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PROBLEMS AND SOLUTIONS IN IDENTIFICATION OF THE PARAMETERS OF MECHANICAL JOINTS

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ABSTRACT

In this work, the problems in the identification of the parameters of mechanical joint directly from the measured frequency response functions (FRFs) of the structure were discussed. The problems include the problem of measurement noise, the problem of using the least squares method and the problem due to the characteristics of the structure itself. The causes of the problems and the associated solutions were discussed by theoretical and experimental examples. The results show that the measurement noise in the FRFs is the basic problem in identification; however, the severity of the noise problems is magnified by the other problems.

INTRODUCTION

The most troublesome problem encountered in the dynamic simulation of a real mechanical system is the difficulty of knowing the accurate system parameters. A real mechanical system usually consists of many components which are connected together through different joints. The dynamic properties of the joints generally are very difficult to know by theoretical methods. Therefore, the experimental identification method becomes an important approach to find the joint properties.

In the past, great efforts have been made in the field of parameter identification. Some of the identification methods were developed to identify the dynamic parameter of the whole structure (Fritzen, 1986; Mottershead and Stanway, 1986; Wang, 1988), some other methods were especially developed for the identification of joint parameters (Yoshimura, 1977, 1979; Yuan and Wu, 1985; Tsai and Chou, 1988). Yoshimura (1977, 1979) proposed an iterative method to identify the joint properties, but the method required considerable computer time due to the iterative procedure. Yuan and Wu (1985) used the finite element method combined with the dynamic data system to identify the joint properties of machine tool. However, the method required the mass, damping and stiffness matrices to form the mathematical model of the whole structure. The method proposed by Tsai and Chou (1988) used the measured frequency response functions (FRFs) of the substructures and whole structure to extract the joint properties. The method is very simple; however, it is too sensitive to measurement error or noise. There are many advantages to use the measured FRFs to extract the joint parameters. However, if the measured FRFs are used to extract the joint parameters, the unavoidable measurement noise in the FRFs could be the biggest trouble, (Juang and Pappa, 1986; Ren and Beard, 1993; Wang and Liou, 1990, 1991, 1993).

Although in the past some methods have been proposed to minimize the noise effect, it is found that, with the same noise level, the accuracy of the identified result is very structure dependent. In other words, in order to improve the accuracy of identification, one could not consider the noise effect only. In this work, the problems and solutions in the identification of the joint parameters were discussed. The accuracy and feasibility of the proposed solutions were verified theoretically and experimentally.

THEORETICAL FORMULATION

A mechanical structure usually consists of many components which are connected together by different joints. Therefore, the whole structure can be divided into two substructures from the joint to be identified. It is assumed that the dynamic behavior of the joint can be modeled as linear spring and damper elements, as shown in Fig.1. The objective of the parameter identification is to extract the joint parameters experimentally from the frequency response functions (FRFs) of the whole structure and the substructures.

With the definition of FRFs, the relation between the displacement vectors and force vectors of substructures 1 and 2, (see Fig. 1), cab be expressed as,

$$\begin{Bmatrix} \{X_e\} \\ \{X_a\} \end{Bmatrix} = \begin{bmatrix} [H_{ee}]_1, [H_{ea}]_1 \\ [H_{ae}]_1, [H_{aa}]_1 \end{bmatrix} \begin{Bmatrix} \{F_e\}_1 \\ \{F_a\}_1 + \{F_j\}_1 \end{Bmatrix} \quad (1)$$

$$\begin{Bmatrix} \{X_b\} \\ \{X_c\} \end{Bmatrix} = \begin{bmatrix} [H_{bb}]_2, [H_{bc}]_2 \\ [H_{cb}]_2, [H_{cc}]_2 \end{bmatrix} \begin{Bmatrix} \{F_b\}_2 + \{F_j\}_2 \\ \{F_c\}_2 \end{Bmatrix} \quad (2)$$

where $\{X_a\}$ and $\{X_b\}$ represent the displacement vectors on the joint interfaces of substructures 1 and 2 respectively, as shown in Fig. 1; $\{X_e\}$ and $\{X_c\}$ represent the displacement vector on all other regions except the joint interfaces of substructures 1 and 2. The vectors $\{F_a\}_1$ and $\{F_e\}_1$ represent the external force vectors acting on the substructure 1, while the vectors $\{F_b\}_2$ and $\{F_c\}_2$ represent the external force vectors acting on the substructure 2. The internal force vectors of the joint are represented by $\{F_j\}_1$ and $\{F_j\}_2$, and they are equal in magnitude, but opposite in direction, i.e.,

