) = 1 + 10x + 15x^2 + 7x^3 + x^4$
$T_5(x) = 1 + 15x + 35x^2 + 28x^3 + 9x^4 + x^5$
... ...

Thus, the polynomial $T_n(x)$ can be written in the form

$$T_n(x) = \sum_{k=0}^{n} a_{n,k}\, x^k, \qquad n = 0, 1, 2, \ldots \qquad (2)$$

where the coefficients $a_{n,k}$, $n = 0, 1, 2, \ldots$, $0 \le k \le n$, fulfill the relation

$$a_{n,k} = 2a_{n-1,k} + a_{n-1,k-1} - a_{n-2,k}, \qquad (3)$$

with $a_{0,0} = 1$ and $a_{1,0} = 1$ as initial values.
Based on (3) the *MNT1* can be constructed. It is presented in *Table*1.

n \ k	0	1	2	3	4	5	6	.Sum of coefficients
0	1							1
1	1	1						2
2	1	3	1					5
3	1	6	5	1				13
4	1	10	15	7	1			34
5	1	15	35	28	9	1		89
6	1	21	70	84	45	11	1	233
...	...	...	...	...	...	...	...	...

Table 1. First modified numerical triangle, *MNT1*

To establish the second numerical triangle *MNT1*, the monic non-zero power polynomials are defined by the recurrence

$$P_{n+2}(x) = (2 + x)P_{n+1}(x) - P_n(x), \quad n = 0, 1, 2, \ldots \qquad (4)$$

with $P_0(x) = 0$ and $P(x) = 1$ as initial elements.
From (4) the following polynomials can be obtained

$$
\begin{aligned}
P_0(x) &= 0 \\
P_1(x) &= 1 \\
P_2(x) &= 2 + x \\
P_3(x) &= 3 + 4x + x^2 \\
P_4(x) &= 4 + 10x + 6x^2 + x^3 \\
P_5(x) &= 5 + 20x + 21x^2 + 8x^3 + x^4 \\
&\ldots
\end{aligned}
$$

From the above expressions the polynomial $P_n(x)$ can be written in the form

$$P_n(x) = \sum_{r=0}^{n} b_{n,r}\, x^r \quad , \qquad n = 0, 1, 2, \ldots \tag{5}$$

where the coefficients $b_{n,r}$, $n = 0, 1, 2, \ldots$, $0 \le r \le n$ are defined by the recurrence

$$b_{n,r} = 2b_{n-1,r} + b_{n-1,r-1} - b_{n-2,r} \tag{6}$$

with $b_{0,0} = 0$ and $b_{1,0} = 1$ as initial values.
Then, based on (6) the *MNT2* can be constructed. It is shown in *Table* 2.

r \ n	0	1	2	3	4	5	Sum of coefficients
0	0						0
1	1						1
2	2	1					3
3	3	4	1				8
4	4	10	6	1			21
5	5	20	21	8	1		55
6	6	35	56	36	10	1	144
...	...	...	...	...	...	…	...

Table 2: Second modified numerical triangle, *MNT2*

Formally, both the *MNT1* and the *MNT2* are apparently similar to the classical Pascal triangle (Ross, 1996), but their elements cannot be evaluated directly by applying the rule corresponding to the classical Pascal triangle (Trzaska, 1995). They must be computed in accordance with recurrence (3) and (6), respectively. The sum of all elements values in a row of *MNT1* or *MNT2* equals to f_{2n} , $n =$ 0, 1, 2, ... , or f_{2n-1} , $n =$ 0, 1, 2, ... , respectively, i. e. they are equal to successive elements of the Fibonacci sequence with even or odd indices, respectively (Bergum, 1994)

$$f_{n+2} = f_{n+1} + f_n , \qquad n = 0, 1, 2, \ldots \tag{7}$$

with $f_0 = 1$ and $f_1 = 1$ as initial values.

Some of the most useful properties of monic power polynomials are following

$$xP_{n+1}(x) = T_n(x) - T_{n-1}(x), \qquad n = 0, 1, 2, \ldots \tag{8}$$

$$P_n(x)T_{n-1}(x) - P_{n-1}T_n(x) = 1, \qquad n = 0, 1, 2, \ldots \tag{9}$$

Properties of monic polynomials were described in detail in Trzaska (1996), Trzaska (1997).

HYPERBOLIC FIBONACCI FUNCTIONS

Hyperbolic Fibonacci functions $sFh(x)$ and $cFh(x)$ are defined as follows

$$
\begin{aligned}
sFh(x) &= \frac{\phi^{2x} - \phi^{-2x}}{\sqrt{5}} , \\
cFh(x) &= \frac{\phi^{(2x+1)} + \phi^{-(2x+1)}}{\sqrt{5}}
\end{aligned} \tag{10}
$$

where $\phi = 1 + \rho \cong 2.618033 \ldots$, and ρ denotes the golden ratio (Trzaska, 1993a).

