Optimal Thermocouple Location in Materials Processing Experiments through Fisher Information Analysis

By Badri Ratnam,* Dr. Kyle Daun† and Dr. Mary Wells‡

In materials processing there is often a need to evaluate the surface condition (e.g. temperature, heat flux, convection coefficient, contact resistance) that cannot be measured directly due to the perturbing effect of probes and adverse conditions that could destroy sensors. We can infer this boundary condition from the transient response of a submerged sensor(thermocouple) (see Fig. 1) using Inverse Heat Conduction (IHC) analysis. An example of application of IHC is inferring the unknown surface heat flux on a slab assuming linear heat conduction. This can be shown to be equivalent [1] to inverting the Duhamel's integral equation, which is a Volterra integral equation of the first kind

$$T_d(t) - T_0 = \int_0^t q_s(t^*) k(t, t^*) dt \tag{1}$$

where T_0 is the initial die temperature, $T_d(t)$ is the die temperature at depth d and time t, q_s is the surface heat flux and $k(t, t^*)$ is the kernel. Equation (1) can be rewritten as a $Ax = b$ linear system where x is the heat flux and b is the temperature. A, known as Stolz matrix, is typically ill-conditioned. Small errors in thermocouple measurement (parametric uncertainty) amplify into large errors in the inferred surface heat flux.

Ill-conditioning can be resolved to an extent by regularization methods such as Tikhonov, TSVD, Beck's future timestep method etc. [2, 7, 1]. Ill-conditioning of the A matrix increases with decreasing Fourier number $\frac{\alpha \Delta t}{d^2}$

*Ph.D. candidate, Mechanical and Mechatronics Department, University of Waterloo
†Assistant Professor, Mechanical and Mechatronics Department, University of Waterloo
‡Associate Professor, Mechanical and Mechatronics Department, University of Waterloo

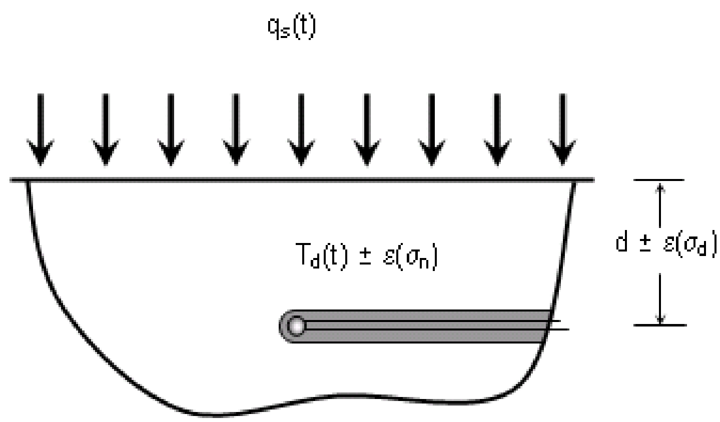

Figure 1: Submerged thermocouple in metal

where α is the thermal diffusivity, d is the thermocouple depth and Δt is the timestep. As Fourier number is inversely proportional to the square of depth, the best location of the thermocouple is close to the surface.

Regularization stabilizes against variations in data due to signal noise but does not address uncertainties in parameters. There are instances where there is uncertainty in *known* parameters. For example, it is difficult to find the exact location of the sensor (thermocouple) without a destructive postmortem analysis. This position error can cause large errors in the inferred data if the sensor is not properly located. There is a need to quantify the combined uncertainty due to position error and measurement noise and determine the optimal sensor location based on the two sources of error.

The Fisher Information Matrix is an ideal tool for such an analysis. The Fisher Information Matrix [3, 4, 5] is the information a random variable has around an unknown parameter. The rationale for the use the Fisher Information Matrix in sensor placement is that the Cramer-Rao theorem sets a lower bound of estimation error of unknown parameters as M^{-1}, where M is Fisher Information Matrix [3] given by

$$M = \sum_{k=1}^{K} \left(\frac{\left(\frac{\partial \phi_k}{\partial u}\right)^2}{S_k + \left(\frac{\partial \phi_k}{\partial b}\right)^2 G} \right) \tag{2}$$

where ϕ_k is the predicted temperature for the k th measurement, u the unknown parameter, $\frac{\partial \phi_k}{\partial u}$ is the sensitivity of measurement to u, $\frac{\partial \phi_k}{\partial b}$ is the sensitivity w.r.t *known* parameter b, G is *known* parameter covariance matrix, and S_k is measurement noise. The denominator of equation (2) can be considered as equivalent noise combining the effects of measurement noise and *known* parameter noise.

