Identification in electric fault arc testing

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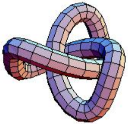

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http://www.tu-chemnitz.de/mathematik/inverse_probleme

Outline

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This talk is joint work with

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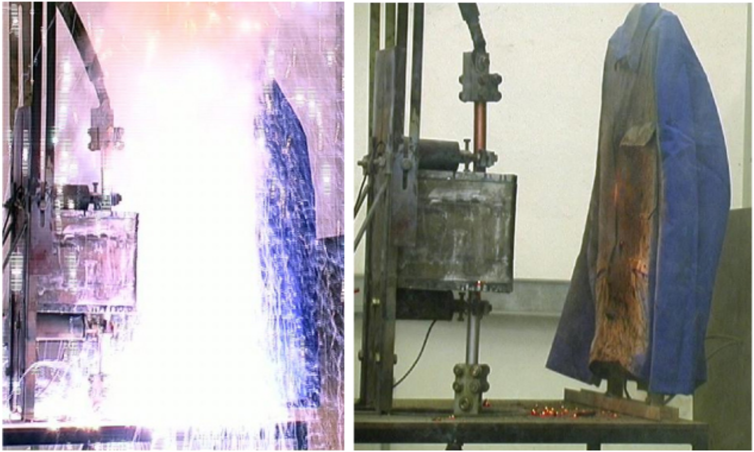
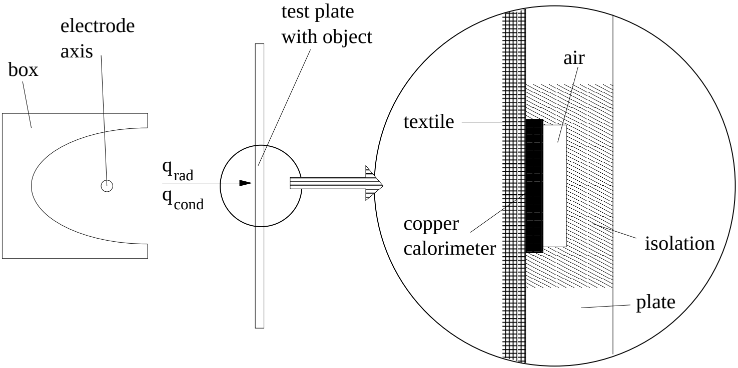
Introduction

Fault arc tests are performed in textile research and certification of protective clothes. The protective clothes are used by people working on electric installations who are exposed to the risk of fault arc accidents, potentially causing injury with heavy burns.



Electric fault arc tests for the certification of protective clothes

There exist different international standards for such arc tests. One particular European test is the so-called CENELEC test (pre-standard ENV 50354:2001). This **box-arc test** method contains a visual assessment (after flaming, hole formation, shrinking etc.) as a qualitative criterion and was extended and improved by including additionally a quantitative measurement of temperatures to get information about transmitted energy.



Schematic test arrangement

An electric arc is fired between two vertically arranged electrodes in a test circuit of defined voltage. After the burning-time interval of $t_p = 0.5s$ the arc is switched off.

A surrounding box focuses thermal arc effects in direction to a test plate with test object, which is arranged in a defined distance to the electrodes. The object consists of a variable number of textile layers stretched onto the test plate and a skin-simulating copper calorimeter embedded by an isolating block in the test plate.

The calorimeter is connected to a thermocouple, and so the calorimeter temperature is measured from the arc ignition $(t = 0)$ until the end of measuring time $t^{end} = 30s$.

Such tests are expensive to realize, and extensive technical equipment is required. A current of 7kA has to be controlled and held stable in a circuit for half a second, the complete arrangement has to sustain temperatures of several thousand degrees. Moreover, the textile or clothing which is tested will be destroyed during the test. For these reasons large series of arc tests, for instance in order to take parametric studies, are not practicable.

This gives rise to the need for a model of the test. A numerical simulation of calorimetric arc effects based on the simulated test was realized at Chemnitz University of Technology in cooperation with the Saxon Textile Research Institute (STFI).

