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APPLICATIONS OF THE BOUNDARY ELEMENT METHOD TO SOME INVERSE PROBLEMS IN ENGINEERING MECHANICS

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ABSTRACT

The paper presents a brief review of current research on computational approach to inverse problems in engineering mechanics. Emphasis is placed on applications of the boundary element method, and the paper reports on some recent applications of such approach to a few classes of the inverse problems which are of practical importance.

INTRODUCTION

Computational methods of analysis based on the finite difference, finite element, and boundary element methods have been so well developed that we can easily solve the initial- and boundary-value problems, which are to be called the direct problems. It has been increasingly attracting the attention of scientists and engineers to apply the computer analysis software well established for the direct problems to the corresponding inverse problems [1-5].

There are many inverse problems around the world, in which we should estimate the reasons from the observed results. However, from the engineering point of view, the inverse problem can be stated such that some information on the initial and/or boundary conditions, domain shapes, material constants, etc. are not known, and this lacking information should be identified by using additional information which is usually provided as measured data.

If we consider a system which is modeled as an initial- and boundary-value problem, we may classify the corresponding inverse problems into the following:

1. Estimation of the initial and/or boundary conditions
2. Determination of domain shapes
3. Estimation of sources
4. Estimation of material constants
5. Estimation of governing differential equations

In this article, we shall first explain fundamentals of the computational approach to the solution of inverse problems using the methods of analysis for the direct problems, and then show several investigations on the inverse problems by author's group. Finally, the paper is concluded by some remarks toward more fruitful and more successful analysis of the inverse problems.

METHODS OF INVERSE ANALYSIS

The inverse problem under consideration is modeled as a parameter identification problem. In a computational approach to this inverse problem, we first assume the values of parameters in an appropriate manner and then carry out analysis of the direct problem. The results obtained are compared with the measured data given as additional information, and the parameter values are then modified so that an appropriate cost function is minimized in an iterative manner. The cost function is usually defined as a square sum of differences between the measured and computed data. We may express the cost function as follows:

$$W = W(\boldsymbol{z}) \tag{1}$$

where $\boldsymbol{z}$ is a vector of parameters, and denoting by M the number of parameters we have

$$\boldsymbol{z} = \{\boldsymbol{z}_1 \ \boldsymbol{z}_2 \ ... \ \boldsymbol{z}_M\}^{\mathrm{T}} \tag{2}$$

where the superimposed T means the transpose of a matrix. We can apply the standard procedures of optimization[6-8] for the solution of the above-modeled inverse problems. The filter theory can also be implemented instead of the optimization procedure as shown in the next section of this article.

In the inverse problems of estimating defects, e.g. cracks or cavities, which is the main subject of NDT (non-destructive testing), we may select as the parameters $\boldsymbol{z}$ the quantities defining the locations and shapes of the defects to be detected. If the defect is modeled as an ellipse or a sphere, the number of parameters can be reduced and then inverse analysis can be easily carried out. Although we may choose all the nodes located on the defect surface as the parameters, it would usually lead us to a large amount of computation time for inverse analysis and to unstable computation which yields less successful results. Hence, such inverse analysis could not be recommended. It is true that the smaller the number of parameters is, the more the performance of inverse analysis is.

The finite element methods have been best developed as computational software for direct problems, and naturally there are many of such investigations on the inverse problems [1-5]. In optimal shape design, however, we have to search the shape in an iterative manner, and careful attention should be paid to re-meshing at each iterative step of inverse analysis so that accuracy of computational results is not reduced by a deformed finite-element mesh. The boundary element methods [e.g., 9-11] can provide a more convenient tool for the problems of shape optimization design, because discretization by the method is confined within the boundary surface. In addition, it is reported that the BEM can give more accurate numerical results than the finite element or finite difference methods, if appropriate care is taken for singular integrals[12,13]. This advantage is very important and makes the BEM more attractive than other methods, because in inverse analysis only a limited number of measurements are available and hence the computational results should be kept to be accurate at each iterative step. From these reasons the boundary element methods have been employed for inverse analyses, and many successful results have been reported in Refs.[1-5, 14].

