SOLUTION OF THE INVERSE PROBLEM OF RADIATIVE PROPERTIES ESTIMATION WITH THE PARTICLE SWARM OPTIMIZATION TECHNIQUE

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ABSTRACT

Several heuristics that mimic natural behaviors have been proposed for the solution of optimization problems. In particular some of the most recent algorithms, classified within the field of swarm intelligence, are based on the observation of social insects like bees, ants, etc. In the last decade of the past century the particle swarm optimization (PSO) technique was introduced for the continuous optimization problem, based on the analogy of bird and fish school behavior. Here we present an implementation of the PSO technique for the solution of the inverse radiative transfer problem of radiative properties estimation. In this approach it is required the solution of the direct radiative transfer problem which is modeled by the linear version of the Boltzmann equation. For that purpose we use a discrete ordinates method combined with the finite difference method. Some general guidelines are proposed and discussed for the PSO implementation that is applied for the estimation of the optical thickness, single scattering albedo, and diffuse reflectivities in a one dimensional plane-parallel participating medium. Test case results demonstrate the feasibility of the use of the proposed methodology.

1. INTRODUCTION

Inverse radiative heat transfer problems have several relevant applications in many different areas such as astronomy, environmental sciences, engineering and medicine [7, 8, 11, 13, 17]. Some outstanding examples are parameter and function estimation for global climate models, hydrologic optics, and computerized tomography [1, 4, 5, 10, 12, 26].

When formulated implicitly [18], inverse problems are usually written as optimization problems. Several heuristics that mimic natural behaviors have been proposed for the solution of optimization problems. In particular some of the most recent algorithms, classified within the field of swarm intelligence [3], are based on the observation of social insects behaviour.

An optimization technique that mimics the flight of a flock of birds, the Particle Swarm Optimization (PSO), has been used to solve continuous optimization problems [27, 28]. An enhanced proposed in [1] was used to solve an inverse radiative transfer problem in which we seek to determine the optical thickness, the single scattering albedo and the diffuse reflectivities at the inner side of the boundaries of a one- dimensional participating medium. As experimental data we consider the intensity of the emerging radiation measured at the boundary surfaces of the medium using only external detectors.

By probing the search space (range of the unknowns) in a random way, a stochastic method, such as PSO, may lead to the vicinity of the global optimum, if it is properly implemented computationally. Nonetheless the computational effort is usually high. Gradient based methods, such as the Levenberg-Marquardt method [20, 21], are usually faster in their convergence, but they may get trapped in the closer local minimum.

Recently, hybrid approaches, coupling stochastic methods and the Levenberg-Marquardt method have been used successfully for the solution of inverse heat transfer problems of parameter estimation [20, 21]: SA-LM (Simulated Annealing and Levenberg-Marquardt) and GA-LM (Genetic Algorithms and Levenberg-Marquardt). Other hybrid strategies combining stochastic and deterministic methods have also been implemented [5]. In such hybrid approach the stochastic method (SA or GA) is run for a small number of individuals and generations (or cycles), requiring therefore a much smaller number of function evaluations. The solution obtained with the stochastic method is then used as the initial guess for the gradient based method. If necessary this approach may be iterated. Artificial Neural Networks (ANN) have also been used for the same strategy of generating a good initial guess for the gradient based method: ANN-LM [22, 23]. Explicit and implicit formulations for the solution of inverse radiative transfer problems have also been combined in the same strategy [18, 19].

In this work, the inverse radiative transfer problem is solved by a PSO implementation without hybridization. This optimization method is able to perform an extensive scanning of the search space, but demanding a low computational effort.

2. MATHEMATICAL FORMULATION OF THE DIRECT AND INVERSE RADIATIVE TRANSFER PROBLEMS

2.1 Direct Problem

In Figure 1 is represented a one-dimensional, gray, homogeneous, isotropically scattering participating medium, of optical thickness τ_0 whose boundaries reflect diffusely the radiation that comes from the interior of the medium. The boundary surfaces at $\tau = 0$ and $\tau = \tau_0$ are subjected to the incidence of radiation originated at external sources with intensities A_1 and A_2, respectively.