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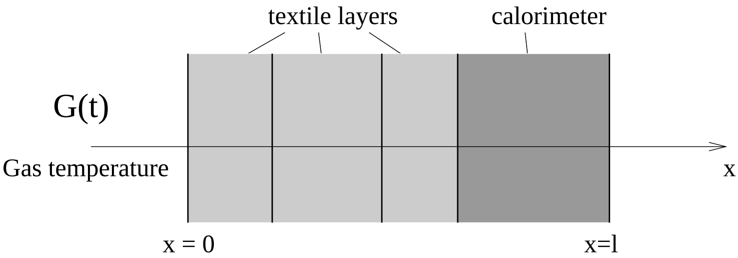
The mathematical model

The underlying model is based on the following assumptions:

- A nonlinear heat equation is set for the heat flux inside the object (textile and calorimeter). An important part of the nonlinearities takes the radiation which is modelled inside the object by a special source term.
- For modelling the boundary conditions a heat transfer proportional to the temperature difference at the interesting material borders is assumed.
- Both boundary conditions and radiation source include a not directly observable gas temperature $G = G(t)$, modelling the influences of hot gas between the arc and the examined object.



1D-model of the object

Notation:

x	1D local coordinate, $x \in (0, l)$,
t	time, $t \in [0, t^{end} = 30s]$,
$u = u(x,t)$	temperature in the object,
$G = G(t)$	temperature of the hot gas,
$C^A(x,t,u)$	apparent heat capacity,
$\kappa(x,u)$	thermal conductivity,
$f_{rad}(x,t,G(t),u(0,t))$	radiation heat source term,
h_0, h_s	heat transfer coefficients,
Q	space-time cylinder $(0,l) \times (0, t^{end})$

G and u relative temperatures w.r.t. ambient temperature T_0.

Structure of the radiation source term:

$$f_{rad}(x,t,G(t),u(0,t))$$
$$= \gamma e^{-\gamma x} \left(q_a(t) + \beta_{Gas}(G(t) + T_0)^4 - \beta_{\mathrm{Obj}} \left((u(0,t) + T_0)^4 - T_0^4 \right) \right)$$

Initial-boundary value problem of forward computations:

$$C^A(x,t,u)\frac{\partial u}{\partial t} - \frac{\partial}{\partial x}\left(\kappa(x,u)\frac{\partial u}{\partial x}\right) = f_{rad}(x,t,G(t),u(0,t)),$$

$$(x,t) \in Q,$$

$$-\kappa(0,u(0,t))\frac{\partial u(0,t)}{\partial x} = h_0(G(t) - u(0,t)), \qquad t \in (0,t^{end}],$$

$$\kappa(l,u(l,t))\frac{\partial u(l,t)}{\partial x} = -h_s u(l,t), \qquad t \in (0,t^{end}],$$

$$u(x,0) = 0, \qquad x \in [0,l].$$

Under some additional assumptions, for example

$$G \in \mathcal{D}_{box} := \left\{ G \in C[0, t^{end}] \, : \, 0 \le G(t) \le G_{max}, \; t \in [0, t^{end}] \right\} ,$$

and using weak formulations the **well-posedness** of the forward problem of computing $u(x, t)$ with $(x, t) \in [0, l] \times [0, t^{end}]$ can be shown.

Moreover, we have a Lipschitz estimate

$$\| u_2 - u_1 \|_{L^2(0,t^{end};H^1(0,l))} \le C \, \| G_2 - G_1 \|_{L^2(0,t^{end})}$$

with a constant $C > 0$ that does not depend on the gas temperature functions G_1 and G_2

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The inverse problem

In order to make forward computations, we need the function G.

- For solving the inverse problem of determining G we make the test without textile layers: **simplified test.**
- Temperatures on the back side of the calorimeter can be measured for this simplified test.

Nonlinear forward operator $F : \mathcal{D}_{box} \to L^2(0, t^{end})$ defined as

$$[F(G)](t) := u(l,t), \quad t \in [0, t^{end}],$$

where $u(l,t)$ means the corresponding function in the following simplified initial-boundary value problem and $u_{data}(t) = u(l,t) + \text{noise}$ expresses the real data.

