# Green's (influence) functions

# and some inverse problems

# in structural mechanics

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**ABSTRACT**

In structural mechanics, the response of a structure (or of its single element) to a concentrated unit force is referred to as the *influence function of a point force*. In mathematics, the influence function is recognized as the *Green's function (matrix)* of a boundary or initial-boundary problem which simulates the equilibrium state of the structure. The role of these functions in both theoretical and applied sciences cannot be overestimated. As to practical applications in engineering, it is worth noting that most of the existing representations of influence functions for structural mechanics related problems are not suitable for immediate use in computations. This makes their computability an important practical issue that is particularly crucial for series representations of influence functions. The point is that such series do not uniformly converge and there are no ways for them to do so due to the singularity of the function that the series represents.

In our earlier works, a special technique was developed that allows one to appropriately break down the series representing an influence function onto two parts which approximate the singular and the regular components of the influence function. With this job done, the part representing the singular component is completely summed up, radically improving the computability of the entire influence function, and a series representing the regular component is uniformly convergent. The technique makes it possible for series representations of influence functions to be productively used in solving a broad spectrum of problems in engineering sciences.

The purpose of this lecture is to demonstrate how the influence (Green's) function-based methods can effectively be used to attack some inverse problem classes in structural mechanics. We will start with discussing the effectiveness of the above mentioned technique making influence functions representations practically usable in a variety of practical statements. As to inverse formulations, our particular focus will be on:

(i) the recovery of a set of forces applied to a beam given the deflection function generated by the forces;
(ii) the recovery of a distributed load for a beam given its deflection;
(iii) optimal control of natural frequencies of a beam;
(iv) finding the equilibrium state of a plate with rigid inclusions;
(v) optimal eigenvalue shape design of Kirchoff plates.

We will conclude the lecture with introducing the extension of the notion of Green's function to regions that are piecewise homogeneous. This notably widens the range of engineering problems that can be handled with the influence function related procedures.