The mathematical model for the interaction of the radiation with the participating medium is given by the linear version of the Boltzmann equation [14],

$$\mu \frac{\partial I(\tau,\mu)}{\partial \tau} + I(\tau,\mu) = \frac{\omega}{2}\int_{-1}^{1} I(\tau,\mu')\, d\mu'$$

$$0 < \tau < \tau_0,\ -1 \le \mu \le 1 \qquad \text{(1a)}$$

$$I(0,\mu) = A_1(\mu) + 2\rho_1 \int_0^1 I(0,-\mu')\,\mu'\,d\mu',$$

$$\mu > 0 \qquad \text{(1b)}$$

$$I(\tau_0,-\mu) = A_2(\mu) + 2\rho_2 \int_0^1 I(\tau_0,\mu')\,\mu'\,d\mu',$$

$$\mu < 0 \qquad \text{(1c)}$$

where I represents the radiation intensity, τ is the optical variable, μ is the cosine of the polar angle, i.e. the angle formed between the radiation beam and the positive τ axis, ω is the single scattering albedo, and ρ_1 and ρ_2 are the diffuse reflectivities at the inner part of the boundary surfaces at $\tau = 0$ and $\tau = \tau_0$, respectively. The other symbols have already been defined.

When the geometry, the boundary conditions, and the radiative properties are known, problem (1) may be solved and the radiation intensity I determined for the whole spatial and angular domains, i.e. $0 \le \tau \le \tau_0$, and $-1 \le \mu \le 1$. This is the so called direct problem.

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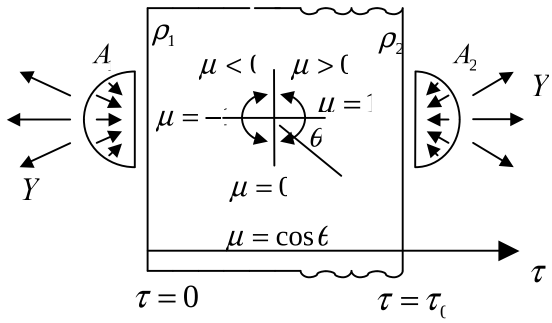

Fig. 1: **R**epresentation of a participating medium (1D), with incidence of radiation. Y represents the intensity of the radiation emerging from the medium, it may be measured by external detectors.

In order to solve problem (1), we use Chandrasekhar‘s discrete ordinates method [6] in which the polar angle domain is discretized as represented in Figure 2, and the integral term (in-scattering) on the right hand side of eqn. (1a) is replaced by a Gaussian quadrature.

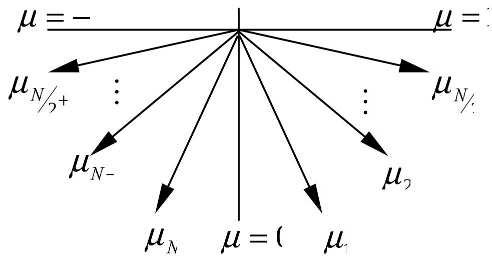

Figure 2. Discretization of the polar angle domain.

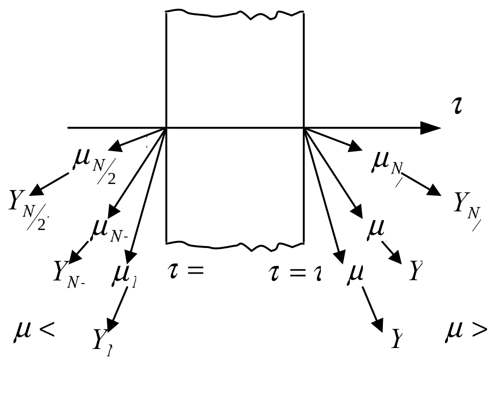

Figure 3. Schematical representation of the experimental data Y_i, $i = 1,2,...,N/2$ acquired at $\tau = \tau_0$, and Y_i, $i = N/2+1$, $N/2+2,...,N$ acquired at $\tau = 0$.

We then used a finite-difference approximation for the terms on the left hand side of eq. (1a), and by performing forward and backward sweeps, from $\tau = 0$ to $\tau = \tau_0$ and from $\tau = \tau_0$ to $\tau = 0$, respectively, $I(\tau,\mu)$ is determined for all spatial and angular nodes of the discretized computational domain.

2.2 Inverse Problem

We now consider that the following vector of radiative properties is unknown

$$\vec{Z} = \{\tau_0, \omega, \rho_1, \rho_2\}^T \tag{2}$$

but experimental data on the intensity of the radiation that leaves the medium is available, i.e. Y_i, $i = 1,2,...,N$. As schematically represented in Figure 3, half of the data is acquired at the boundary $\tau = 0$, and half at $\tau = \tau_0$, using only external detectors.