Simplified version of initial-boundary value problem:

$$C_{\text{Cu}} \frac{\partial u}{\partial t} - \frac{\partial}{\partial x}\left(\kappa_{\text{Cu}} \frac{\partial u}{\partial x}\right) = f_{rad}(x, t, G(t), u(0, t)), \qquad (x, t) \in Q,$$

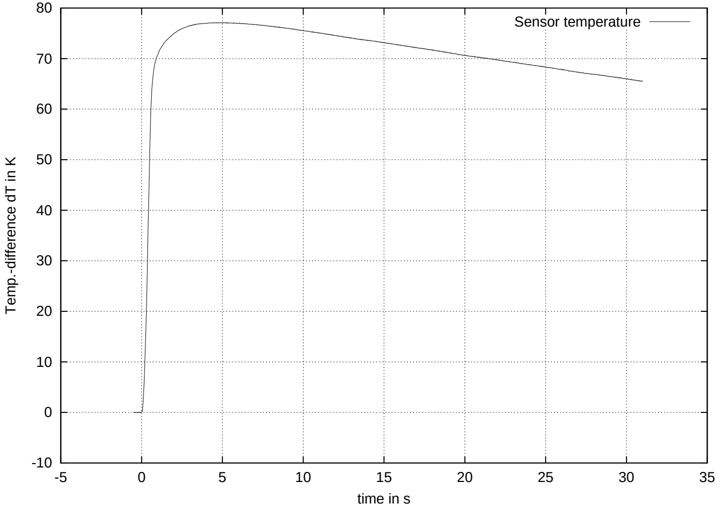
$$-\kappa_{Cu} \frac{\partial u(0, t)}{\partial x} = h_0(G(t) - u(0, t)), \qquad t \in (0, t^{end}],$$

$$\kappa_{Cu} \frac{\partial u(l, t)}{\partial x} = -h_s u(l, t), \qquad t \in (0, t^{end}],$$

$$u(x, 0) = 0, \qquad x \in [0, l].$$

Parameter specification for real data experiment:

$$
\begin{aligned}
l &= 1.6 mm,\\
\kappa_{\mathrm{Cu}} &= 392 W \cdot m^{-1} K^{-1},\\
C_{\mathrm{Cu}} &= 3.4265 \cdot 10^{6} J \cdot m^{-3} K^{-1},\\
\gamma &= 2.05 \cdot 10^{5} m^{-1},\\
q_a(t) &= 7.055 \cdot 10^{4} W \cdot m^{-2} \cdot \chi_{[0,t_p]}(t),\\
\beta_{Gas} &= 1.114 \cdot 10^{-9} W \cdot m^{-2} K^{-4},\\
\beta_{\mathrm{Obj}} &= 4.72 \cdot 10^{-8} W \cdot m^{-2} K^{-4},\\
h_0 &= 40 W \cdot m^{-2} K^{-1},\\
h_s &= 15 W \cdot m^{-2} K^{-1}.
\end{aligned}
$$



Calibration measurements of calorimeter temperature

Approaches for approximate solutions

- Least-squares approach without regularization

$$\|F(G) - u_{data}\|^2_{L^2(0,t^{end})} \to \min, \quad \text{s.t.} \quad G \in \mathcal{D}_{box}$$

- Second order Tikhonov regularization

$$\|F(G)-u_{data}\|^2_{L^2(0,t^{end})} + \alpha \|G''\|^2_{L^2(0,t^{end})} \to \min, \;\; \text{s.t.} \;\; G \in \mathcal{D}_{box}$$

- Descriptive regularization: I. only decay of G on $[2t_p, t^{end}]$, II. multi-parameter approach (monotonicity and convexity)

$$\|F(G)-u_{data}\|^2_{L^2(0,t^{end})} + \alpha \|G''\|^2_{L^2(0,t^{end})} \to \min, \;\; \text{s.t.} \;\; G \in \mathcal{D}_{descr}$$

A case study with synthetic data:

$$G^*(t) := \begin{cases} 125\left(4-(t-2)^2\right), & t \le 3s \\ 345\exp\left(\frac{3-t}{4}\right)+30, & t > 3s \end{cases}$$

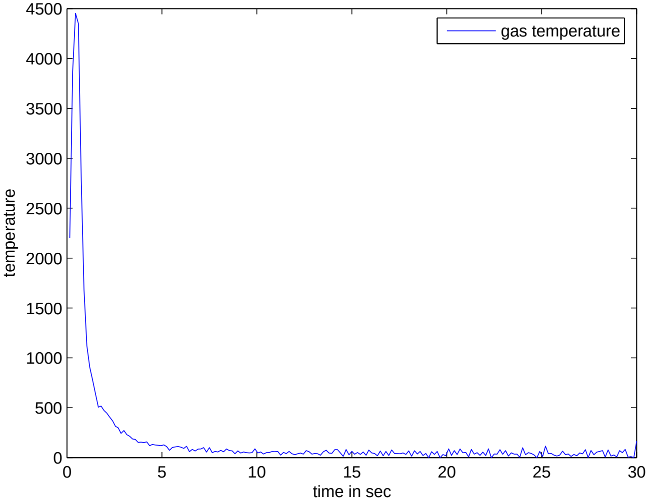
	Discrepancy method		Quasi-optimality criterion	
	α	$\frac{\|G_\alpha - G^*\|}{\|G^*\|}$	α	$\frac{\|G_\alpha - G^*\|}{\|G^*\|}$
$\delta = 10^{-4}$	$2.8243 \cdot 10^{-5}$	0.0112	$1.1973 \cdot 10^{-6}$	0.0056
$\delta = 10^{-3}$	$1.668 \cdot 10^{-3}$	0.0202	$4.2391 \cdot 10^{-4}$	0.0152