It is easy to demonstrate that when a discrete variable k I is used then the functions $sFh(k)$ and $cFh(k)$ in terms of corresponding elements of the Fibonacci sequence

$$f(p + 1) = f(p) + f(p - 1), \quad p = \ldots -3, -2, -1, 0, 1, 2, 3, \ldots \tag{11}$$

with $f(0) = 0$ and $f(1) = 1$ can be written in the formulas

$$sFh(k) = f(2k), \qquad cFh(k) = f(2k + 1) \tag{12}$$

The generalized Fibonacci hyperbolic functions $cFh_Q(k)$ and $sFh_Q(k)$ can be regarded as generating functions for polynomials $T_k(Q)$ and $P_k(Q)$, $k = 0, 1, 2, \ldots$, respectively

$$cFh_Q(k) = Q^k\, T_k(Q), \qquad sFh_Q(k) = Q^k\, P_k(Q) \tag{13}$$

where Q means complex matrix or scalar parameter.

For $Q = 1$ there are usual Fibonacci hyperbolic functions

$$cFh_Q(k)_{|Q=1} = cFh(k), \qquad sFh_Q(k)_{|Q=1} = sFh(k) \tag{14}$$

Moreover, it is evident that Fibonacci hyperbolic functions and modified numerical triangles above presented can be very useful for practical problem studies.

CONSTRUCTION OF ALGORITHMS

On the basis of the stationary 2-D space-continuous system described by the Poisson equations with specified boundary conditions, the computational algorithms are elaborated for solving direct and inverse problems with special regard to identification problems. This system is described by the second order partial differential equation

$$\frac{\partial^2 u(x,y)}{\partial x^2}+\frac{\partial^2 u(x,y)}{\partial y^2}=f(x,y) \qquad (15)$$

where $u(x, y)$ is the potential function and $f(x, y)$ is the function of field sources distribution.

At first a direct problem will be solved. This problem consists in finding a solution of equation (15) which is the potential function u for known function f and for given boundary conditions. To solve this problem a discretization using the finite difference method (Dahlquist, 1974) and an expanding the function f with a use of the Fourier series (Potter, 1973), for parameter $n = 1, 2, \ldots, N - 1$ are done. The boundary conditions for function f are in the form

$$f_{m,0} = f_{m,N} = 0. \qquad (16)$$

The discrete values of a field sources function can be calculated from following formula

$$f_{m,n} = \sqrt{2}\sum_{k=1}^{M-1} F_m(k)\sin\frac{k\pi n}{M}, \quad m = 1, 2, \ldots, M-1. \qquad (17)$$

In the same way a solution for the potential function u can be represented as follows

$$u_{m,n}=\sqrt{2}\sum_{k=1}^{M-1} U_m(k)\sin\frac{k\pi n}{M}. \qquad (18)$$

After modification with use some trigonometric identities, the second order difference equation can be established

$$\left[\frac{1}{h^2}(U_{m+1}(k)-2U_m(k)+U_{m-1}(k)-(4\sin^2\frac{k\pi}{2N})U_m(k))\right]=F_m(k) \qquad (19)$$

where $m = 1, 2, \ldots, M - 1$, values M and N define the limits of the space.

To complete equation (19), the boundary conditions are formulated as follows

$$U_0(k) = 0 \text{ and } U_M(k) = C_k \qquad (20)$$

where constants C_k (values of $U_M(k)$) are calculated from condition $u_{M,n} = 0$, $n = 1, \ldots, N$.

For the new parameter $q_k = 4\sin^2\frac{k\pi}{2N}$ and on the basis of equation (4) generating $P_n(q)$ polynomials, the solution of equation (19) is obtained in the form

$$U_m(k)=P_m(q_k)U_1(k)+\sum_{l=1}^{m-1} P_{m-l}(q_k)h^2F_l(k), \quad m = 2, 3, \ldots, M-1. \qquad (21)$$

From (21) the values $U_m(k)$ can be found in all nodes of discretization. The values $U_1(k)$ in the equation (21) are calculated from the boundary conditions

$$u_{0,n} = 0 \text{ and } u_{M,n} = 0, \quad n = 0, 1, \ldots, N. \qquad (22)$$

When $N = M$ and $n = 1$, the second equation shown above is appeared as follows

$$0 = u_{M,1} = \sqrt{2}\sum_{k=1}^{M-1} U_M(k)\sin\frac{k\pi}{M}. \qquad (23)$$

Doing similarly for $n = 2, \ldots, M - 1$, the system of $M - 1$ equations can be obtained to calculate a set of coefficients $U_M(k)$, $k = 1, 2, \ldots, M - 1$.