From the experimental data available, we then try to obtain estimates for the unknown radiative properties. This is the inverse radiative transfer problem we want to solve.

As the number of experimental data, N, is usually larger than the number of unknowns, we may formulate the inverse problem as a finite dimensional optimization problem in which we seek to minimize the cost function (also known as objective function) given by the summation of the squared residues between calculated and measured values of the radiation intensity,

$$Q(\vec{Z}) = \sum_{i=1}^{N} \left[I_{calc_i}(\tau_0, \omega, \rho_1, \rho_2) - Y_i \right]^2 \tag{3}$$

For the solution of the inverse problem described here, we have used a stochastic method, the Particle Swarm Optimization (PSO).

3. PARTICLE SWARM OPTIMIZATION

One of the main streams in artificial life is to understand how real world animals behave as part of a swarm and to try to mimic this behavior in an algorithm. Some aspects on such behavior must be abstracted in order to obtain rules that are feasible to be implemented in an algorithm. Even when the individual behavior is simple, the collective behavior can be very complex. This is the case of PSO.

Boyd e Richerson [29] have studied the decision making process in human beings and observed that decisions are taken based on the personal experience, but also on the neighbors' experience. This feature was exploited in the PSO algorithm and applied to the behavior of the birds. It is assumed that the behavior of the flock is a consequence of the effort of each bird in keeping an optimal distance from the neighboring birds.

The aesthetical choreography of a flock of birds was studied by zoology and computer science researchers in order to know what are the rules that provide for the synchronous flight of the flock even subjected to successive changes of direction.

In the PSO, a flock of birds is represented in a n-dimensional search space. The position of each agent/bird *i* at iteration *k* is given by its vector of Cartesian coordinates $\mathbf{X}_i^k$. At every iteration, that corresponds to an unitary amount of time, the flock of birds evolve as a consequence of the update of the positions of each bird. The update of position of agent/bird *i* is calculated using its current velocity vector $\mathbf{V}_i^k$, which is also updated at every iteration as a function of its previous position $\mathbf{X}_i^{k-1}$ and velocity $\mathbf{V}_i^{k-1}$.

The position of each bird represents a possible solution in the allowed search space. The evaluation of each bird is performed at every iteration by means of an objective function F(**X**). Each bird stores its best position $\mathbf{X}_i^{pbest}$, that corresponds to the better evaluation obtained by itself. This information is due to its own experience. Every bird also knows the best evaluation obtained by the flock until the moment, $\mathbf{X}^{gbest}$, that correponds to the experience of the group. At every iteration, the velocity vector $\mathbf{V}_i^{k-1}$ of each bird *i* is updated in function of the following variables:

- its previous position $\mathbf{X}_i^{k-1}$
- its previous velocity $\mathbf{V}_i^{k-1}$
- the distance vector defined by its previous position and its $\mathbf{X}_i^{pbest}$
- the distance vector defined by its previous position and flock's $\mathbf{X}^{gbest}$

The new (current) position $\mathbf{X}_i^k$ is defined by applying the current velocity operator to previous position $\mathbf{X}_i^{k-1}$. Actually, for a unitary time step, this is equivalent to add this velocity to the previous position in order to obtain the current position.

$$\mathbf{X}_i^k = \mathbf{X}_i^{k-1} + \mathbf{V}_i^k \tag{4}$$

In the PSO, the following equation defines the current velocity of each bird:

$$V_i^k = c_1.V_i^{k-1} + c_2.rand_1\,(\mathbf{X}_i^{pbest} - \mathbf{X}_i^{k-1}) + c_3.rand_2\,(\mathbf{X}^{gbest} - \mathbf{X}_i^{k-1}) \tag{5}$$

where $rand_1$ and $rand_2$ are random numbers between 0 and 1 and three positive real numbers, denoted learning parameters, must be chosen:

c_1: parameter that express the trust of the bird in itself;

c_2: parameter that express the trust of the bird in its experience;

c_3: parameter that express the trust of the bird in the experience of the flock.

The above learning parameters c_2 and c_3, weight the stochastic accelerations towards positions $\mathbf{X}_i^{\mathbf{pbest}}$ and $\mathbf{X}^{\mathbf{gbest}}$, respectively [28]. In terms of behavior, the parameter c_2 represents the cognitive factor associated to its best former experience, while the parameter c_3 represents the social factor associated to the best former experience of the group. It is common to assign the same value to these two parameters [30,31].

A general description of the **PSO** algorithm follows.