Errors of Tikhonov regularization for different parameter choice strategies

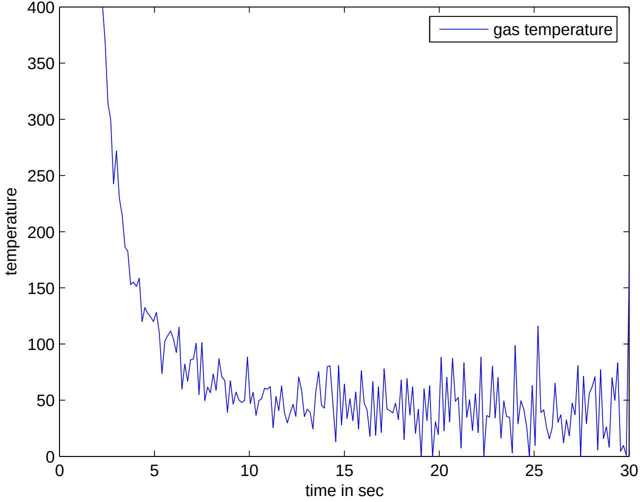
	Least-squares	Tikhonov regularization		Descriptive regularization	
δ	$\frac{\|G_\alpha - G^*\|}{\|G^*\|}$	α	$\frac{\|G_\alpha - G^*\|}{\|G^*\|}$	α	$\frac{\|G_\alpha - G^*\|}{\|G^*\|}$
0	$6.7512 \cdot 10^{-5}$	–	–	–	–
10^{-4}	0.0266	$1.1973 \cdot 10^{-6}$	0.0056	$9.6977 \cdot 10^{-7}$	0.0046
10^{-3}	0.1475	$4.2391 \cdot 10^{-4}$	0.0152	$3.4337 \cdot 10^{-4}$	0.0136

Error comparison for different approximate solution approaches

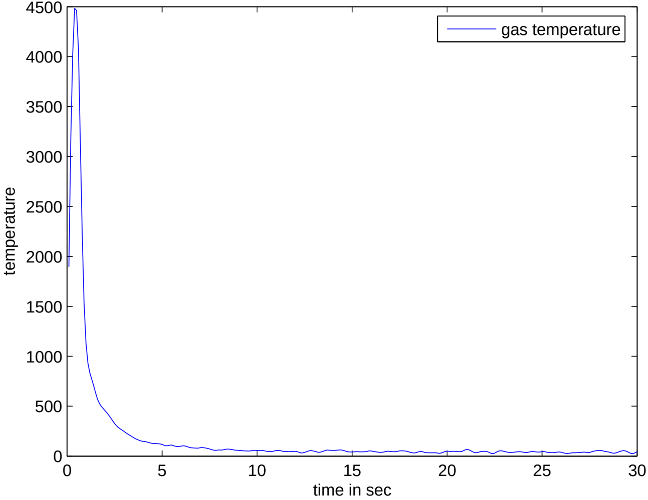
Now real-life data reconstructions:



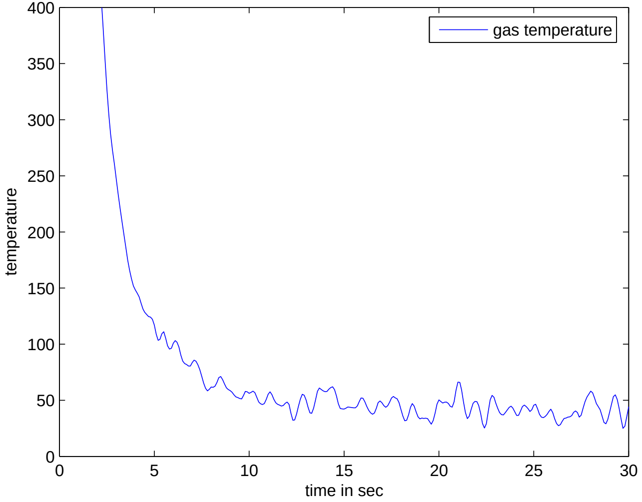
Gas temperature least-squares reconstruction without regularization