$$\{F_j\}_1 = -\{F_j\}_2 \quad (3)$$

The displacement vectors of the joint interfaces are related to the joint force by,

$$\{X_b\} - \{X_a\} = [H_j]\{F_j\}_1 \quad (4)$$

with

$$[H_j] \equiv [P_j]^{-1}$$

$$= \begin{bmatrix} k_1 + j\omega d_1, 0, ... 0 \\ 0, k_2 + j\omega d_2, ... 0 \\ \vdots \\ 0 ...., k_n + j\omega d_n \end{bmatrix}^{-1}$$

where $j = \sqrt{-1}$ and k_1, k_2, ...k_n, d_1, d_2....d_n are the spring and damping coefficients of the joint, as shown in Fig. 1.

If the whole structure is considered, the relation between the displacement vector and the external force vector can be expressed as,

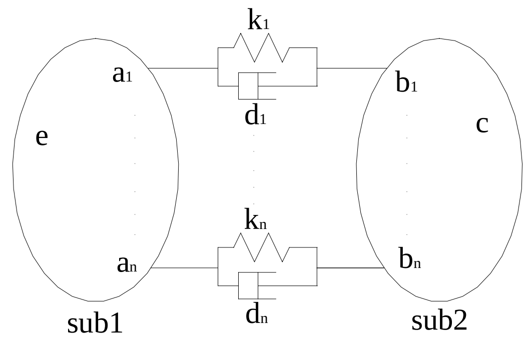

Fig.1 Two substructures connected by joint elements k_i and d_i .

$$\begin{Bmatrix} \{X_e\} \\ \{X_a\} \\ \{X_b\} \\ \{X_c\} \end{Bmatrix} = \begin{bmatrix} [H_{ee}], [H_{ea}], [H_{eb}], [H_{ec}] \\ [H_{ae}], [H_{aa}], [H_{ab}], [H_{ac}] \\ [H_{be}], [H_{ba}], [H_{bb}], [H_{bc}] \\ [H_{ce}], [H_{ca}], [H_{cb}], [H_{cc}] \end{bmatrix} \begin{Bmatrix} \{F_e\}_1 \\ \{F_a\}_1 \\ \{F_b\}_2 \\ \{F_c\}_2 \end{Bmatrix} \quad (5)$$

From the method of substructure synthesis, it can be proved (Wang and Liou, 1990) that the FRFs of the whole structure in Eq. (5) can be expressed in terms of the FRFs of the substructures and the joint matrix $[H_j]$ in Eq. (4). For instance,

$$[H_{ee}] = [H_{ee}]_1 - [H_{ea}]_1 [H_B]^{-1} [H_{ae}]_1 \quad (6a)$$

$$[H_{aa}] = [H_{aa}]_1 - [H_{aa}]_1 [H_B]^{-1} [H_{aa}]_1 \quad (6b)$$

$$[H_{ba}] = [H_{bb}]_2 [H_B]^{-1} [H_{aa}]_1 \quad (6c)$$

$$\vdots$$

with

$$[H_B] = [H_{aa}]_1 + [H_{bb}]_2 + [H_j] \quad (7)$$

Equation (6) contains three different matrices, i.e., the FRFs of substructures, FRFs of the whole structure, and the joint matrix $[H_j]$. Therefore, if the FRFs of the substructures and the whole structure are known by experimental measurement, then the only unknowns in Eq. (6) are the joint parameters in $[H_j]$. Theoretically, the joint parameters in $[H_j]$ can easily be obtained from Eq.(6), provided that the FRFs are known. In practice, it is very difficult to obtain the correct parameters from Eq. (6) because many inverse operations on the matrices should be taken. A small error in the matrices can cause the result to be faulty. For instance, one can derived the unknown matrix $[H_j]^{-1} = [P_j]$ directly from Eq. (6c) as (Tsai and Chou, 1988),

$$[P_j] = [H_{aa}]^{-1} [H_D] [H_{aa}]^{-1} \quad (8)$$

with

$$[H_D]=(([H_{aa}]_1-[H_{aa}])^{-1} \\ -[H_{aa}]_1^{-1}([H_{aa}]_1+[H_{bb}]_2)[H_{aa}]_1^{-1})^{-1}$$

Although Eg.(8) is very complicated, if all the FRFs are exactly correct, the joint parameters can exactly be obtained from Eq. (8) because there is no approximation in deriving Eq.(8). In practice, the measured FRFs can't be free from noise or error. A small noise in the FRFs may cause the identified results to deviate from the correct values drastically because there are too many inverse operations on the FRF matrices. In the past, many efforts have been done by many researches in different ways in order to obtain the accurate parameters from Eq. (8) or other similar equations. In order to reduce the number of inverse operation on the matrices, Wang and Liou (1990) developed a new identification formula from Eqs. (6b) and (6c) as,

$$[P_j]=-([H_{aa}]_1+[H_{bb}]_2)^{-1} \\ ([H_{aa}]_1+[H_{ba}]-[H_{aa}])([H_{ba}]-[H_{aa}])^{-1} \tag{9}$$