INVERSE ANALYSIS VIA BEM AND FILTER THEORY

In the filter theory, it is assumed that measured data includes errors with a Gaussian distribution. There is the following relationship between the observation vector $\boldsymbol{y}$ of measured data, and the state vector $\boldsymbol{z}$ of the parameters corresponding to the unknown information to be identified, that is,

$$\boldsymbol{y}_k = \boldsymbol{h}(\boldsymbol{z}_k) + \boldsymbol{v}_k \tag{3}$$

The nonlinear function is expanded into a Taylor series with respect to the state vector, and higher-order terms are neglected. Thus, we can obtain a linearized relation of equation (3) as follows:

$$\eta_k = \boldsymbol{y}_k - \boldsymbol{h}(\hat{\boldsymbol{z}}_{k-1}) + \boldsymbol{H}_k \hat{\boldsymbol{z}}_{k-1} \tag{4}$$

where k denotes iteration counter, and

$$\boldsymbol{H}_k = \left[\frac{\partial h_i(\boldsymbol{z}_k)}{\partial z_j} \right]_{\boldsymbol{z}_k = \hat{\boldsymbol{z}}_{k-1}} \tag{5}$$

This implies that the assumed values of the parameters can be modified using the following relation:

$$\hat{\boldsymbol{z}}_k = \hat{\boldsymbol{z}}_{k-1} + \boldsymbol{K}_k \left[\boldsymbol{y}_k - \boldsymbol{h}(\hat{\boldsymbol{z}}_k) \right] \tag{6}$$

where $\hat{\boldsymbol{z}}_k$ is an estimated set of the parameters $\boldsymbol{z}$ at the kth iteration, and $\boldsymbol{K}_k$ is the filter gain which is given for the extended Kalman filter [15,16] by

$$\boldsymbol{K}_k = \boldsymbol{P}_{k/k-1}\boldsymbol{H}_k^{\mathrm{T}}\left[\boldsymbol{H}_k\boldsymbol{P}_{k/k-1}\boldsymbol{H}_k^{\mathrm{T}} + \boldsymbol{R}_k\right]^{-1} \tag{7}$$

and for the projection filter [16,17] by

$$\boldsymbol{K}_k = \left[\boldsymbol{H}_k^{\mathrm{T}}\boldsymbol{R}_k^{-1}\boldsymbol{H}_k\right]^{-1}\boldsymbol{H}_k^{\mathrm{T}}\boldsymbol{R}_k^{-1} \tag{8}$$

In the above expressions, $\boldsymbol{P}_{k/k-1}$ is the covariance of the estimation errors of parameters at iteration $k-1$, and $\boldsymbol{R}_k$ the covariance of measurement errors at iteration k.

In the inverse analysis using the filter theory mentioned above, the sensitivity matrix $\boldsymbol{H}_k$ is computed by means of the finite difference method, so that the boundary element method is twice applied to compute the physical quantities at each iteration.

The main flow of the proposed inverse analysis is illustrated in Fig.1. It is an easy matter to replace the filter algorithm shown in the figure with the standard optimization technique[18].