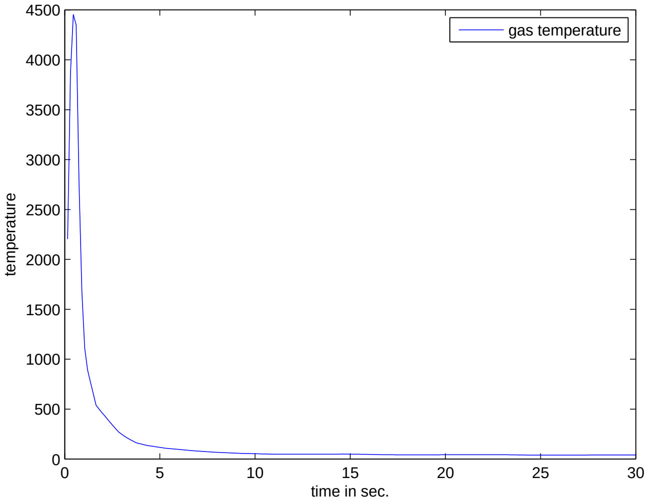
Gas temperature reconstruction without regularization (zoom)



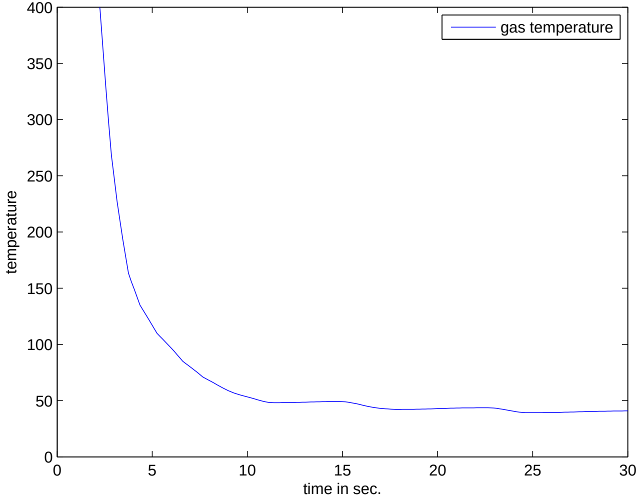
Gas temperature based on second order Tikhonov regularization



Gas temperature second order Tikhonov regularization (zoom)



Gas temperature obtained by descriptive regularization - realized as a multi-parameter approach



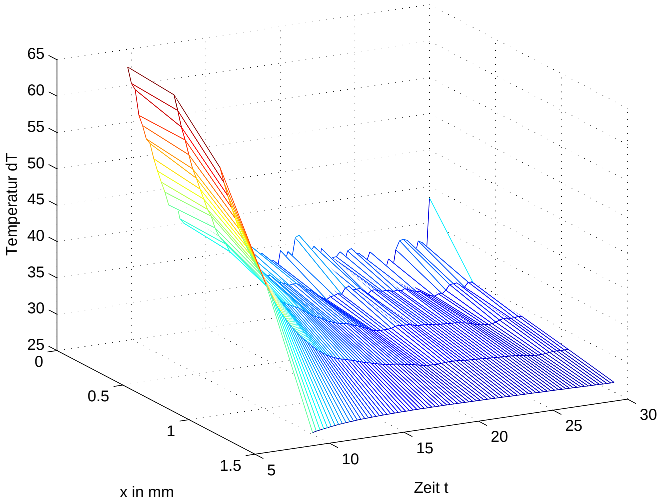
Gas temperature obtained by descriptive regularization (zoom)