It has been demonstrated that Eq. (9) is less sensitive to noise than Eq. (8). However, our experiences show that the accuracy of the identified result by using Eq. (9) is structure dependent. In other words, noise is not the only consideration in improving the accuracy of identification. Some other problems in identification should be considered, as discussed in what follows.

PROBLEMS IN PARAMETER IDENTIFICATION

As mentioned, if the FRFs in Eq. (9) are exactly correct, the joint parameters can be exactly identified by Eq. (9) because there is no approximation in deriving Eq. (9). However, measurement noise is unavoidable in practice. A small noise level, for instance 2% random noise, may cause the error of the identified result to be higher than 100% in some structures. As to the question why a small noise level may cause the result to be drastically faulty? This question has been discussed by Wang and Liou (1991) from the mathematical point of view. There are two inverse operations on the FRF matrices in Eq. (9), the FRF matrices may become ill-condition in some frequency ranges. It is well known that a small perturbation on an ill-conditioned matrix may cause the inverse matrix to deviate from the exact value drastically. The concept of condition number of a matrix has been proposed (Wang and Liou, 1993) to eliminate the FRF data in the ill-conditioned matrices. Although the concept of condition number can improve the accuracy of identification, the experiences show that the accuracy is structure dependent. That means the ill-condition is not the only problem in using Eq. (9). The term ($[H_{ba}]$-$[H_{aa}]$) in Eq.(9) represents physically the relative deflection between the joint interfaces "a" and "b" when an external force is applied at joint interface "a". This can be explained by examining the relation in Eq.(5). If only the external force $\{F_a\}_1$ is applied to the whole structure, then from Eq.(5) one can obtain,

$$\{X_a\}=[H_{aa}]\{F_a\}_1 \tag{10}$$
$$\{X_b\}=[H_{ba}]\{F_a\}_1 \tag{11}$$

From Eqs. (10) and (11) one can obtain

$$\{X_b\}-\{X_a\}=([H_{ba}]-[H_{aa}])\{F_a\}_1 \tag{12}$$

From Eq.(12) one can explain the reason why the accuracy of identification by using Eq.(9) is structure dependent. Fig. 2 shows two different structures containing the same joint element. The first structure consists of two long and slender beams while the second structure consists of two short beams. Assume that the stiffness of the joint, i.e., the "k" in Fig. 2, is very high relative to the bending stiffness of the slender beams. One can expect that the relative deflection between the joint interfaces , i.e., $\{X_b\}$-$\{X_a\}$, of structure 1 may be smaller than that of structure 2 in most low frequency ranges. According to the relation of Eq.(12), the difference $[H_{ba}]$-$[H_{aa}]$ may be very small for structure 1. As a result, if the $[H_{ba}]$ and $[H_{aa}]$ of structure 1 are polluted by noise, then the difference $[H_{ba}]$-$[H_{aa}]$ is dominated by noise. From the above discussion one can expect that the joint parameter can be identified more accurately from structure 2 than from structure 1 provided that the absolute noise level in the FRFs is the same for both structures. In the next section we will give an example to demonstrate that the accuracy of identification can be improved by modifying the test structure properly.

Another problem in using FRFs to identify the joint parameter is caused by the order of magnitude of the FRFs. A typical FRF is shown in Fig. 3. One can find that the orders of magnitude vary drastically with frequencies. Because $[P_j]$ is a diagonal matrix, as derived in Appendix, Eq.(9) can be arranged as a set of linear equations as,

$$[Q]_{n\times n}\{P\}_{n\times 1}=\{U\}_{n\times 1} \tag{13}$$

where {P} contains the joint parameters to be identified, i.e.,

$$\{P\}_{n\times 1}=\begin{Bmatrix} k_1+j\omega d_1 \\ k_2+j\omega d_2 \\ \vdots \\ k_n+j\omega d_n \end{Bmatrix}$$

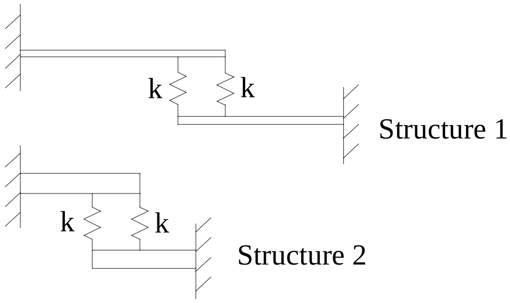

Fig.2 Two different structures with the same joint element.

