Bezier splines and sensitivity analysis use for inverse geometry and boundary problems

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The continuous casting process of metals and alloys is nowadays frequently utilized in metallurgical industry, and more general in material engineering. Typically, the liquid material flows into the mould (crystallizer) having the walls cooled by flowing water. The solidifying ingot is pulled out by the withdrawal rolls. Below the mould the side surface of the ingot is very intensively cooled by the water flowing out of the mould and being sprayed over the surface.

Definition of the process conditions and an accurate determination of the interface location between the liquid and solid phases are very important for the design of cooling system as well as the quality of the casted material. This problems were the objectives of the works dealing with the boundary and the geometry inverse problems. The boundary inverse problem consists in determination of heat flux distribution along outer boundary of the ingot. The numerical procedures were built on the strength of sensitivity analysis and boundary element method [6]. The influence of the number and accuracy of measurements were tested. In this work, numerically simulated temperature measurements were used.

The geometry inverse problem concern the estimation of phase change front location. The publications [3, 2] include details of the solving algorithms and the method of modeling the boundary shape. Especially much attention was paid to the application of the Bezier splines for the phase change boundary approximation [5]. The way of using sensitivity analysis taking into account the sensitivity coefficients determination (in case of quadratic or cubic boundary elements application) is widely presented in works [3, 5].

Presented paper proposes an algorithm being a combination of both boundary and geometrical inverse problem in one task. The heat flux distribution along outside surface of the ingot and the phase change front location is estimated at the same time. Additionally the dependence of the final results on the number, location and accuracy of measurements was investigated.

Mathematical description of the considered problem consists of:

- diffusion-convection governing equation in solid phase. This equation contains partial differential operator acting on temperature $T(\mathbf{r})$

$$\nabla^2 T(\mathbf{r}) - \frac{1}{a}\, v_x \frac{\partial T}{\partial x} = 0 \tag{1}$$

  where v_x stands for the casting velocity and a is the thermal diffusivity

- boundary conditions defining heat transfer processes along the boundary involving melting temperature along the phase change front.

In the inverse analysis, the location of the phase change front (where the temperature is equal to the melting temperature) and the boundary conditions along outer surface are unknown. This means that the mathematical description needs to be supplemented by measurements because it is incomplete. Usually, some temperatures, U_i, measured at selected locations are provided. If these sensors are placed inside the body the values U_i are referred to as *internal temperature responses*. The ill-posed nature of all inverse problems causes that the number of temperature sensors should be appropriate to make these problems overdetermined. The use of Bezier function allows to reduce profoundly the total number of design variables, which results in fewer number of required measurements.

The objective is to estimate the identified values uniquely describing the phase change front location and the heat flux distribution along Γ_{BCD}. These values are collected in the vector $\mathbf{Y} = [y_1, \ldots, y_{2n+m}]^T = [v_1, \ldots, v_{2n}, q_1, \ldots, q_m]^T$.

Components v_i are connected with the front location and they are the coefficients of the Bezier spline control points, components q_j are connected with the heat flux distribution.

The present inverse problem is solved by building up a series of direct solutions which gradually approach the correct values of design variables. This procedure is split into boundary and geometry parts and can be expressed by the following main steps:

1. make the boundary problem well-posed. That means that the mathematical description of the thermal process is completed by assuming arbitrary but known values $\mathbf{Y}^*$ (as required by the direct problem).
2. *geometrical part* - solve the direct problem obtained above and calculate temperatures $\mathbf{T}^*$ at the sensor locations; compare these temperatures and measured values $\mathbf{U}$ and modify the assumed data v_j^*, $j = 1, 2, \ldots, 2n$ keeping q_k^*, $k = 1, 2, \ldots, m$ unchanged -this part is solved like a typical inverse boundary problem [6]
3. point 2. should be repeated until v_j converged
4. *boundary part* - ones the previous step is completed continue iterations and compare $\mathbf{T}^*$ and measured values $\mathbf{U}$ and modify the assumed data q_k^*, $k = 1, \ldots, m$ keeping v_j^*, $j = 1, 2, \ldots, 2n$ unchanged - typical inverse geometry problem [3, 5]
5. if it is necessary the external loop (points 2. to 4.) can be repeated

The modification of quantity $\mathbf{Y}^*$ in this work utilizes the concept of sensitivity coefficients matrix $\mathbf{Z}$. Elements of this matrix are derivatives of the measured temperature T with respect to an assumed (and then identified) elements of input data $\mathbf{Y}$. Through their definition sensitivity coefficients received obvious and practical meaning. They point out directly which members of $\mathbf{Y}$ should be modified and how much they should be changed to allow the best matching of calculated and measured temperatures.

It was also found that in order to guarantee appropriate accuracy of calculations a geometrical representation of the phase change front should be smooth. This feature can be easily satisfied by use of Bezier cubic function. This is demonstrated by 2-D numerical example from continuous casting of copper.

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