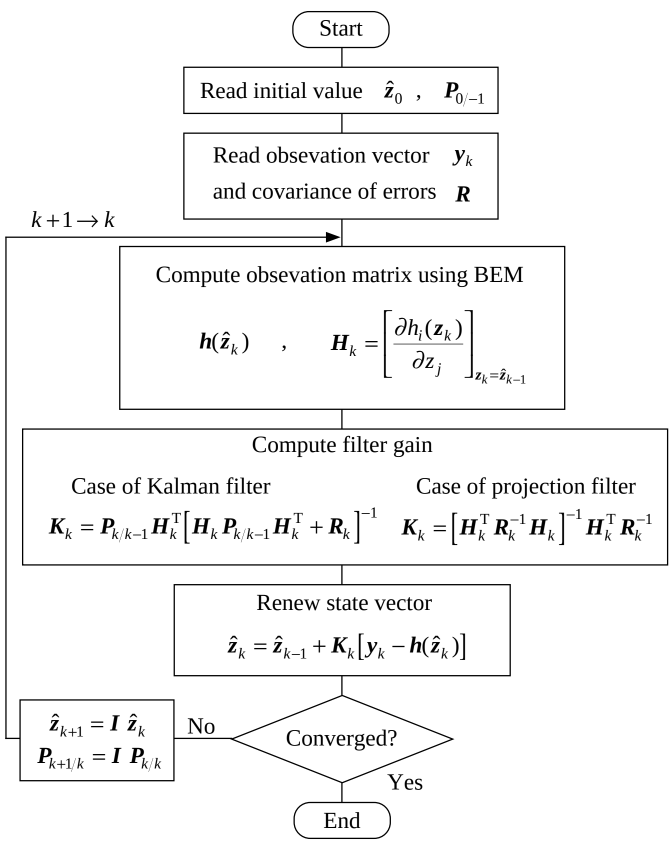


Fig. 1 Main flow of inverse analysis

DEFECT DETECTION AS INVERSE ELASTO-DYNAMIC PROBLEM

To evaluate safety or reliability of structural components, it is important to estimate non-destructively the location and shape of an internal cavity or crack. Here, we shall introduce one of such investigations[18-21] using the boundary element method for steady-state elastodynamics.

For the elastodynamic inverse problem under consideration, we can use the measured data on displacements, strains, or natural frequencies as additional information. In the following, an investigation[21,22] will be shown in which the displacement responses are measured and available for inverse analysis.

It is assumed that displacements at some selected points on the boundary are measured when the structural component is subjected to a time-harmonic excitation. Using these data as additional information we want to detect the position and shape of an internal cavity. The cost function is defined as a square sum of the differences between the measured displacements and computed ones by the boundary element method. The inverse problem is then solved by minimizing the following cost function:

$$W = \sum_{n=1}^{M}\sum_{j=1}^{d}\left(u_j^n - \overline{u_j^n}\right)\left(u_j^n - \overline{u_j^n}\right) \tag{9}$$

where u_j is a displacement component, and the superimposed bar denotes the measured one.

There are investigation[18-20] of this problem using the conjugate gradient method of optimization. In the following, however, we show another study using the Kalman filter and the BEM[21,22]. To demonstrate the usefulness of the inverse analysis method, numerical experiment is carried out for a few simple examples in two-dimensional problems. It is assumed that the shape of cavity is elliptic and hence the parameters to be estimated can be expressed in terms of five parameters as follows:

$$\boldsymbol{z} = \left\{x_1 \; x_2 \; a \; b \; \theta\right\}^{\mathrm{T}} \tag{10}$$

where x_1 and x_2 are the coordinates of the center point of the cavity, a and b are the lengths of two principal axes of the ellipse, and θ the angle of the short axis with the axis x_1.

In Fig.2 is shown the numerical example with one elliptical cavity in the rectangular component of horizontal side 300[mm]

$\times$ vertical side 200[mm]. The component is fixed on the bottom side and subjected to a time-harmonic excitation p at point F1, and the cavity to be estimated is located as shown in the figure with the parameters $\boldsymbol{z} = \{-70\ -20\ 20\ 10\ 45°\}$. Computation of inverse analysis is started by assuming the cavity as a circle located at the center point of the component. The material constants of isotropic elasticity and other computational data are assumed as follows:

Young's modulus E=210[GPa]
Poisson's ratio $\nu = 0.3$
mass density $\rho = 7.85 \times 10^3 [\mathrm{kg/m^2}]$
time-harmonic excitation $p = 0.15\exp(i\omega t)$ [GPa]
angular frequency of excitation
$$\omega = 10,000[\mathrm{rad/s}] \cong 1,600[\mathrm{Hz}]$$
covariance of measurement errors $\sigma^2 = 1.0 \times 10^{-6}$

It is further assumed that the displacements are measured at the six points on the boundary as shown in Fig.2.

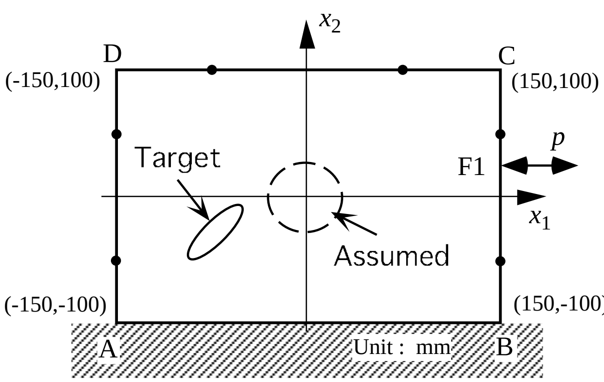


Fig.2 Rectangular component with elliptical cavity

In the numerical experiment, the measured data on displacements are given by the computational results obtained by the boundary element analysis using the target values of parameters $\boldsymbol{z} = \{-70\ -20\ 20\ 10\ 45°\}$. Table 1 summarizes the results obtained, which shows that after 17 iterations a satisfactory estimation can be obtained. It is interesting to point out that inverse analysis was sometimes not successful when the defect was assumed at some different positions. In such cases, the so-called multiple excitation method[21,22] is very useful to strengthen additional information for inverse analysis. This method uses measured data obtained under a set of different excitations for inverse analysis.