Outline

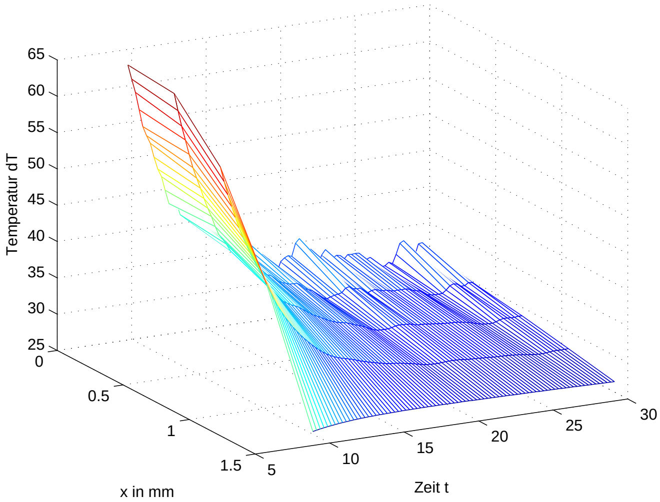
Forward computations with textile layers

For the three obtained approximate gas temperature functions we make forward computations with textile layers.

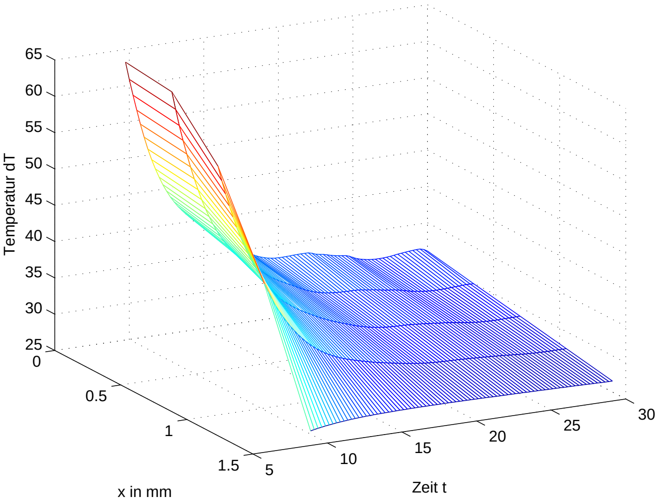
We use two textile layers both with a thickness of 0.7 mm.



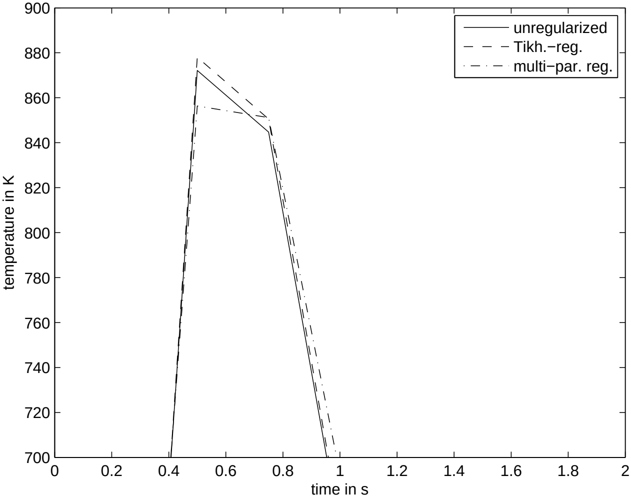
Forward computations with textile layers based on unregularized gas temperature



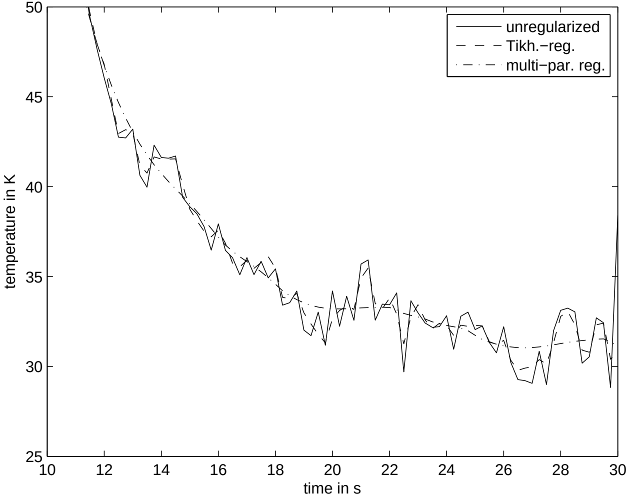
Forward computations with textile layers based on Tikhonov regularized gas temperature



Forward computations with textile layers based on descriptively regularized gas temperature (multi-parameter approach)



Variation of temperature peak for different reconstructions at location $x = 0$



Variation of temperature decay for different reconstructions at location $x = 0$

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