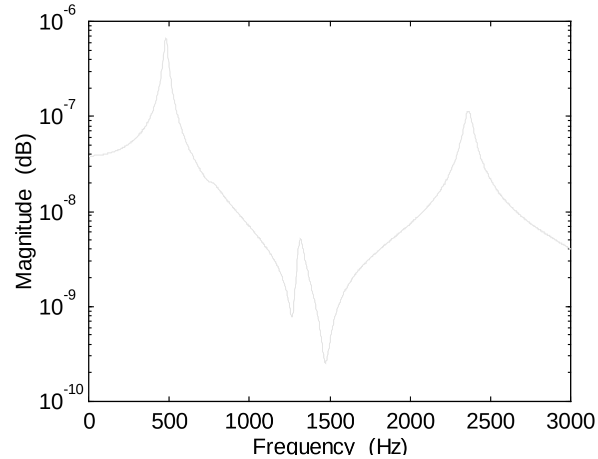

Fig.3 A typical FRF of structure.

Note that the matrix [Q] and vector {U} are derived from the FRFs matrices. So, [Q] and {U} are function of frequency. If the joint parameters are frequency independent, then the joint parameters can be obtained from Eq. (13) by direct matrix inversion. However, in practice, in order to reduce the effect of random noise, the data of FRFs at many frequencies should be used. For instance, if the FRFs are known at some discrete frequencies, ω_1, ω_2,......ω_m, then for each frequency one can have a set of n simultaneous equations like Eq. (13), i.e.,

$$\begin{bmatrix} Q(\omega_1) \\ Q(\omega_2) \\ \vdots \\ Q(\omega_m) \end{bmatrix}_{mn\times n} \begin{Bmatrix} P \end{Bmatrix}_{n\times 1} = \begin{Bmatrix} U(\omega_1) \\ U(\omega_2) \\ \vdots \\ U(\omega_m) \end{Bmatrix}_{mn\times 1} \tag{14}$$

or in compact form as,

$$[\underline{Q}]_{mn\times n}\{P\}_{n\times 1} = \{\underline{U}\}_{mn\times 1} \tag{15}$$

The overdetermined equations can then be solved by the least squares method as,

$$\{P\} = \left([\underline{Q}]^T[\underline{Q}]\right)^{-1}[\underline{Q}]^T\{\underline{U}\} \tag{16}$$

Theoretically, if the number of equations is larger than the number of unknowns, i.e., mn>>n, then the least squares method can effectively smooth the random noise in the equations. For instance, the FRFs are generally measured by spectrum analyzer with 800 lines resolution. In other words, the number of m in Eq.(15) can be as large as 800. However, our experiences by theoretical simulation show that the random noise in the FRFs can't be effectively smoothed by using Eq.(16) even though the number of equations is two orders higher than the number of unknows. Some efforts have been made to understand the reason why a random noise in the FRFs can't be effectively smoothed by the least squares method. This will be explained in what follows. As mentioned, the order of magnitude of the FRFs vary drastically with frequencies, so are the matrix [Q] and vector {U} in Eq.(14). In other words, the coefficients of the linear simultaneous equations in Eq.(14) vary drastically with frequencies. The coefficients of some equations are very large while the coefficients of some other equations are very small. The following simultaneous equations are a typical example.

$$90000x + 7000y = 76000 \tag{17a}$$
$$0.1x + 2y = -39 \tag{17b}$$
$$0.3x + 0.1y = 1 \tag{17c}$$
$$72500x + 8500y = 555000 \tag{17d}$$
$$2x - 0.1y = 22 \tag{17e}$$

One can obtain the unknowns x, y by solving any two equations as x=10, y=-20. If the coefficients are now perturbed by random noise, and the least squares method is used to solve the overdetermined equations. Then , one can find that the solution is mainly determined by Eqs.(17a) and (17d), the other equations have very little effect on the solution. Eq.(14) has the same problem. The unknown parameters are actually determined only by some equations with large coefficients. As a result, the number of effective equations could be only somewhat larger than the number of unknows. That is the reason why the random noise can't be smoothed by the least squares method because from statistical point of view the number of the effective equations is not enough. In order to overcome this problem, a normalized procedure is proposed in this work. Eq.(13) is normalized by a matrix $[W]^{-1}$ as,