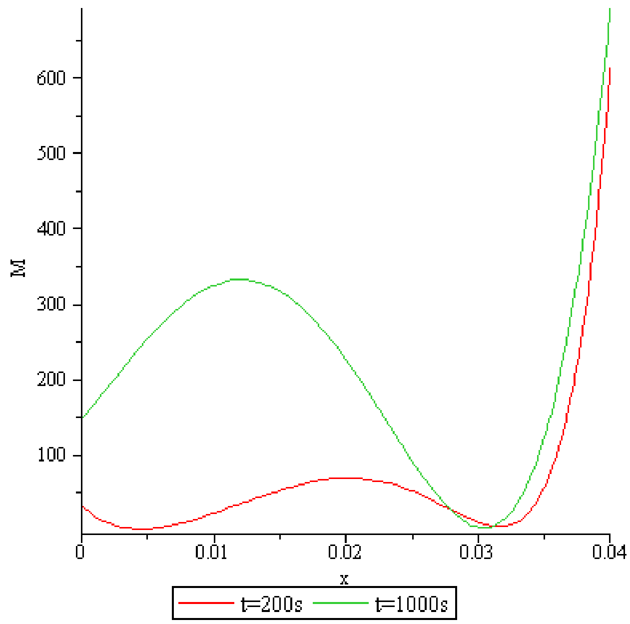

Figure 2: Fisher Information Matrix

An example of the application of the Fisher Information Matrix given in Emery *et al.* [3] (that we have reproduced here) is for a slab of thickness $L = 0.04$, convective heat transfer coefficients $h_0 = 5$ and $h_L = 20$ at $x = 0$ and $x = L$ respectively, fluid temperature 1000° C, thermal conductivity $k = 1$, initial temperature 0° C, $\rho C = 10^6$ (where ρ and C are the density and heat capacity respectively). All units are SI. The plots of the Fisher Information Matrix at different locations at $t = 200s$ and $t = 1000s$ using equation (2) for measurement noise $\sigma_n = 1\%$ are given in Fig. 2. This indicates that the optimum location of the sensor (and maximum M) is $x \approx 0.22$ and $x \approx 0.13$ for $t = 200s$ and $t = 1000s$ respectively. This considers only measurement noise but can be easily extended to include noise in the *known* parameters.

In Hot Forming Die Quenching (HFDQ) [6], the as-formed material properties greatly depend on the microstructure (bainite, martensite) which in turn depends on the cooling rate of the thin blank. Hence, it is critical to characterize the heat transfer coefficient between the blank and die. We will recover the heat transfer coefficient using IHC techniques. Using the Fisher Information analysis, we will show the optimum location of the sensor (thermocouple) in the die is closest to the surface. This may in part be due to the slow cooling rate. We will adapt this analysis to other quenching/thermal processing problems with a higher cooling rate.

References

[1] Oleg Mikhailovich Alifanov. *Inverse Heat Transfer Problems.* Springer Verlag.

[2] James Vere Beck, Ben Blackwell, and Charles R. St. Clair. *Inverse Heat Conduction: Ill-posed Problems.* Wiley-Interscience.

[3] A.F. Emery and T.D. Fadale. The effect of imprecisions in thermal sensor location and boundary conditions on optimal sensor location and experimental accuracy. *Journal of Heat Transfer*, 119:661–665, 1997.

[4] A.F. Emery and T.D. Fadale. Uncertainties in parameter estimation: the optimal experiment design. *International Journal of Heat and Mass Transfer*, 43:3331–3339, 2000.

[5] A.F. Emery, T.D. Fadale, and A.V. Nenarokomov. Two approaches to optimal sensor locations. *Journal of Heat Transfer*, 117:373–379, 1995.

[6] P. kerstrom and M. Oldenburg. Austenite decomposition during press hardening of a boron steelcomputer simulation and test. *Journal of Materials Processing Technology*, 174:399–406, 2006.

[7] M. Necati Ozisik and Helcio R. B. Orlande. *Inverse Heat Transfer: Fundamentals and Applications.* Taylor and Francis.