Step 1: Setting of initial conditions for the flock; for each bird, the position ($\mathbf{X}_i^0$) and velocity ($\mathbf{V}_i^0$), are randomly generated, given suitable ranges;

Step 2: Evaluation of the objective function F(**X**) for each bird of the flock; the positions $\mathbf{X}_i^{\mathbf{pbest}}$ and $\mathbf{X}^{\mathbf{gbest}}$ are eventually updated;

Step 3: Update of the velocities of each bird of the flock using Eq. (5);

Step 4: Update of the positions of each bird of the flock using Eq. (4), in order to obtain the new positions $\mathbf{X}_i^k$;

Step 5: Check of the stopping criteria; if it is not verified, return to step 2 for the next iteration.

The stopping criteria can be defined in a suitable manner. In this work, it is employed a threshold to be reached by the objective function. Other options include a limit number of iterations or a limit time.

4. RESULTS AND DISCUSSION

As in most of optimization algorithms, the quality of the solution obtained is related to the proper choice and fine tuning of the control parameters. For the PSO implementation, we have considered flocks with different number of birds (100, 1000 or 2000), different seeds for the generation of random numbers (33, 57 or 99), and different sets of learning parameters ({0.2,0.2,2.0}, {0.5,0.5,0.5} or {1.0,0.2,0.2}). These sets of learning parameters were chosen from previous tests in which standard functions were minimized using PSO. The first parameter weights the influence of the last position of the bird, the second, the best position of the bird, while the third, the best position of the band. Therefore, the chosen sets are intended to represent different schemes for this weighting.

We are interested in the estimation of the four unknown radiative properties given in Eq. (2). The range for each of the unknowns was taken as the interval (0, 1), the physical bounds of all the unknowns but the optical thickness. Synthetic experimental data were employed. They are calculated from the exit radiation intensities using the exact values of the radiative properties. In all test cases we have considered noiseless data.

In order to evaluate the performance of the PSO minimizer we chose a relatively difficult test case with

$$\vec{Z}_{exact} = \{\tau_0, \omega, \rho_1, \rho_2\}^T = \{1.00, 0.50, 0.10, 0.95\} \quad (7)$$

The incident radiation was taken as $A_1 = 1.0$ and $A_2 = 0.0$ in Eqs. (1b) and (1c), respectively. The main difficulty for the solution of the inverse radiative transfer problem considered in this work is related to the estimation of ρ_1, since its effect will be sensed by the external detectors only after the radiation goes into the medium at $\tau = 0$, is reflected at $\tau = \tau_0$ and is then both transmitted and reflected at $\tau = 0$. This difficulty is confirmed by the sensitivity analysis related to this particular unknown.

Table 1 presents the set of estimated values of τ_0, ω, ρ_1 and ρ_2, that yielded minimum cost considering a flock of 100 birds for different seeds and different sets of learning parameters. The values are followed by the corresponding value of the cost function, $Q(\vec{Z})$ defined by Eq. (3), and the amount of processing time and evaluations of this equation, and consequently of the direct model, that were required to reach that cost. Certainly, when a sub-optimal estimation was found, i.e. when the value of the cost function was greater than 1.0E-10, the PSO performed more evaluations that required more time until user-termination, but only time and number of evaluations until best solution was found are shown in the table.

The best set of learning parameters ({0.2,0.2,2.0} and its corresponding best seed (33) yielded the optimal solution in less than 9 seconds, being executed on a AMD Athlon 1.67 GHz IA-32 single-processor machine. However, flocks of 100 birds are not sufficient for solving this inverse problem: results for other sets of learning parameters are much worse and more prone to the influence of the seed that is used for the random number generation. As expected, the analysis of the results presented in Table 1 show that the poorest estimates are related to the unknown ρ_1. An important point is that the best results were obtained using a set of learning parameters that weights more the experience of the whole flock.

Table 1: Results for 100 birds, with different sets of learning parameters and seeds: set of estimated parameters, with their values of the objective function, processing time and number of evaluations of the direct model.