The above approach is extended to detection of crack, and it is discussed in [20-22] how to select the measured data in consideration of sensitivities with respect to the parameters to improve the robustness of inverse analysis. However, it should be mentioned that the electrical- potential methods of inverse analysis[23] are most successfully applied to detection of cracks in structural components of electricity-conducting materials.

Table 1 Estimated results

Parameters	Target values	Estimated	Iterations
x_1	-70	-69.985	17
x_2	-20	-19.978	
a	20	20.007	
b	10	9.994	
θ	45°	44.908°	

ESTIMATION OF EROSION CURVE IN BLAST FURNACE REFRACTORY

Now, we shall try to identify the erosion curve of the refractory in the blast furnace hearth. From a macroscopic point of view, the problem of the blast furnace under consideration can be modeled as a steady-state heat conduction problem in an axisymmetric body subject to the boundary conditions of axisymmetric distribution. When the boundary element method is applied to this problem, the boundary integral equation for three-dimensional problems is transformed into the two-dimensional boundary integral equation on the meridian of the axisymmetric body. The boundary element methods have been already established for the axisymmetric problems[24,25]

For the inverse problem under consideration, Yoshikawa et al.[26] already reported on an inverse analysis method in which an optimal combination of parameters was found under a limited number of parameter values. There can be a more sophisticated approach which uses a filter theory to take account of measurement errors in the inverse analysis. The author presents such an approach, in which the boundary element method with quadratic interpolations and the filter theory are combined to use for the inverse analysis of the problem[27]. In this article, only the outline of the investigation will be shown. The two filter theories, Kalman and projection filters, are used for inverse analysis. Numerical experiment is carried out, and the results obtained are discussed whereby the advantages and the disadvantages of the filter theories are revealed.

Analysis Model

Figure 3 shows the blast furnace hearth which is assumed to be rotationally symmetric about Z axis. The erosion curve is considered as the isothermal curve of the solidification temperature of molten metal (1150 °C). We consider the steady-state heat conduction problem subject to the Dirichlet boundary condition on the internal surface of the erosion curve. The other boundary conditions and constants in heat transfer are assumed as follows:

1) The heat conduction coefficient of refractory is $\lambda = 13[\mathrm{W/mK}]$.
2) The side wall is subject to a boundary condition of heat transfer in which the ambient temperature is $30[°\mathrm{C}]$ and the heat transfer coefficient is $70[\mathrm{Wm^2K}]$.
3) The bottom surface is subject to a boundary condition of heat transfer in which the ambient temperature is $35[°\mathrm{C}]$ and the heat transfer coefficient is $70[\mathrm{Wm^2K}]$.
4) The upper surface is subject to an adiabatic condition in which $q = 0$.

As the parameters to be identified, we take the distance between a fixed point on the Z axis and a point on the internal boundary which lies on the ray issued from the fixed point with a given angle as shown in Fig.3. The erosion curve is drawn by the C-spline functions using the estimated values of the parameters. It is assumed that 61 thermocouples are located on the side wall and on the bottom surface and that these measurements can be used for inverse analysis. In general, however, inverse analysis is likely to be unstable or ill-posed, when the number of measurements used for analysis is too small or too large. For numerical analysis, we shall use only the number of 7 among these measured data, which are chosen in the order of higher sensitivity with respect to the parameter values at each iteration. That is, the seven measuring points which have largest absolute values of sensitivity defined in equation (5) by $\left|\partial h_j/\partial z_i\right|$. These points are different at each iteration, but in this numerical example no appreciate difference can be observed. Hence, the seven measuring points are selected as points having highest sensitivities with respect to the initial parameter values, as shown in Fig.3 by small circles on the boundary. In addition, we assume that the measurement error is within 10 % of the highest temperature measured at the seven points mentioned above, and that the covariance of estimation errors is 1.0×10^{-4}.