$$[W]^{-1}[Q]\{P\} = [W]^{-1}\{U\} \tag{18}$$

where

$$[W] = \begin{bmatrix} W_1, & 0, & \cdots & 0 \\ 0, & W_2, & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0, & 0, & \cdots & W_n \end{bmatrix}$$

$$[Q] = \begin{bmatrix} q_{11}, & q_{12}, & \cdots & q_{1n} \\ q_{21}, & q_{22}, & \cdots & q_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ q_{n1}, & q_{n2} & \cdots & q_{nn} \end{bmatrix}$$

$$W_i = (q_{i1}^2 + q_{i2}^2 + ...q_{in}^2)^{1/2}, i = 1,2,....n.$$

The normalized equation was then used to form Eq.(14). After the normalization, the difference of the orders of magnitude of the coefficients between equations is reduced. Then, the least squares solution (i.e., Eq. (16)) is meaningful from the statistical point of view. In the next section, an example will be given to demonstrate the proposed method.

RESULTS OF NUMERICAL SIMULATION

Because measurement noise is always unavoidable in practice, in the following simulation, noise was added to the FRFs to simulate the practical situation. A random noise with Gaussian distribution (zero mean, variance σ_n^2) was added to the FRFs to simulate the measurement noise. If $H_{ij}(\omega)$ represents the FRF between the *i*th and *j*th degrees of freedom of a structure, then the noise level E is defined as,

$$E^2 = \frac{\sigma_n^2}{\left|H_{ij}(\omega)\right|_{\max}^2} \tag{19}$$

where $|H_{ij}(\omega)|_{\max}$ represents the maximum absolute value of $H_{ij}(\omega)$ in the frequency range of interest. If a noise level is given, then a set of random number with Gaussian distribution can be generated by a computer program. Note that the FRF is complex, the random noise should be added to the real and imaginary parts of the FRF, respectively.

The first simulated structure consists of two beams connected together by two linear joints, as shown in Fig. 4. The whole structure was approximated by finite beam elements. The geometry and material data of the structure is given in Table 1. Note that a structural damping with proportional form, D=αM+βK, was used to simulate the damping capability of the beams. If the joint parameters k_1, k_2, d_1, and d_2 are given, then the equation of motion of the whole structure and the substructures can be known. Note that the equation of motion is

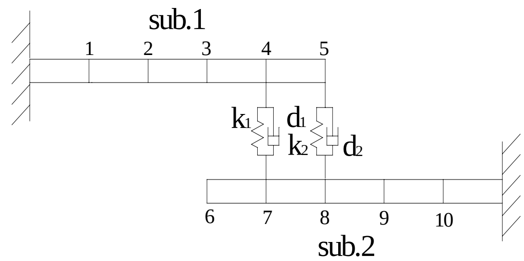

Fig.4 Simulated structure, k_1, k_2, d_1, d_2 are the joint parameters to be identified.

Table 1 Geometry and material data of the simulation example.

L(*m*)	B(*m*)	h(*m*)	E(N/m^2)	ρ(kg/m^3)	α	β	Noise Level
0.06	0.04	0.003	2.1×10^{11}	7850	0.03	4×10^{-9}	5%

used only to generate the FRFs. If the joint parameters are identified by experimental method, one does not need the equation of motion of the structure. The main purpose of this example is to demonstrate that if the stiffness of the joints is relatively higher than the bending stiffness of the beams, the properties of the joints can't be identified accurately by the method (Wang and Liou, 1990) without the pre-normalization of the FRFs. The maximum frequency of interest and the frequency resolution were set to be 2000Hz and 5Hz, respectively. The joint stiffness was assumed to be k_1=5×10^6N/m, k_2= 1×10^7N/m, which was very stiff relative to the bending stiffness of the beam. According to Eq.(14), the number of frequencies which can be used is 400=2000Hz/5Hz., i.e., m=400 in Eq.(14). Because there are only two complex parameters to be identified, i.e., $k_1+j\omega d_1$, and $k_2+j\omega d_2$, the number of equations is far higher than that of unknows. So, the least squares method (i.e., Eq.(16)) was used to smooth the random noise. The identified result is shown in Table 2. One can find that the result is very poor and unreasonable; for instance the k_1 value become negative. The reason for this unacceptable result is due to the fact that the value of $[H_{ba}]$-$[H_{aa}]$ in Eq.(9) is mainly dominated by noise because of the high stiffness of the joints. If the joint stiffness is not too high in comparison with the stiffness of the structure, the parameters can be identified accurately by using Eq.(16). An example is given in Table 3. In this example, the structure and noise level are the same as that of Table 2 except that the stiffness and damping of the joints are reduced to be k_1 =k_2 = 5×10^5N/m and d_1=d_2=75N·s/m. One can see that the joint parameters can be accurately identified even the FRFs are polluted with high noise level, i.e., 5%.