Then after a substitution of these coefficients to the equation (21) for $m = M$, a set of coefficients $U_1(k)$, $k = 1, 2, \ldots, M - 1$ can be found. Next, on the basis of equation (18), (21) the solution of equation (15) can be calculated as a set of potential function values at nodes of discretization.

The above algorithm of calculation was implemented in the form of a computer program to analyse the system described by Poisson equation (15). The results of calculations are presented as graphs of potential function for different steps of discretization. Also the analytical solution if exists can be given on the input. It is used to calculate an error's distribution served for the comparison between the calculated and exact solution.

After some modifications of the above algorithm, another algorithm was constructed to solve inverse problem of the system described by equation (15). The task consists in the calculation of unknown field sources function f for known the potential function u and given boundary conditions described by equations (16), (22). The solution is calculated using elaborated algorithm in two stages. Within the first stage, the task consists in calculation of the matrix of Fourier series coefficients for discrete values of potential function $u_{m,n}$ in accordance with equation (18). For example, when m=1 the connection between coefficients $U_1(k)$ in demand and values $u_{1,n}$ can be presented in the matrix equation

$$\mathbf{U}_1 = \frac{1}{\sqrt{2}}\mathbf{S}^{-1}\mathbf{T}_1, \qquad (24)$$

where $\mathbf{U}_1$ is a vector which consists of coefficients $U_1(k)$ for $k = 1, 2, \ldots, N - 1$, the vector $\mathbf{T}_1$ consists of values of potential

function $u_{1,n}$ for n = 1, 2, ... , N - 1, and the matrix **S** is defined by suitable values of the *sine* function.
During the second stage of calculations, the matrix **F** of Fourier series coefficients is determined for the field sources function development. Values of elements in rows 1 to M - 2 can be calculated from following formula

$$F_l(k)=\frac{U_{l+1}(k)-P_{i+1}(q(k))U_1(k)-\sum_{i=1}^{l-1}P_{l+1-i}(q(k))h^2F_i(k)}{P_1(q(k))h^2} \quad (25)$$

Whereas the calculations M-1 row of matrix **F** are done on the basis of boundary conditions (22).

The last operation is a determination the matrix $\mathbf{f}_p$ which corresponds with the matrix **f** in the algorithm for a direct problem. For calculated elements of matrix **F,** the elements of matrix $\mathbf{f}_p$ can be constructed on the basis of following equation

$$f_p(m,n)=\sqrt{2}\sum_{k=1}^{M-1}F_m(k)\sin\frac{k\pi n}{M} \quad (26)$$

where m, n means succeeding row and column respectively.
The elaborated algorithms were established in the MATLAB language for PC computer. In order to test an accuracy of calculations, the computer simulations were carried out with special functions called the *benchmark functions* in the form

$$f_1(x, y) = 100\,(y^2 - y + x^2 - x),$$

$$f_2(x, y) = 300xy(x^2 + y^2 - 2),$$

$$f_3(x, y) = -\,2\cos x\sin y,$$

for different steps of discretization.

Some examples of benchmark functions are shown in Fig. 1 and Fig.2.

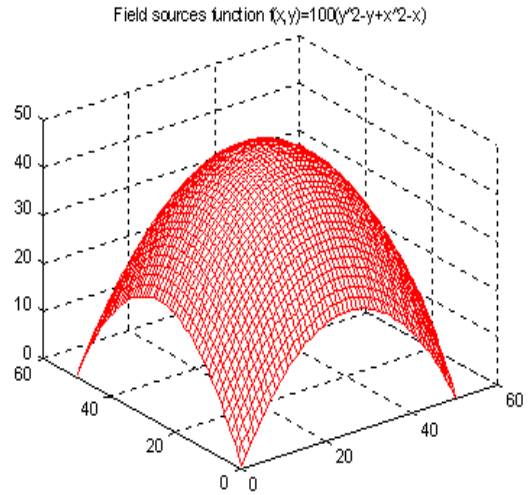

Fig. 1. Benchmark source function f_1

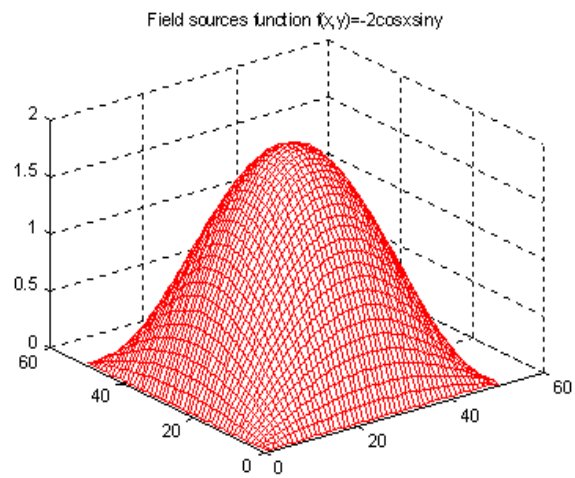

Fig. 2. Benchmark source function f_3

The corresponding potential functions illustrated exact solutions of the Poisson equation (15) are presented in Fig. 3 and Fig. 4.