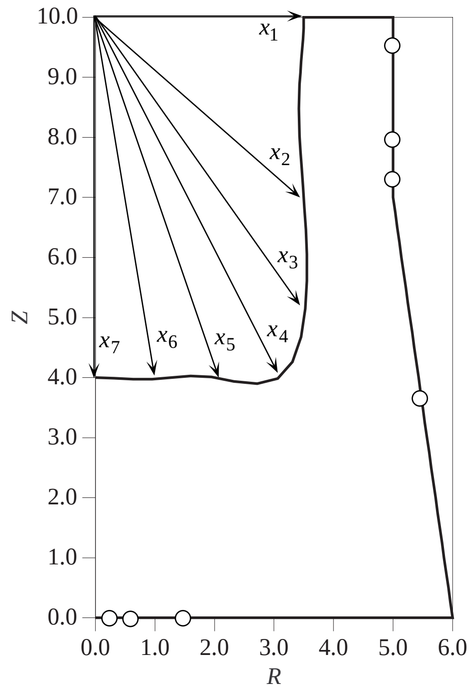


Fig.3 Analysis model of blast furnace hearth

In Fig.4 is shown one of the numerical results obtained using the Kalman filter when the initial values of parameters are not appropriately assumed and hence the final estimation is less satisfactory, in particular, near the symmetrical axis where the estimated erosion curve is not fitted to the target geometry. In this computation, calculation has been terminated after 100 iterations, although no convergence is realized. If the projection filter is used in the inverse analysis under the same assumption for the initial values of parameters, worse results are obtained rather than the case of Kalman filter. It may imply that the projection filter is rather sensitive for the initial values of parameters than the Kalman filter. This property will be investigated in the following.

Now, we shall carry out inverse analysis using the two filter theories for the sake of comparison. We perform inverse analysis with the initial values deviating from the target values between -15% and +15%. Two typical initial assumptions of the erosion curve with deviations 10% and +10% from the target values are shown in Fig.5.

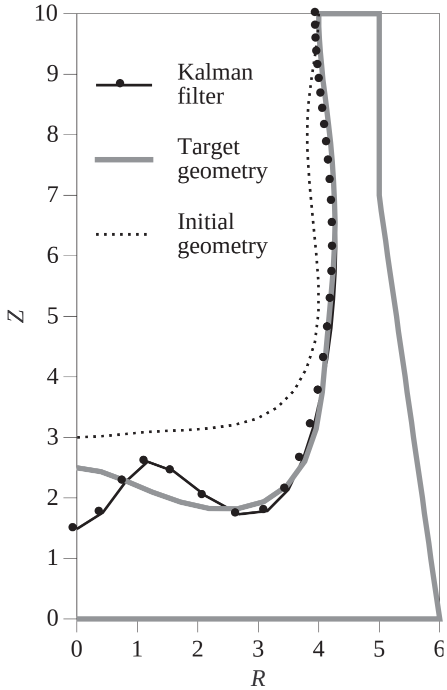


Fig.4 Typical example of estimated results

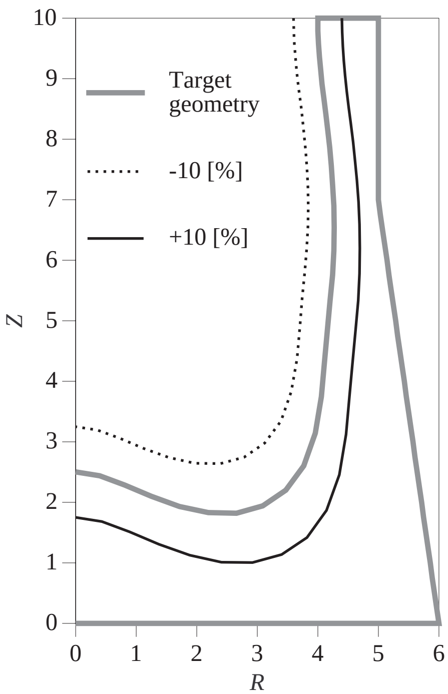


Fig.5 Initial guesses of erosion curve

Table 2 Difference between another initial geometries

Difference to the target geometry [%]	Kalman filter	Projection filter
+5	○ (33)	○ (3)
+10	△	○ (4)
+15	△	○ (7)
-5	○ (8)	○ (3)
-10	○ (30)	○ (7)
-11	○ (38)	×
-12	○ (67)	×
-13	△	×
-14	△	×
-15	×	×

The numerical results are summarized in Table 2, in which computation is terminated after 100 iterations. In this table, the symbol O denotes convergence after the iteration number shown in parentheses, while the symbol × does no convergence and the symbol Δ does no convergence but indicates possibility of convergence after a large number of iterations. From Table 2, it can be seen that the Kalman filter is rather tough than the projection filter: Even if the initial guess of parameters is not so good, Kalman filter could still give an approximate solution of the inverse problem.