Table 2 Identified result by traditional method.

Properties	Parameters	Exact Value	Identified Values	Error(%)
Stiffness (N/m)	k_1	5,000,000	−24822	−100.5
	k_2	10,000,000	43957	−99.6
Damping (N-s/m)	d_1	200	0.05	−99.9
	d_2	300	−0.22	−100.1

Table 3 Identified accuracy is improved by reducing the relative stiffness of the joints.

Properties	Parameters	Exact value	Identified Values	Error(%)
Stiffness (N/m)	k_1	500,000	466400	−6.7
	k_2	500,000	488580	−2.3
Damping (N-s/m)	d_1	75	77.8	3.7
	d_2	75	72.6	−3.2

As discussed, the problem caused by too low relative deflection between the joint interfaces can be overcome by modifying the structure and by the method of pre-normalization of the FRFs. In the following example, the proposed normalization method is used to improve the accuracy of identification. The structure, joint parameters and noise level are the same as that of Table 2. The [Q] matrix in Eq.(13) was first normalized according Eq.(18), and then the normalized [Q] was used to form the overdetermined set of equations. The overdetermined equations were then solved by Eq.(16). The number of discrete frequencies i.e., m, in Eq.(14) is 400=2000Hz/5Hz. The identified result is shown in Table 4. The accuracy of the result is significantly improved in comparison with that of Table 2. Note that the normalization process can only increase the number of “effective” equations in the set of Eq. (14), and as a result, improve the effectiveness of the least squares method. The normalization can’t change the fact that the matrix difference $[H_{ba}]$- $[H_{aa}]$ is still dominated by noise. This is the reason why the result of Table 4 still has significant error. We believe that if the condition of the test structure is not modified, the joint parameters can not be identified with reasonable accuracy no matter what kind of identification algorithm is applied. Therefore, in the following example, we will demonstrate that the accuracy of identification can be improved by properly modifying the structure. The purpose of the modification is to increase the relative deflection between the joint interfaces in the frequency range of interest so that the difference, $[H_{ba}]$- $[H_{aa}]$, would not be dominated by noise. The general rule to achieve this aim is to increase the stiffness of the structure. Therefore, the simplest way to modify the structure of Fig. 4 is to fix the substructure 2 completely. The modified model is shown in Fig. 5. Although, in practice, it is impossible to fix a structure completely, the model of Fig. 5 means that the stiffness of the substructure 2 is very high in comparison with substructure 1 and the joints, and can be considered as rigid in the frequency range of interest.

Table 4 Identified accuracy is improved by the proposed pre-normalization method. (in comparison with Table 2)

Properties	Parameters	Exact value	Identified Values	Error(%)
Stiffness (N/m)	k_1	5,000,000	3516717	−29.7
	k_2	10,000,000	5941174	−40.6
Damping (N-s/m)	d_1	200	289	44.3
	d_2	300	366	22.1

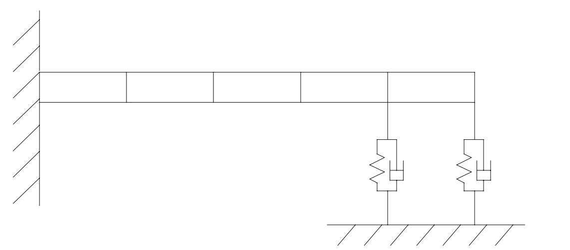

Fig.5 Model of Fig.4 with completely fixed substructure 2.

The result of identification with the model of Fig.5 is shown in Table 5. Note that the normalization process was also applied in the identification process. One can see that the result is improved significantly in comparison with the result of Table 4. The reason for this improvement is that the information of the deformation at the joint interfaces can be “observed” more clearly by the FRFs so that the effect of noise can be reduced. In other words, the information of the deformation at the joint interfaces of Fig.4 is immersed in the noise while that of Fig. 5 is only perturbed by noise. The result demonstrates that the accuracy of parameter identification can be significantly improved by properly modifying the dynamic conditions (includes the boundary condition, the stiffness, the mass, et. al.) of the structure.

The results of the simulated examples clearly indicate the problems in parameter identification of mechanical joints by using the noise-contaminated FRFs and the associated solutions to these problems. In the next section, an experimental example will be given to verify the feasibility of the proposed methods.

EXPERIMENTAL RESULTS AND DISCUSSIONS

Test Structure

In this section, the test structure will be described first, and then the experimental result will be discussed. Because the data need by the proposed method are the FRFs, the measurement instrumentation is very simple. It includes the vibration sensors, impact hammer and a FFT analyzer.