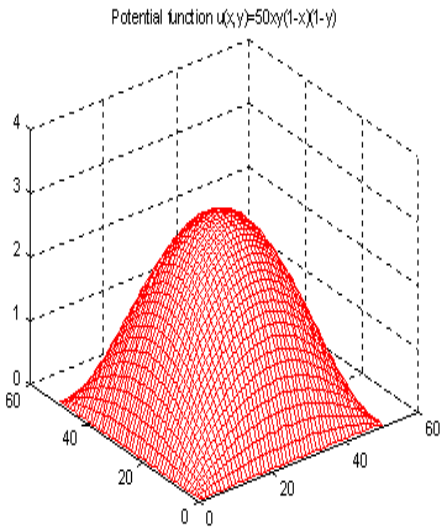

Fig. 3. Analytical solution of Poisson equation for f_1

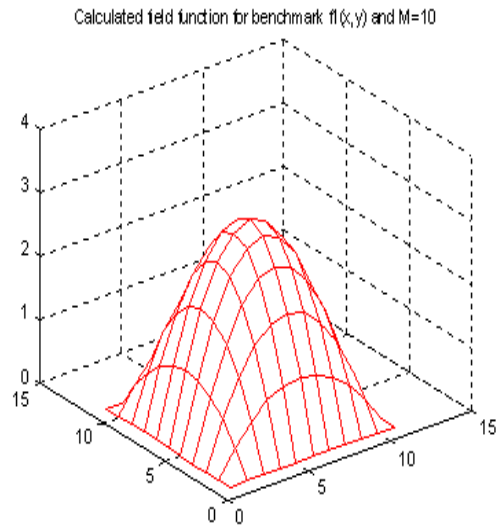

Fig. 5. Calculated potential function for M = 10

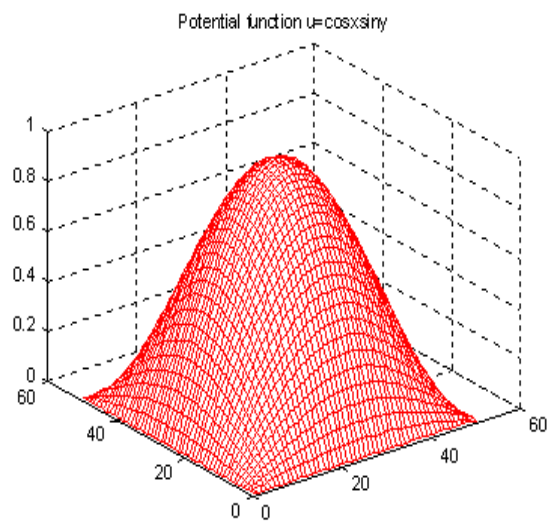

Fig. 4. Analytical solution of Poisson equation for f_3

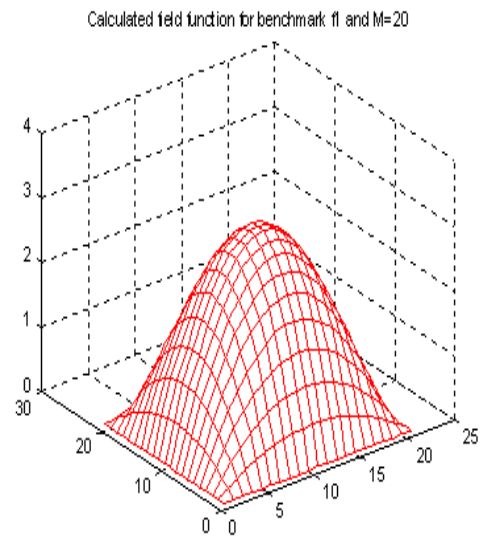

Fig. 6. Calculated potential function for M = 20

Calculated potential functions were obtained for different steps of discretization. For example, results of computer simulations for benchmark f_1 are shown in Fig 5 (M=10) and in Fig 6 (M=20) to illustrate an effect of increase knots number of a discretization grid.

The analytical solutions agree quite well with ones obtained by using the elaborated algorithms of numerical calculations. This proves the efficiency of the established method in the practical use.

The detailed calculations are done in Rydygier, 1998b.