APPLICATION TO TEMPERATURE CONTROL

Study on Known Heat Input

Most of control problems are formulated into the inverse problems which can be solved by the method of inverse analysis discussed in this article. As one of such examples, we now consider the control of temperature in a solid under heat conduction. The problem is stated as in Fig.6: We want to control the temperature on part Γ_r of the boundary as required, by changing the temperature or heat flux on the boundary part Γ_c . The problem of this temperature control is then formulated into such an inverse problem that the optimal controllable heat load should be found by minimizing a cost function representing the differences between the required and calculated temperatures on the boundary part Γ_r . For these active control problems, the boundary element method can be successfully used together with the optimization procedure or also the filter theory. In the following, a couple of investigations along this line are briefly explained .

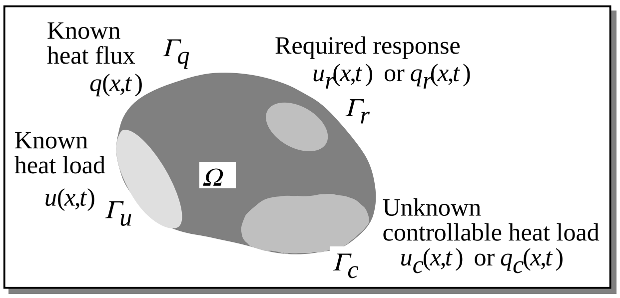


Fig.6 Schematic illustration of temperature control

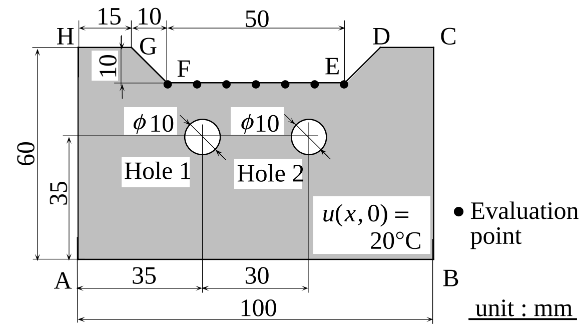
We can treat the problem of temperature control in such a way that after assuming the controlling heat load on Γ_c, we compute the responses on Γ_r by the boundary element method and calculate the cost function defined by

$$W=\sum_{l=1}^{L}\sum_{d=1}^{D}\left(\frac{u(x_l,t_d)-u_r(x_l,t_d)}{u_r(x_l,t_d)}\right)^2 \tag{11}$$

where $u(x_l,t_d)$ denotes the temperature at evaluation point x_l at time t_d, and $u_r(x_l,t_d)$ the required temperature on the boundary part Γ_r. Then, the parameter values of control heat load are modified by the standard optimization technique or by the inverse analysis method as shown in Fig.1. In the following, we shall show some of the numerical results[28-30] obtained using the boundary element method based on the Laplace transform.

Numerical simulation on the temperature control in an injection mold of plastics is presented. A two-dimensional model of injection mold with two equal holes is considered as shown in Fig.7. The side DEFG is in contact with injected plastics. We want to keep the temperature on the surface FE uniformly distributed, by changing the temperature of the two holes. The required temperature on EF is assumed as 30 °C. Seven evaluation points are taken on the side EF, and 31 points on the time axis. It is assumed that the temperature on the surfaces of two holes is uniform during the whole time and at the beginning of inverse analysis it is 30 °C. The history of temperature of molten plastics is assumed as shown in Fig.8. Without any control, the history of temperature on EF can be calculated as shown in Fig.9.

The change of temperature on the holes surface is expressed in terms of B-spline functions, and their coefficients are the parameters to be estimated by inverse analysis. In Fig.10 is shown the optimized history of temperature on the cooling surface obtained by inverse analysis. Figure 11 shows the history of temperature on EF after optimally controlled.