Table 5 Identified accuracy is improved by the modification of the dynamic condition of structure.

Properties	Parameters	Exact value	Identified Values	Error(%)
Stiffness (N/m)	k_1	5,000,000	5363750	7.3
	k_2	10,000,000	93135436	-6.9
Damping (N-s/m)	d_1	200	231	15.5
	d_2	300	336	12

The test structure consists of two cantilever beams connected together by a single bolted joint. The specification of the bolt is M6x1, and the applied torque is 15kgf·cm. The bolted joint was modeled as linear spring and damping elements with stiffness and damping coefficients k and d to be identified. The measured frequency range of the FRFs is 0.2000Hz, and the frequency resolution is 2.5Hz. In other words, there are 800 data in each spectrum.

Results and Discussions

Because the number of joint to be identified is only one, the number “n” in Eq.(13) is equal to one. The parameters were first identified by using Eq.(16) directly without pre-normalization of the FRFs. As mentioned, there are 800 data in each spectrum, the number “m” in Eq.(14) is 800. In other words, we use 800 equations to solve one unknown by the least squares method. The identified result is not shown here because the result is nonrepeatable and unreasonable, i.e., the value of stiffness is negative. This result is expected because the test structure was so designed that the stiffness of the joint was far higher than the bending stiffness of the beams in the measured frequency range 0.2000Hz. To improve the result, the proposed normalization procedure was applied. The result is shown in Table 6. Because it lacks another reliable method to identify the exact property of the bolted joint, there is no exact value for comparison in Table 6. However, one believes that the result of Table 6 is not reliable because the value of damping is negative. The above results indicate that the proposed normalization procedure can only partially improve the accuracy of identification by increasing the effectiveness of the least squares method; however, it can’t change the fact that the measured $[H_{ba}]-[H_{aa}]$ are immersed in noise. Besides the improvement of the measurement method, the only method to improve the signal to noise ratio is to modify the dynamic conditions of structure, as discussed in theoretical example. There are many different possibilities to modify the test structure; however, the basic principle is to increase the relative deflection between the joint interfaces in the frequency range of measurement. In this example, a simple method was used, namely, a point mass was added to the substructure 2, near to the joint, to increase the deflection of joint interfaces. The parameters of the joint was identified from the new FRFs of

Table 6 Experimental result of a single bolted joint only by pre-normalization.

	k(N/m)	d(N-s/m)
Result	50,859,352	-3,817

the modified structure. The result is shown in Table. 7. As mentioned, it lacks another reliable method to know the exact value of the joint parameters, there is no exact value for comparison. In order to know the accuracy of the identified result, the identified parameters were used with the measured FRFs of the substructures to synthesize the FRFs of the total structure. One of the synthesized FRF is shown in Fig. 6 in comparison with the measured one. Note that the measured FRFs of the substructure was more or less contaminated by noise so that the synthesized FRF would not exactly match with the measured one even if the exact values of the parameters were used. Therefore, the difference between the synthesized and the measured FRF in Fig.6 is not all due to the error of the joint parameters. Although so far we don’t know exactly the error of the identified result, it is sure that the result of Table 7 is better than that of Table 6. It should be pointed out that it lacks a general, reliable method to verify the identified parameters is also a problem in parameter identification.

CONCLUSIONS

The dynamic behavior of a mechanical structure is strongly affected by the properties of mechanical joints. There are many different methods using the measured FRFs directly or indirectly to identify the joint parameters; however, most of the methods suffer seriously from the problem of unavoidable measurement noise. Although the previous works (Wang and Liou, 1990, 1991, 1993) have proposed some methods to minimize the effect of noise, a further investigation indicates that noise problem is not the only consideration in improving the accuracy of identification. In this work, the causes of some other problems and the associated solutions were discussed. The theoretical and experimental results demonstrate that the proposed normalization procedure can improve the effectiveness of the least squares method to smooth the random noise. The results also show that if the structure is properly modified to increase the relative deflection of the joint interfaces in the frequency range of interest, the accuracy of identification can be significantly improved.

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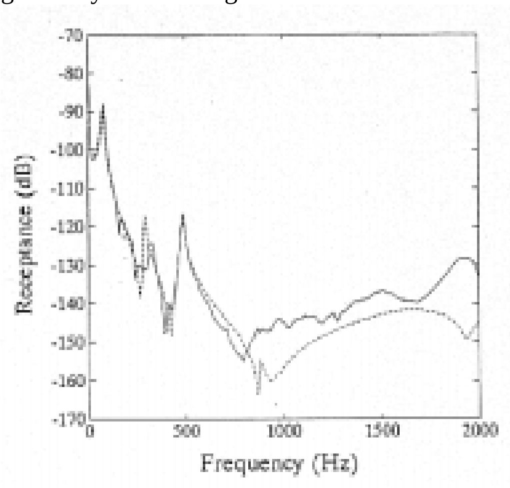

Fig.6 Comparison of the synthesized(-----) and measured (——) FRF.