IDENTIFICATION OF FIELD SOURCES

The elaborated algorithms for solving inverse problems were tested using the experimental data to verify numerical calculations. These experimental data were obtained from measurements of potential distribution on the thin conductive plate or thin conductive layer placed on a plate of perfect insulator (Trzaska and Rydygier, 1998).

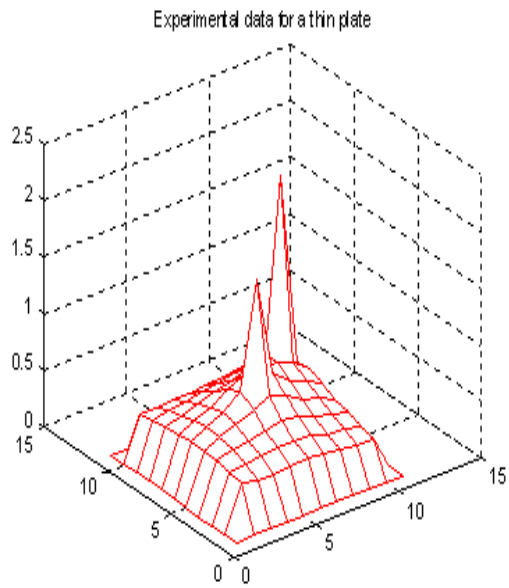

Fig. 7. Experimental data for two sources

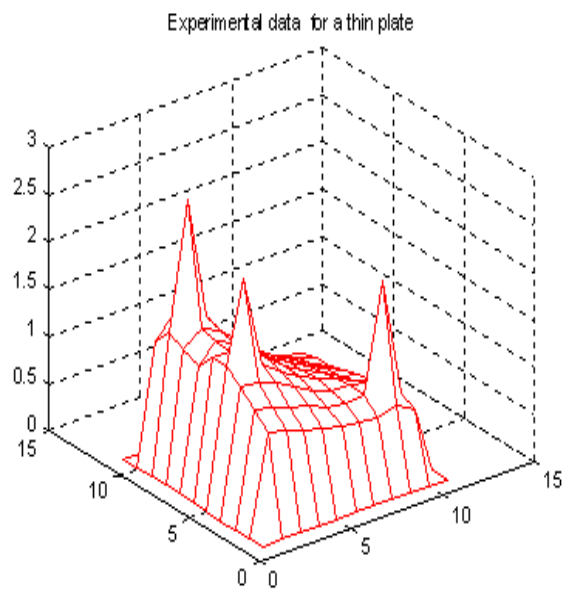

Fig. 8. Experimental data for three sources

In investigated systems, the point current constraints are sources of potential field. For example several sets of experimental data are shown in Fig. 7 (for two sources), in Fig. 8 (for three sources). The input data were treated like given values of potential function at nodes of discretization within the investigated domain.

The values of boundary conditions were placed inside the program. The calculated field sources' functions are shown in Fig. 9 and in Fig. 10.

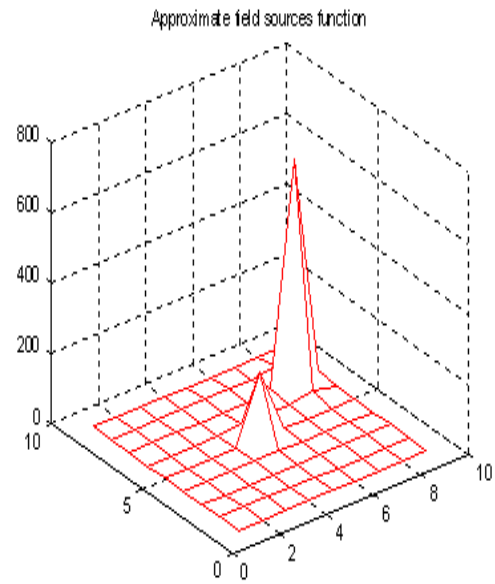

Fig. 9. Identification of two sources

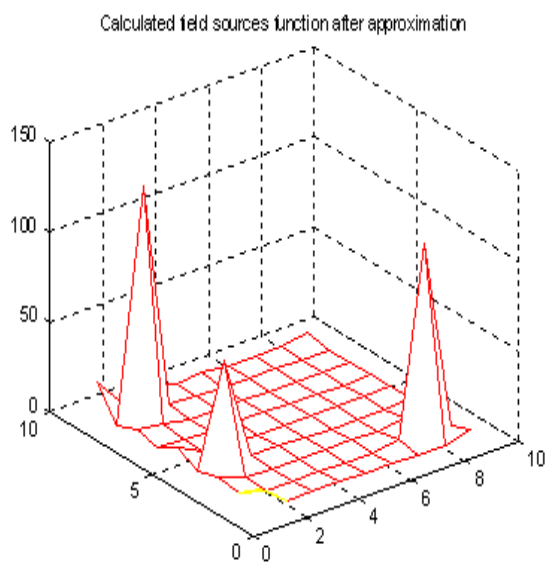

Fig. 10. Identification of three sources

A set of values determining discrete spatial distribution of field sources function is obtained as a result of calculations with the combinatorial method. A location of field sources was defined on the basis of points placed inside the domain which correspond to the maxima of field sources function. Next, the maximum values of calculated functions were used to estimate intensities of sources.