Boundary Conditions
BC, HA: $q=0$
DEFG: $h(t)=240\,\mathrm{W/m^2°C}$, $u_a=f(t)$
AB, CD, GH: $h(t)=2100\,\mathrm{W/m^2°C}$, $u_a=30$°C

Fig.7 Analysis model of injection mold

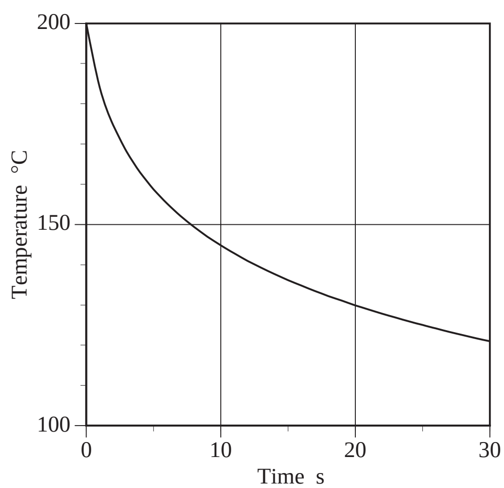


Fig.8 Temperature change in injected plastics

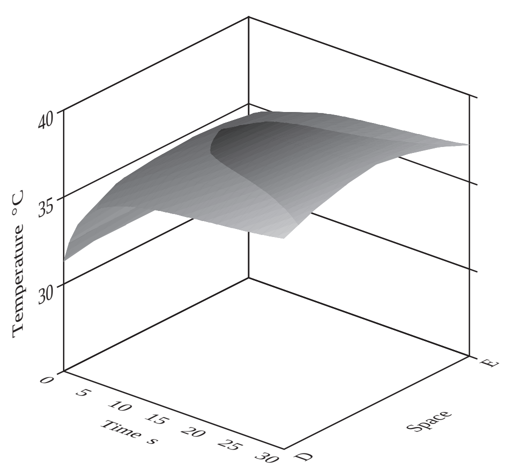


Fig.9 Temperature history on EF without control

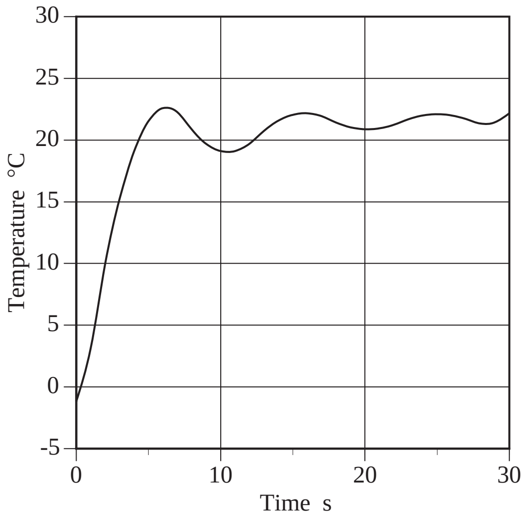


Fig.10 Change in temperature on cooling surface of two holes for optimal control

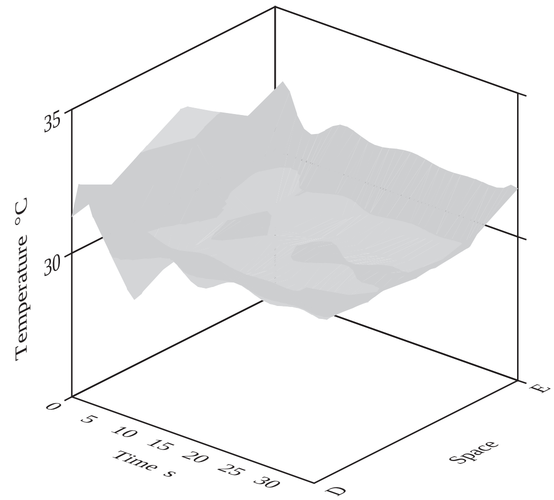


Fig.11 Optimally controlled temperature on EF

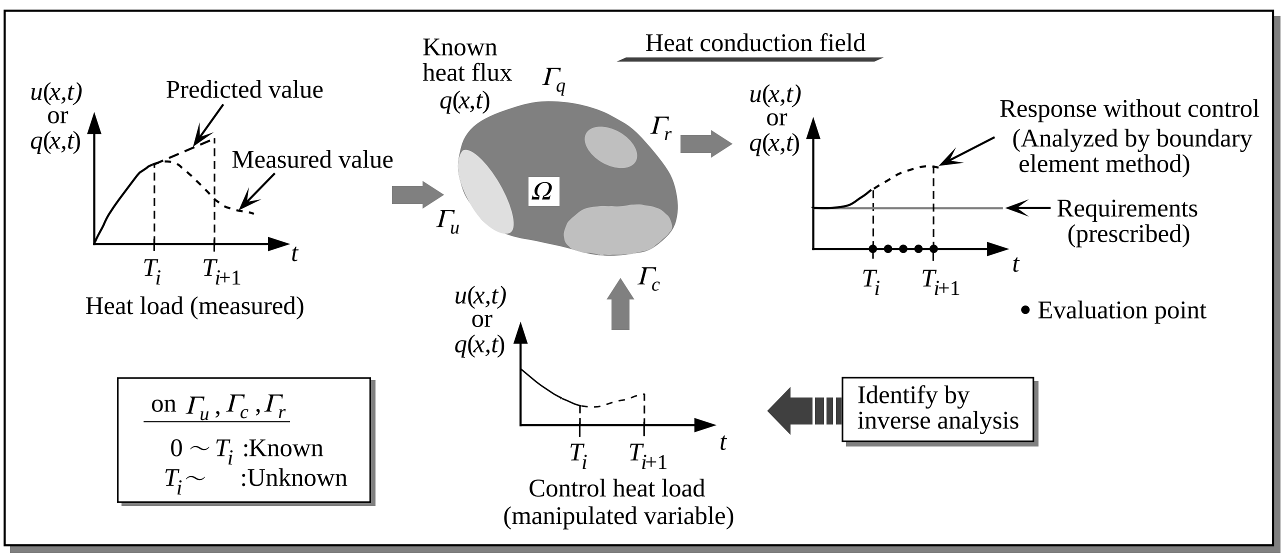


Fig.12 Schematic illustration of temperature control for unknown heat input

Study on Unknown Heat Input

It is interesting to note that the same inverse analysis method can be applied to more complicated cases in which the input heat load on the boundary part Γ_u is not known[31].

In Fig.12 is illustrated a schematic view of the temperature control for the cases of unknown heat input. We have to take account of the temperature control for each of small time intervals. The control heat load is approximated as constant or smooth by B-splines during the time interval, as shown in Fig.13, and then the inverse analysis mentioned in the previous section is carried out. A few examples are simulated by means of this procedure and the results obtained are discussed, whereby the usefulness of the inverse analysis is demonstrated. Because of space limitation, their details can not be presented in this article. Those who are interested in this approach are kindly asked to see the original paper[31].