100 birds	τ_0	ω	ρ_1	ρ_2	$Q(\vec{Z})$, Eq. (3)	Time(s)	eval
{ 0.2 , 0.2 , 2.0 }							
seed 33	0.99994	0.5000	0.10002	0.94999	4.0122E-10	8.66	1590
seed 57	0.99980	0.50031	0.10109	0.95004	7.9529E-10	22.40	3975
seed 99	0.99985	0.50027	0.10097	0.95005	7.2561E-10	37.56	6573
{ 0.5 , 0.5 , 0.5 }							
seed 33	0.99916	0.50134	0.10478	0.95019	1.5129E-08	104.6	18218
seed 57	0.88494	0.77433	0.72890	0.98127	5.2750E-04	304.84	52685
seed 99	0.98649	0.52206	0.17522	0.95308	3.9807E-06	188.28	32678
{ 1.0 , 0.2 , 0.2 }							
seed 33	0.97866	0.50773	0.14027	0.95023	1.7679E-05	24.34	4283
seed 57	0.99971	0.55373	0.27292	1.0000	3.3719E-03	25.72	4561
seed 99	0.99777	0.55163	0.26936	1.0000	3.3732E-03	33.45	5884
Exact	**1.00**	**0.50**	**0.10**	**0.95**	**0.00**		

Table 2: Results for 100 birds, with different sets of learning parameters and seeds: set of estimated parameters, with their values of the objective function, processing time and number of evaluations of the direct model.

1000 birds	τ_0	ω	ρ_1	ρ_2	$Q(\vec{Z})$, Eq. (3)	Time(s)	eval
{ 0.2 , 0.2 , 2.0 }							
seed 33	0.99991	0.50016	0.10057	0.95004	8.4289e-10	67.75	12736
seed 57	1.00000	0.50018	0.10053	0.95001	7.6907E-10	40.71	8036
seed 99	0.99994	0.50022	0.10071	0.95003	6.057E-10	54.05	10260
{ 0.5 , 0.5 , 0.5 }							
seed 33	0.99984	0.50027	0.10093	0.95003	7.0006e-10	546.21	94069
seed 57	0.99983	0.50032	0.10112	0.95004	8.4499E-10	364.62	64249
seed 99	0.99991	0.50027	0.10090	0.95004	9.6085E-10	226.49	40023
{ 1.0 , 0.2 , 0.2 }							
seed 33	0.99962	0.50696	0.12024	0.95130	1.7008E-06	193.78	34583
seed 57	0.99725	0.50736	0.12311	0.95180	2.5660E-06	139.10	24762
seed 99	0.99872	0.50272	0.10801	0.95053	7.7596E-07	85.09	15729
Exact	**1.00**	**0.50**	**0.10**	**0.95**	**0.00**		

Table 3: Results for 100 birds, with different sets of learning parameters and seeds: set of estimated parameters, with their values of the objective function, processing time and number of evaluations of the direct model.

.2000 birds	τ_0	ω	ρ_1	ρ_2	$Q(\vec{Z})$, eqn. (3)	Time(s)	Eval
{ 0.2 , 0.2 , 2.0 }							
seed 33	0.99983	0.50024	0.10087	0.95003	5.2516E-10	106.56	20364
seed 57	0.99998	0.50010	0.10029	0.94999	5.4930E-10	114.22	21654
seed 99	0.99981	0.50031	0.10110	0.95005	8.8029E-10	91.17	17672
{ 0.5 , 0.5 , 0.5 }							
seed 33	0.99993	0.50028	0.10089	0.95004	9.3544E-10	215.93	39439
seed 57	0.99981	0.50032	0.10110	0.95004	9.9527E-10	357.84	63104
seed 99	0.99995	0.50017	0.10056	0.95001	6.5325E-10	221.37	40348
{ 1.0 , 0.2 , 0.2 }							
seed 33	0.99818	0.50142	0.10622	0.95010	1.6337E-07	444.55	78734
seed 57	0.99226	0.51443	0.14776	0.95188	2.0802E-06	315.04	56510
seed 99	0.99262	0.50533	0.12073	0.95072	2.0696E-06	802.04	141051
Exact	**1.00**	**0.50**	**0.10**	**0.95**	**0.00**		

Tables 2 and 3 are similar to Table 1, but for flocks of 1000 and 2000 birds, respectively. It can be seen that even using 2000 birds, a solution with objective function value in the order of 10-6 can be reached for any seed or set of learning parameters in less than 15 minutes on the same machine. It can be concluded that a larger flock gives robustness to the PSO since more birds are able to perform a better scanning of the search space.

6. CONCLUSIONS

In the present work the PSO yielded good estimates for the radiative properties of a one-dimensional participating medium using the measured data of the intensity of the radiation acquired only by external detectors. The use of flocks with more birds gave more robustness to the PSO. In future works it is intended to repeat these tests using noisy data and to

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