During experimental verification, some detailed problems have been solved. These problems are connected with a data treatment like a two-dimensional interpolation and a smoothing of scattered data as well as an approximation of function

circumscribed the field sources distribution. These approximation procedures were used in order to stabilize the results as a form of regularization method (Engl, 1996).

Together with the estimation limits of a step of discretization, the different approximation procedures were used special methods leading to the self-regularization (Kurpisz, 1995). The correct results were obtained for approximation procedure elaborated on the basis of an inverse distance method named also the Shepard's method (Allasia, 1992, Gordon, 1978). Exemplary results of self-regularization method are the graphs of approximated field sources function presented in Fig 9 and Fig. 10. These graphs correspond to the sets of experimental data presented above. After the experimental verification, it should be noted that for solving inverse problems the algorithm using the combinatorial method allows to determine both localization and intensities of field sources with good accuracy.

Detailed calculations are presented in Rydygier (1998b.)

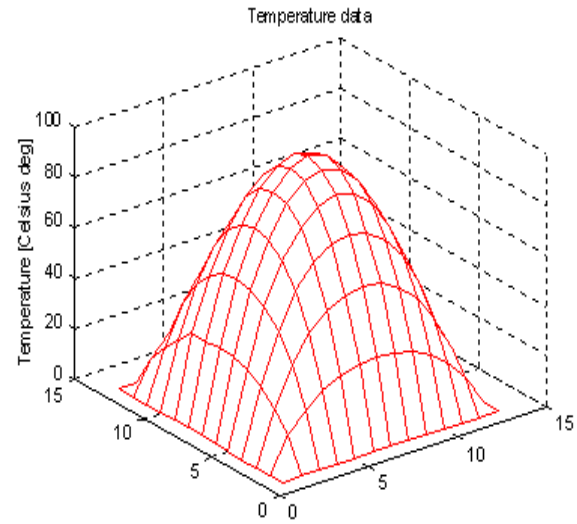

Fig. 11. Temperature data for a tungsten rod

PRACTICAL INVERSE PROBLEMS

After the experimental verification, the elaborated combinatorial method was tested for detailed problems. Especially, the heat transfer problem was examined for a resistance sintering of a tungsten rod. It corresponds to a tungsten rod manufacturing that is widely used in the practice. The investigated system is described by the Poisson equation

$$\frac{\partial^2 T(x,y)}{\partial x^2} + \frac{\partial^2 T(x,y)}{\partial y^2} = q \qquad (27)$$

where q = const, $T(x, y)$ means temperature distribution on a rectangular plane of cross-section of a rod.

The equation (27) is completed with zero-value boundary conditions. In the resistance sintering problem, heat is generated by an electrical current.

The exemplary temperature data is shown in Fig. 11 for a rod of 0.011 × 0.011 × 0.446 m sizes. This is a distribution of temperature refer to a plane of cross-section at z = 0.2355 m (the z axis is putted along the length of a rod). The inverse problem for the system described by equation (27) consists in a calculation a field sources function q. Calculations were done using square net of 15 × 15 nodes for discretization. As a result of calculations, the constant field sources function was obtained.

The calculated function q is shown in Fig. 12.

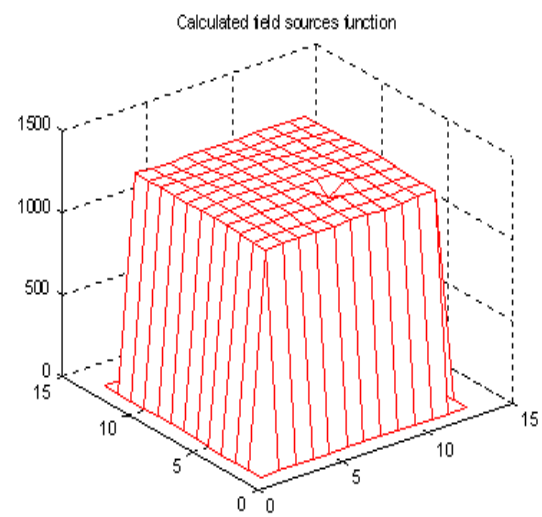

Fig. 12. Calculated sources function

The conclusion on the investigated above problem is that an estimation of field sources distribution can't be done only on the basis of experimental data of potential function. The reliable identification of field sources can be realized on the basis of field sources function obtained as a suitable inverse problem solution.

