Modal Decomposition Applied to Heat Conduction

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Premise

Heat conduction problem has approximate discrete model

$$\underline{A}\,\dot{\underline{y}} = \underline{B}\,\underline{y}$$

where $\underline{y}$ is a vector of measurable temperatures.

Expect characteristic solutions of the form (similar to separation of variables in a continuous heat conduction problem)

$$\underline{y}_i(t) = c_i e^{-\lambda_i t}\underline{u}_i$$

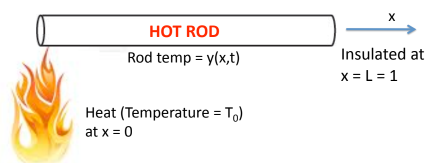
λ_i = decay rate, $\underline{u}_i$ = mode shape

Goal

Estimate characteristic parameters λ_i and mode shapes $\underline{u}_i$ of heat conduction from sampled temperature time-history data, means removed.

Simulation Example: "Hot Rod"

Heated rod, suddenly applied temperature T_0 at left end, insulated at right end. Type X12B10T0 [2].



Modal Decomposition [1]

Ensemble matrices

$\underline{Y}$ has columns $\underline{y}(j\Delta t)$ $\quad j = 1,\dots,n$

$\underline{V}$ has columns $\dot{\underline{y}}(j\Delta t)$ $\quad j = 1,\dots,n$

Correlations $\quad \underline{R} = \underline{Y}\,\underline{Y}^T/n \qquad \underline{N} = \underline{Y}\,\underline{V}^T/n$

Eigenvalue problem (matrix form)

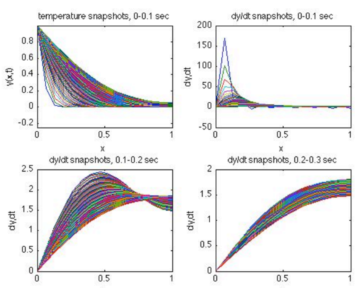
$$\underline{R}\Psi\Lambda = \underline{N}\Psi$$

Modal matrix $\quad \underline{U} = \Psi^{-T}$

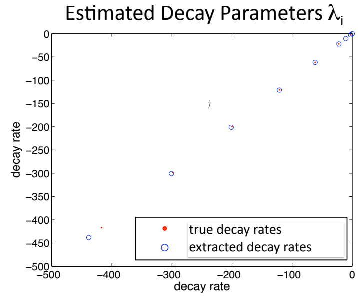
Modal vectors $\underline{u}_i$ are columns of $\underline{U}$

Modal parameters are eigenvalues $\underline{\Lambda}$

Temperature Snapshots

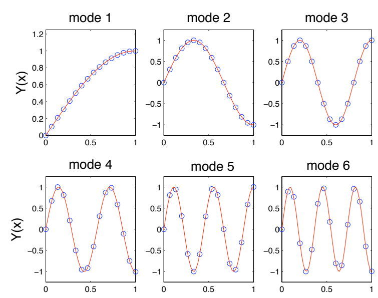


Estimated Decay Parameters λ_i



Estimated Mode Shapes

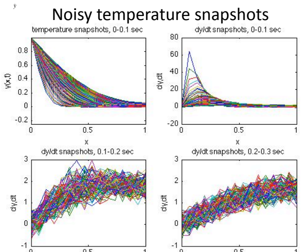
--- true modes, o estimated modes



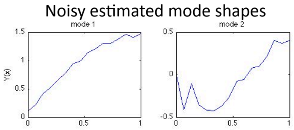
Effects of Noise

Uniformly distibuted between $\pm 2^{-8}$. One or two modes estimated.

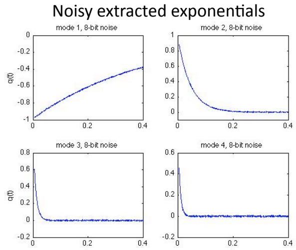
Noisy temperature snapshots



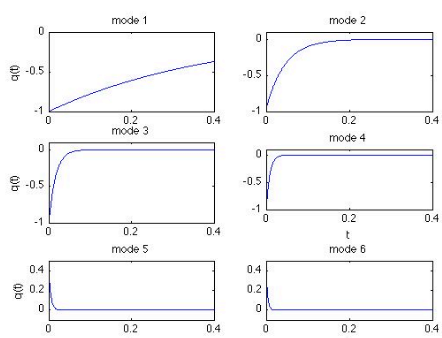
Noisy estimated mode shapes



Noisy extracted exponentials



Extracted Exponentials



Summary and Conclusion

- Heat conduction problems have characteristic modes and decay rates
- A state-variable modal decomposition method has been adopted to extract the modal information from time-history temperature data
- The method was tested on a simulated heated rod
- With no noise, the method works very well.
- With added noise, the slowest modes are recoverable
- Future work can address coping with noise and applying to experiments

Acknowledgement

This material is related to work supported by the National Science Foundation under Grant No. CMMI-0727838. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the National Science Foundation.

References

1. Feeny, B. F., and Farooq, U., 2008. "A nonsymmetric state-variable decomposition for modal analysis," *Journal of Sound and Vibration*, **310** (4-5), pp. 792–800.
2. Beck, J. V., Cole, K. D., Haji-Sheikh, A., and Litkouhi, B., 1992, *Heat Conduction Using Greens Functions*, Hemisphere Press, Washington.