A Method for Analyzing the Stability of Noniterative Inverse Heat Conduction Algorithms

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Summary

A simple method for analyzing the stability of noniterative inverse heat conduction algorithms is proposed. Developed for application to linear matrix normal equations in the unknown instantaneous surface heat flux distribution, the technique first introduces a temperature measurement error vector, $\delta\mathbf{Y^1}$, at an arbitrary instant, $t = t^1$. The initial error is then tracked in time, producing a propagating error in the computed instantaneous temperature distribution, $\delta\theta^{\mathbf{n+1}}$ (at $t = t^{n+1}$). It is shown that $\delta\theta^{\mathbf{n+1}}$ is related to the error at $t = t^n$ via $\delta\theta^{\mathbf{n+1}} = \mathbf{G} \cdot \delta\theta^{\mathbf{n}}$, where $\mathbf{G}$ is a global propagation matrix determined by the global mass and stiffness matrices and a dimensionless Fourier number. Defining a stable algorithm as one in which the propagating error decays with increasing time, then a unambiguous stability criterion emerges: if the spectral radius, ρ_G, of the propagation matrix is less than 1, the method is stable. By contrast, if $\rho_G > 1$, the method is unstable. The proposed approach is illustrated using two simple 1-D inverse heat conduction problems.