Another detailed problem is a use of combinatorial method for 2-D systems that have a complicated shape. The problem from elastostatics is considered for torsion of a metal I-bar. The 2-D system taken from the practice is described by the Poisson equation

$$\frac{\partial^2 \psi(x,y)}{\partial x^2} + \frac{\partial^2 \psi(x,y)}{\partial y^2} = q \qquad (28)$$

where $\psi(x, y)$ means an auxiliary function connected with a torsion angle on a cross-section plane of a bar, q is a constant function.

The equation (28) is completed with zero-value boundary conditions. The input data on a cross-section plane of the I-bar of 16'' × 6'' sizes are shown in Fig. 13. The calculated field sources function q is shown in Fig. 14. Calculations are done using a net of 19 × 19 nodes for discretization.

The application of a combinatorial method to 2-D systems with complicated shapes consists on a substitution zero-values for the nodes placed beyond the limits of a cross-section domain. Then calculations were made like for square domain of discretization.

It should be noted that the proposed approach of calculations for complicated shapes is simple and effective. In the event of disturbances, the additional calculations with a use of smoothing and approximation procedures must be done.

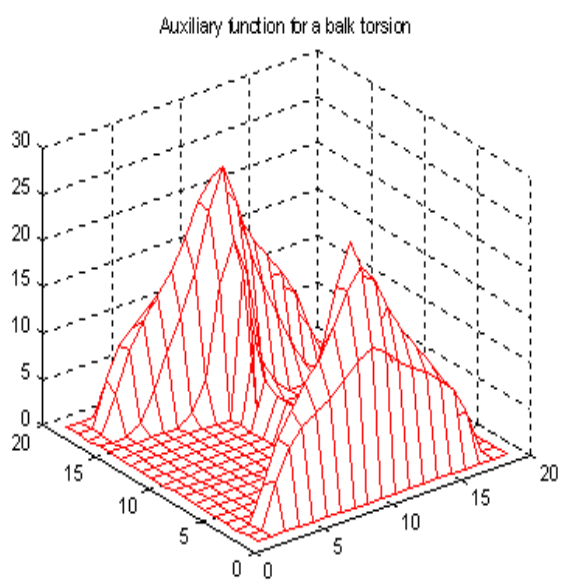

Fig. 13. Input data for a torsion of the I-bar

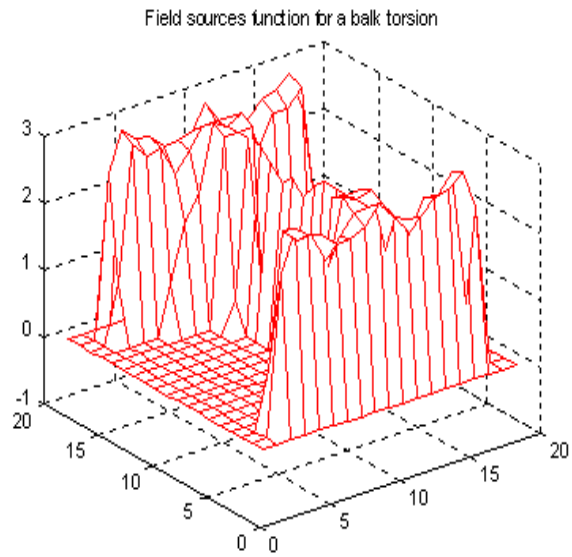

Fig. 14. Field sources function for a torsion of the I-bar

CONCLUSIONS

The new approach to solve inverse problems is named the combinatorial method. Numerical algorithms are constructed using monic power polynomials generated by modified numerical triangles. After a comparison of the combinatorial method with another numerical methods used to solve different inverse problems (Anger, 1990, Botkin, 1995, Flis, 1996, Huang, 1992, Isaacs, 1996, Malyshev, 1989, Rydygier, 1998a, 1998b), it can be found that this new method is effective and easy to use. The advantages of combinatorial method are simplicity of calculations on account of a use of recurrence equations defining monic polynomials, a high accuracy of calculations, and ease to implement the numerical algorithms in the MATLAB package.

The elaborated combinatorial method can be applied to determine inverse heat sources which are solutions of different practical heat transfer problems (Malyshev, 1989, Matrin, 1996) and to locate corrosion domains on iron and carbon steel surfaces (Inglese, 1997, Isaacs, 1996). Results of presented research can be utilized to improve usable properties of metal plates in production process in the industry. Also the results of this work can be used to build integrated computer systems for identification of thin layers properties in particular the heterogeneous spots in their structure.