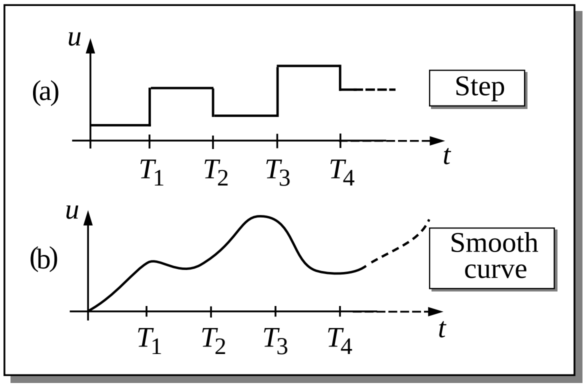


Fig.13 Approximation of control temperature via two ways

CONCLUDING REMARKS

The inverse analysis presented in this article can also be successfully applied to many inverse problems in engineering mechanics. Basically, in the inverse analysis methods, we have to calculate the gradient of a cost function or sensitivities of the measured data with respect to the parameters to be estimated. If the values of parameters are assumed to be near the exact values, inverse analysis can be very successfully carried out. However, inverse analysis would be less successful, difficult or almost impossible, when the initial assumption of the parameters is not suitable to perform inverse analysis. This occurs frequently in analyses of almost all inverse problems. Therefore, it is inevitably required to access the parameter values close to the exact ones before applying the inverse analysis method discussed here. For this purpose, we have to use *a priori* information as much as possible. It would also be useful from the engineering point of view that we first try to obtain an approximate solution under rough assumptions or constraints on the range of parameter values. To this end, genetic algorithms or other methods based on knowledge engineering seem to be very promising. Then, we apply the present method of inverse analysis to get a refined solution of the inverse problem. If this *two-step* solution procedure is applied to the inverse problems at hand, we could almost always obtain the satisfactory solutions.

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