Table 7 Experimental result of a single bolted joint by both pre-normalization of FRFs and modification of the structure.

	k(N/m)	d(N-s/m)
Result	21,500,828	1,363

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APPENDIX

Equation (9) can be rewritten as

$$
\begin{aligned}
&([H_{aa}]_1 + [H_{bb}]_2)[P_j]_{n\times n} \\
&= -([H_{aa}]_1 + [H_{ba}] - [H_{aa}])([H_{ba}] - [H_{aa}])^{-1}
\end{aligned}
\qquad \text{(A-1)}
$$

Let

$$
([H_{aa}]_1 + [H_{bb}]_2) \equiv \begin{bmatrix} q_{11}, q_{12}, \ldots\ldots, q_{1n} \\ q_{21}, q_{22}, \ldots\ldots, q_{2n} \\ \vdots \\ q_{n1}, q_{n2}, \ldots\ldots, q_{nn} \end{bmatrix}_{n\times n} \qquad \text{(A-2)}
$$

and

$$
\begin{aligned}
&-([H_{aa}]_1 + [H_{ba}] - [H_{aa}])([H_{ba}] - [H_{aa}])^{-1} \\
&\equiv \begin{bmatrix} u_{11}, u_{12}, \ldots\ldots, u_{1n} \\ u_{21}, u_{22}, \ldots\ldots, u_{2n} \\ \vdots \\ u_{n1}, u_{n2}, \ldots\ldots, u_{nn} \end{bmatrix}_{n\times n}
\end{aligned}
\qquad \text{(A-3)}
$$

Then Eq. (A-1) can be written as

$$
\begin{bmatrix} q_{11}, q_{12}, \ldots\ldots, q_{1n} \\ q_{21}, q_{22}, \ldots\ldots, q_{2n} \\ \vdots \\ q_{n1}, q_{n2}, \ldots\ldots, q_{nn} \end{bmatrix}
\begin{bmatrix} p_1, 0, \ldots\ldots, 0 \\ 0, p_2, \ldots\ldots, 0 \\ \vdots \\ 0, \ldots\ldots, p_n \end{bmatrix}
=
\begin{bmatrix} u_{11}, u_{12}, \ldots\ldots, u_{1n} \\ u_{21}, u_{22}, \ldots\ldots, u_{2n} \\ \vdots \\ u_{n1}, u_{n2}, \ldots\ldots, u_{nn} \end{bmatrix}
\qquad \text{(A-4)}
$$

with

$$
p_i = k_i + j\omega d_i, \qquad i = 1,2,\cdots n
$$

or

$$\begin{bmatrix} q_{11}p_1, q_{12}p_2, \ldots\ldots, q_{1n}p_n \\ q_{21}p_1, q_{22}p_2, \ldots\ldots, q_{2n}p_n \\ \vdots \\ q_{n1}p_1, q_{n2}p_2, \ldots\ldots, q_{nn}p_n \end{bmatrix} = \begin{bmatrix} u_{11}, u_{12}, \ldots\ldots, u_{1n} \\ u_{21}, u_{22}, \ldots\ldots, u_{2n} \\ \vdots \\ u_{n1}, u_{n2}, \ldots\ldots, u_{nn} \end{bmatrix} \quad \text{(A-5)}$$

Eq. (A-5) can be rearranged as

$$\begin{bmatrix} q_{11}, q_{12}, \ldots\ldots, q_{1n} \\ q_{21}, q_{22}, \ldots\ldots, q_{2n} \\ \vdots \\ q_{n1}, q_{n2}, \ldots\ldots, q_{nn} \end{bmatrix}_{n\times n} \begin{Bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{Bmatrix}_{n\times 1} = \begin{Bmatrix} u_{11} + u_{12} + \ldots\ldots + u_{1n} \\ u_{21} + u_{22} + \ldots\ldots + u_{2n} \\ \vdots \\ u_{n1} + u_{n2} + \ldots\ldots + u_{nn} \end{Bmatrix}_{n\times 1}$$

or in a compact form as

$$[Q]_{n\times n}\{P\}_{n\times 1} = \{U\}_{n\times 1} \quad \text{(A-6)}$$