REFERENCES

Akgul, M., Hamacher, H. W., Tufekci, S., 1992, *Combinatorial Optimization. New Frontiers in Theory and Practice*, NATO ASI Series 82, Berlin, Springer-Verlag.

Allasia, G., 1992, "Some Physical and Mathematical Properties of Inverse Distance Weighted Methods for Scattered Data Interpolation", *Calcolo*, Vol. 29, pp. 97-109.

Anger, G., 1990, *Inverse Problems in Differential Equations*, Berlin, Akademie-Verlag.

Bergum, G. E., Horadam, A. F., Philippou, A. N., 1994, *Applications of Fibonacci Numbers*, Vol. 1-6, Santa Clara, Fib. Assoc. Press.

Botkin, N. D., 1995, "Identification of Unknown Parameters for Heat Conductivity Equations", *Numer. Funct. Anal. and Optimiz.,*Vol. 16, pp. 583-599.

Dahlquist, G., Bjorck, A., 1974, *Numerical Methods*, New Jersey, Prentice Hall.

Engl, H. W., Hanke, M., Neubaer , A. ,1996, *Regularization of Inverse Problems*, Dordrecht, Kluwer.

Flis, J., Zakroczymski, T., 1996, "Impedance Study of Reinforcing Steel in Simulated Pore Solution with Tannin", *J. Electrom. Society*, Vol. 143, pp. 2458-2464.

Gordon, W. J., Wixon, J. A., 1974, "Shepard Method of 'Metric interpolation' to Bivariate and Multivariate Interpolation", *Math. Comp.*, Vol. 32, pp. 253-264.

Groetsch, Ch. W., 1993, *Inverse Problems in the Mathematical Sciences*, Braunschweig/Wiesbaden, Vieweg.

Huang, C. H., Ozisk, M. N., 1992, "Inverse Problem of Determining the Unknown Strength of an Internal Plane Heat Source", *J. The Franklin Institute*, Vol. 329, pp. 751-764.

Inglese, G., 1997, "An Inverse Problem in Corrosion Detection", *Inverse Problems*, Vol. 13, pp. 977-994.

Isaacs, H.S., Shum-Ming, Jovancicevic, V., 1996, "Location of Corrosion and Carbon Steel Surfaces with Crevices", *J. Electrochem. Society*, Vol. 143, pp. 178-180.

John, F., 1978, *Partial Differential Equations*, New York, Springer-Verlag.

Kaczorek, T., 1985, *Two-dimensional Linear Systems*, Berlin, Springer-Verlag.

Kurpisz, K., Nowak, A. J., 1995, *Inverse Thermal Problems*, Southampton, Com. Mech. Publ.

Malyshev, I., 1989, "An Inverse Source Problem for Heat Equations", *J.Mathematical Analysis and Applications*, Vol. 142, pp. 206-218.

Martin, T. J., Duklikravich, G. S., 1996, "Inverse Determination of Boundary Conditions and Sources in Steady Heat Generation", *J. Heat Transfer*, Vol. 118, pp. 546-554.

Neittaanmaki, P., Rudnicki, M., Sawini, A., 1996, *Inverse Problems and Optimal Design in Electricity and Magnetism*, Oxford, Clarendon Press.

Potter, D., 1973, *Computational Physics*, N.Y., J. Wieley.

Ross, K .A., Wright, CH. R. B., 1992, *Discrete Mathematics*, New Jersey, Prentice Hall.

Rydygier, E., 1998a, "Inverse Problems Solved with Artificial Intelligence Methods", *Proc. 6th European Congress on Intelligent Techniques and Soft Computing*, Aachen, pp. 1284-1288.

Rydygier, E, 1998b, *Field Sources Identification Using a Combinatorial Method*, Doctor's Thesis, Warsaw University of Technology, Warsaw, Poland.

Trzaska, Z., 1993a, "Modified Numerical Triangles and the Fibonacci Sequence", *The Fibonacci Quarterly*, Vol. 32, pp. 124-129.

Trzaska, Z., 1993b, "Numerical Triangles, Fibonacci Sequence and Ladder Networks: Some Further Results", *Appl. Math. Let.*, Vol. 6, pp. 55-61.

Trzaska, Z., 1997, "New Approach to Continuous-Discrete Systems", *Bulletin of the Polish Academy of Sciences*, Vol. 45, 3, pp. 433-443.

Trzaska, Z., Rydygier, E., 1998, "Identification of Field Sources in 2-D Systems Using a Combinatorial Method", *Proc. 5th International Symposium on Methods and Models in Automation and Robotics*, Miedzyzdroje, Poland, pp. 129-134.

Tikhonov, A. N., Goncharsky A. V., Stepanov, V. V., Yagola, A. G., 1995, *Numerical Methods for the Solution of Ill-posed Problem*, Dordrecht, Kluwer.